$$\begin{bmatrix} FAB \\ MAB \\ FBA \\ MBA \end{bmatrix} = \begin{bmatrix} A & B & -A & B \\ B & D & -B & E \\ -A & -B & A & -B \\ B & E & -B & D \end{bmatrix} \begin{bmatrix} UVA \\ URA \\ UVB \\ URB \end{bmatrix}$$

Fig.1.

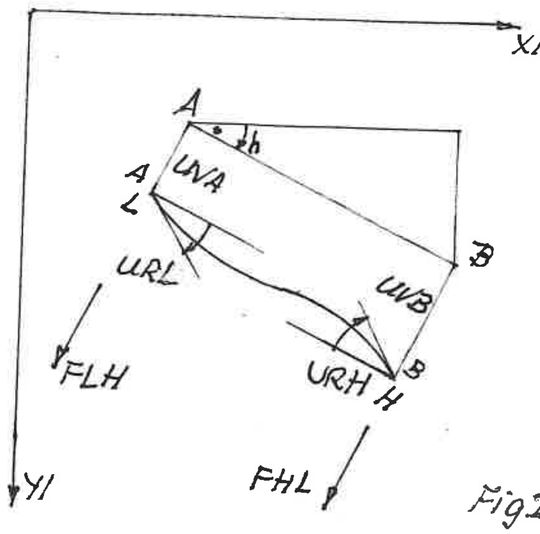
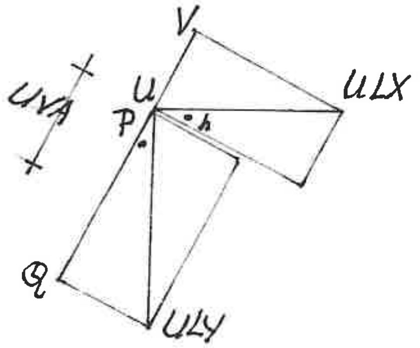


Fig2.



C= PQ/ULY S= UV/ULX

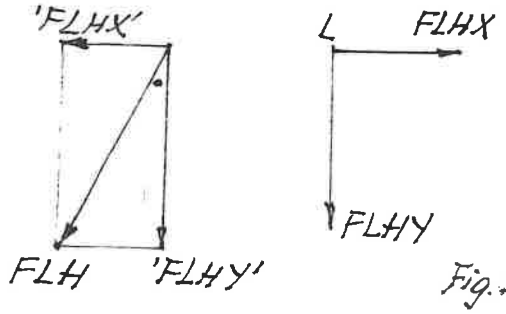


Fig.3.

6. Determination of member stiffness matrix S5 of a frame.

Fig.1
Starting point is the horizontal beam/member, of which both member ends are rigidly connected with the joints. Of which the member ends, and joints, can displace only vertically and of which the joints can rotate.
The relation between member end forces and member end moments and joint rotations was found like shown on the left. With

A=12EI/L1^3, B=6EI/L1^2, D=4EI/L1, E=2EI/L1.

Fig.2.
The member ends, joints, now do not only displace vertically but also horizontally, the components of UVA and UVB.
The horizontal beam of page 41 is now drawn under an angle with the assumptions for the member end slope deflections, being the joint rotations URA and URB, from now on URL and URH, the member end displacements UVA and UVB perpendicular to the member axis, and
the member end forces FAB and FBA, and
the member end moments MAB and MBA, from now on FLH, FHL, MLH and MHL.

Fig.3 and 2
The horizontal member end forces FLHX and FHLX are assumed directed to the right, and the vertical member end forces FLHY and FHLY are assumed downward, and the
member end moments MLH and MHL to the right.
The first equation for member end A, is number L, of the given member is.

FLH= A*UVA +B*URA -A*UVB +B*URB. 1)

UVA consists of the componentsU LX en ULY.
Cos(h)=PQ/ULY or PQ=Cos(h)*ULY or PQ=**C***ULY.
Sin(h)=UV/ULX or UV=Sin(h)*ULX or UV=**S***ULX.
With the figures follow
UVA= C*ULY -S*ULX or UVA= -S*ULX +C*ULY
and in similar way UVB= -S*UHX +C*UHY.
Put in equation 1) it gives

FLH= A(-S*ULX+C*ULY) +B*URL

-A(-S*UHX+C*UHY) +B*URH. 1')

Sin(h)= 'FLHX'/FLH or
'FLHX'=Sin(h)*FLH or 'FLHX'= S*FLH.

Member end forces FLHX and FLHY drawn with their assumed directions.

FLHX and 'FLHX' are opposite directed, so

FLHX= -'FLHX' or FLHX= -S*FLH, is -S times 1').

FLHX= -S(A(-S*ULX+C*ULY) +B*URL
-A(-S*UHX+C*UHY) +B*URH).
Then the first of six equations becomes

FLHX= A*S^2*ULX -A*S*C*ULY -B*S*URL

-A*S^2*UHX +A*S*C*UHY -B*S*URH. (1)

$$
\begin{bmatrix} FLHX \\ FLHY \\ MLH \\ FHLX \\ FHLY \\ MHL \end{bmatrix} = S5 \begin{bmatrix} ULX \\ ULY \\ URL \\ UHX \\ UHY \\ URH \end{bmatrix} \qquad \begin{bmatrix} FLHX \\ FLHY \\ 0 \\ FHLX \\ FHLY \\ 0 \end{bmatrix} = S5 \begin{bmatrix} ULX \\ ULY \\ 0 \\ UHX \\ UHY \\ 0 \end{bmatrix}
$$

f u

$$
\begin{bmatrix}
A*S^2 & -A*S*C & -B*S & -A*S^2 & A*S*C & -B*S \\
-A*S*C & A*C^2 & B*C & A*S*C & -A*C^2 & B*C \\
-B*S & B*C & D & B*S & -B*C & E \\
-A*S^2 & A*S*C & B*S & A*S^2 & -A*S*C & B*S \\
A*S*C & -A*C^2 & -B*C & -A*S*C & A*C^2 & -B*C \\
-B*S & B*C & E & B*S & -B*C & D
\end{bmatrix}
$$

S5 for deformation by bending

$$
\begin{bmatrix}
R*C^2 & R*S*C & 0 & -R*C^2 & -R*S*C & 0 \\
R*S*C & R*S^2 & 0 & -R*S*C & -R*S^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-R*C^2 & -R*S*C & 0 & R*C^2 & R*S*C & 0 \\
-R*S*C & -R*S^2 & 0 & R*S*C & R*S^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

S5 for deformation by tension/compress.

The second of the 6 equations, with FLHY.

FLH= A(-S*ULX+C*ULY) +B*URL

-A(-S*UHX+C*UHY) +B*URH 1')

Cos(h)= 'FLHY'/FLH or
'FLHY'=Cos(h)*FLH or 'FLHXY'= C*FLH.

FLHY and 'FLHY' are equal directed, so
FLHY= 'FLHY' of FLHY= C*FLH, is C times 1').

FLHY= C(A(-S*ULX+C*ULY) +B*URL
-A(-S*UHX+C*UHY) +B*URH). Then is

FLHY= -A*S*C*ULX +A*C^2*ULY B*C*URL

+A*S*C*UHX -A*C^2*UHY B*C*URH. (2)

Next the third equation with moment MLH.
For the horizontal beam was found

MLH= B*ULY +D*URL -B*UHY +E*URH also applicable for the beam under angle h, so that with

ULY= -S*ULX +C*ULY, preceding page, and
UHY= -S*UHX +C*UHY follows

MLH= B(-S*ULX+C*ULY) +D*URL
-B(-S*UHX+C*UHY) +E*URH).

Then the third of 6 equations becomes

MLH= -B*S*ULX +B*C*ULY D*URL

+B*S*UHX -B*C*UHY E*URH. (3)

The second trio equations can be found in the same way. On the left the the stiffness factors are placed in stiffness matrix S5.
The member with hingy member ends is like a truss member. The member ends are no real joints and therefore no joint rotations, and therefore are the third row and column, and the sixth row and column of S5 filled with zeros.
Deformation due to bending has no connection with deformation due to tension/compression. Therefore the elements of both matrices can be added and arises the member stiffness matrix shown here below.

$$
\begin{bmatrix} FLHX \\ FLHY \\ MLH \\ FHLX \\ FHLY \\ MHL \end{bmatrix} =
\begin{bmatrix}
R*C^2+A*S^2 & R*S*C-A*S*C & -B*S & -R*C^2-A*S^2 & -R*S*C+A*S*C & -B*S \\
R*S*C-A*S*C & R*S^2+A*C^2 & B*C & -R*S*C+A*S*C & -A*S^2-A*C^2 & B*C \\
-B*S & B*C & D & B*S & -B*C & E \\
-R*C^2-A*S^2 & -R*S*C+A*S*C & B*S & R*C^2+A*S^2 & R*S*C-A*S*C & B*S \\
-R*S*C+A*S*C & -R*S^2-A*C^2 & -B*C & R*S*C-A*S*C & R*S^2+A*C^2 & -B*C \\
-B*S & B*C & E & B*S & -B*C & D
\end{bmatrix} \cdot
\begin{bmatrix} ULX \\ ULX \\ URL \\ UHX \\ UHY \\ URH \end{bmatrix}
$$

A= 12*EI/L1^3 B= 6*EI/L1^2 D= 4*EI/L1 E= 2*EI/L1 R=EA/L1 C =Cos(h) S=Sin(h)

FLHX		R*C^2+A*S^2	R*S*C-A*S*C	0	-R*C^2-A*S^2	-R*S*C+A*S*C	-B*S		ULX
FLHY		R*S*C-A*S*C	R*S^2+A*C^2	0	-R*S*C+A*S*C	-A*S^2-A*C^2	B*C		ULX
MLH		0	0	0	0	0	0		URL
FHLX	=	-R*C^2-A*S^2	-R*S*C+A*S*C	0	R*C^2+A*S^2	R*S*C-A*S*C	B*S	.	UHX
FHLY		-R*S*C+A*S*C	-R*S^2-A*C^2	0	R*S*C-A*S*C	R*S^2+A*C^2	-B*C		UHY
MHL		-B*S	B*C	0	B*S	-B*C	D		URH

A= 3*EI/L1^3 B= 3*EI/L1^2 D= *EI/L1 R=EA/L1 C =Cos(h) S=Sin(h)

If member end with the lowest member end number L is a hinge and member end with the highest member end number H a rael joint then there will be a joint rotation URH but no joint roptation URL. The concerning third row and third column are filled with zeros.

FLHX		R*C^2+A*S^2	R*S*C-A*S*C	-B*S	-R*C^2-A*S^2	-R*S*C+A*S*C	0		ULX
FLHY		R*S*C-A*S*C	R*S^2+A*C^2	B*C	-R*S*C+A*S*C	-A*S^2-A*C^2	0		ULX
MLH		-B*S	B*C	D	B*S	-B*C	0		URL
FHLX	=	-R*C^2-A*S^2	-R*S*C+A*S*C	B*S	R*C^2+A*S^2	R*S*C-A*S*C	0	.	UHX
FHLY		-R*S*C+A*S*C	-R*S^2-A*C^2	-B*C	R*S*C-A*S*C	R*S^2+A*C^2	0		UHY
MHL		0	0	0	0	0	0		URH

A= 12*EI/L1^3 B= 6*EI/L1^2 D= 4*EI/L1 E= 2*EI/L1 R=EA/L1 C =Cos(h) S=Sin(h)

If member end H is a hinge and member end L a real joint then there is a joint rotation URL but no joint rotation URH. The concerning sixth row and sixth column are filled with zeros.

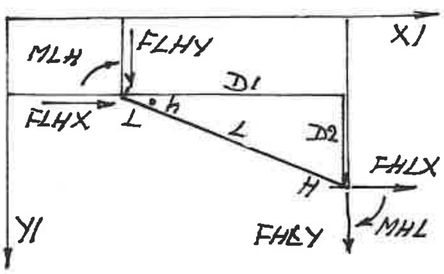


Fig.3.

A1	A2	A3	-A1	-A2	A3
A2	A4	A5	-A2	-A4	A5
A3	A5	D	-A3	-A5	E
-A1	-A2	-A3	A1	A2	-A3
-A2	-A4	-A5	A2	A4	-A5
A3	A5	E	-A3	-A5	D

A= 12EI/L^3
B= 6EI/L^3
D= 4EI/L
E= 2EI/L

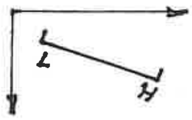


Fig.4.

A1	A2	.	-A1	-A2	A3
A2	A4	.	-A2	-A4	A5
.	.	.	.	.	.
-A1	-A2	.	A1	A2	-A3
-A2	-A4	.	A2	A4	-A5
A3	A5	.	-A3	-A5	D

A= 3EI/L^3
B= 3EI/L^2
D= 3EI/L

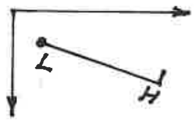


Fig.5.

A1	A2	A3	-A1	-A2	.
A2	A4	A5	-A2	-A4	.
A3	A5	D	-A3	-A5	.
-A1	-A2	-A3	A1	A2	.
-A2	-A4	-A5	A2	A4	.
.	.	.	.	.	.

A= 3EI/L^3
B= 3EI/L^2
D= 3EI/L

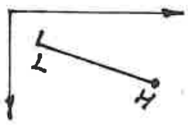


Fig.6.

RC^2	RSC	.	-RC^2	-RSC	.
RSC	RS^2	.	-RSC	-RS^2	.
.	.	.	.	.	.
-RC^2	-RSC	.	RC^2	RSC	.
-RSC	-RS^2	.	RSC	RS^2	.
.	.	.	.	.	.

R=EA/L
C=D1/L1
S=D2/L1

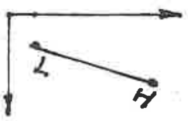


Fig.7.

Fig.3.

Member end L with coordinates X1(L) and Y1(L) and member end H with X1(H) and Y1(H).
The member ends are rigidly connected with the joints indicated with the short little stripes at the member ends.

D1=X1(H)-X1(L)   and   D2=Y1(H)-Y1(L).

Staaflengte L1=Sqr(D1^2+D2^2).

C=cos(h)   C=D1/L1   and   S=Sin(h)   S=D2/L1.

f		S5						u
FLHX		A1	A2	A3	-A1	-A2	A3	ULX
FLHY		A2	A4	A5	-A2	-A4	A5	ULY
MLH	=	A3	A5	D	-A3	-A5	E	URL
FHLX		-A1	-A2	-A3	A1	A2	-A3	UHX
FHLY		-A2	-A4	-A5	A2	A4	-A5	UHY
MHL		A3	A5	E	-A3	-A5	D	URH

Fig.4.
The elements of S5 are represented with A1, A2, A3, A4 and A5, with a mius sign - if applicable.

A1= R*C^2 +A*S^2,   A2= R*S*C -A*S*C,

A3= -B*S,   A4= R*S^2 +A*C^2,   A5= B*C.

For the case if both member ends are rigidly connected with the joints apply

A=12*EI/L^3   B=6*EI/L^2   D=4*EI/L   E=2*EI/L.

Fig.5 en 6.
One of both member ends is hingy connected with the joint, or, is a hinge.
The same formulas for A1 to A5 apply but because of the member end hinge now

A=3*EI/L^3   B=3*EI/L^2   D=3*EI/L.

Fig.5.
Member end L a hinge. This member has no joint L, then there is no unknown joint rotation URL. See f = S5 u here above. The converning row and column are filled with zeros, third row and third column.

Fig.6.
Member end H a hinge. There is no unknown joint rotation URH, the sixth row and sixth column are filled with zeros.

Fig.7.
Both member ends a hinge. Then the formulas for the elements are like those of a truss member, R*C^2, R*S*C etc.

If L and H are exchanged then the same considerations, start matrix in accordance with the assumptions is always the matrix of figure 4. C and S alter depending on the coordinates and determining also the elements of S5.

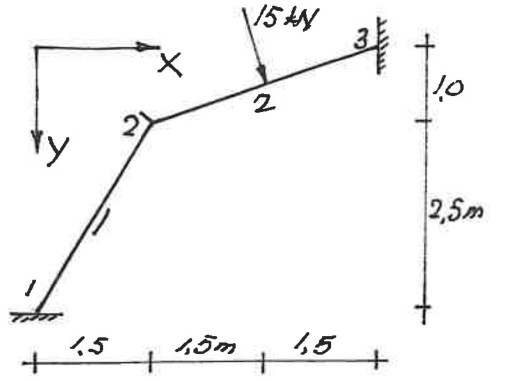


Fig.1.

Member 1.

R='EA/L'= EA/2,92= 0,342 EA
R=(0,342)50EI= 17,1 EI

A=12*EI/L^3= 12*EI/(2,92^3)= 0,482 *EI
B= 6*EI/L^2= 6*EI/(2,92^2)= 0,704 *EI

D= 4*EI/L= 4*EI/2,92= 1,370 *EI
E= 2*EI/L= 2*EI/2,92= 0,685 *EI

Member 2.

D1=X1(3)-X1(2)= 4,5-1,5= 3,0 m
D2=Y1(3)=Y1(2)= 0,0-1,0=-1,0 m
L1=Sqr(3,0^2+(-1,0)^2)= 3,16 m
C=3,0/3,16= 0,949 S=-1,0/3,16=-0,316

R='EA/L'= EA/3,16= 0,316 EA EA=50EI
R=(0,316)50EI= 15,8 EI

A=12*EI/L^3= 12*EI/(3,16^3)= 0,380 *EI
B= 6*EI/L^2= 6*EI/(3,16^2)= 0,601 *EI
D= 4*EI/L= 4*EI/3,16= 1,266 *EI
E= 2*EI/L= 2*EI/3,16= 0,633 *EI

With the formulas for A1, A2, A3, A4 and A5 like calculated on the right.
A1= 14,270 *EI A2= 2,624 *EI
A3= 0,190 *EI A4= 1,920 *EI
A5= 0,570 *EI

Joint loads of joint 2 and 3 due to member load force of 15 kN.

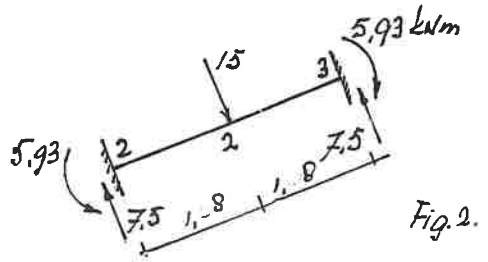


Fig.2.

With the formulas page follow the forces 7,50 kN and moments 5,93 kNm. These forces resolved into horizontal and vertical components give on joint 2 and 3 (7,5/3,16)*3,0= 7,12 kN downward and (7,5*/3,16)*1,0=2,37 kN to the right, see next page.

Example.

Fig.1.
Two members and three joints. Both members with bending stiffness EI, strain siffness EA=50EI.
X1(1)= 0,0 X1(2)= 1,5 X1(3)= 4,5 m
Y1(1)= 3,5 Y1(2)= 1,0 Y1(3)= 0,0 m

Member 1. D1=X1(H)-X1(L)= 1,5-0,0= 1,5 m
D2=Y1(H)-Y1(L)= 1,0-3,5=-2,5 m
L1=Sqr(1,5^2+(-2,5)^2)= Sqr(8,5)= 2,92 m

C=D1/L1= 1,5/2,92= 0,514
S=D2/L1=-2,5/2,92=-0,856

A1= R*C^2 +A*S^2=
= 17,1EI*(0,514^2) +0,482EI*(-0,856^2)
= 4,518EI +0,353EI= 4,871 *EI
A2= R*S*C-A*S*C=
= 17,1EI(-0,856)(0,514)-0,482EI(-0,856)(0,514)
=-7,524EI +0,212EI= -7,312 *EI

A3=-B*S =-0,704EI*(-0,856)= 0,603 *EI
A4= R*S^2 +A*C^2=
= 17,1EI*(-0,856^2) +0,482EI*(0,514^2)
= 12,530EI +0,127EI= 12,657 *EI
A5= B*C= 0,704EI*(0,514)= 0,362 *EI

	1	2	3	4	5	6
1	4871	-7312	603	-4871	7312	603
2	-7312	12657	362	7312	-12657	362
3	603	362	1370	-603	-362	685
4	-4871	7312	-603	4871	-7312	-603
5	7312	-12657	-362	-7312	12660	-362
6	603	362	685	-603	-362	1370

x EI/1000 S51

	4	5	6	7	8	9
4	14267	-4624	190	-14267	4624	190
5	-4624	1920	570	4624	-1920	570
6	190	570	1266	-190	-570	633
7	-14267	4624	-190	14267	-4624	-190
8	4624	-1920	-570	-4624	1920	-570
9	190	570	633	-190	-570	1266

x EI/1000 S52

$$\begin{bmatrix} F12X \\ F12Y \\ M12 \\ F21X \\ F21Y \\ M21 \end{bmatrix} = S51 \begin{bmatrix} U1X \\ U1Y \\ UR1 \\ U2X \\ U2Y \\ UR2 \end{bmatrix} \qquad \begin{bmatrix} F23X \\ F23Y \\ M23 \\ F32X \\ F32Y \\ M32 \end{bmatrix} = S52 \begin{bmatrix} U2X \\ U2Y \\ UR2 \\ U3X \\ U3Y \\ UR3 \end{bmatrix}$$

member 1 member 2

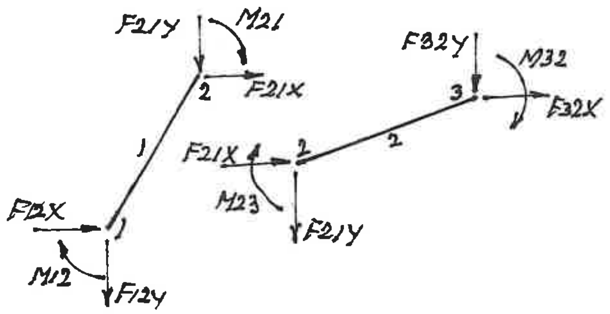


Fig.3.

The separated members 1 and 2 with the on the member ends acting forces and moments with their assumed directions.

Fig.4.

On separated joint 2 act member end forces as large as but opposite directed, the force of 7,12 kN downward, 2,37 kN to the right and the moment of 5,93 kNm to the right, due to the member load of 15 kN.

On the separated joint 3 act 7,12 kN downward, 2,37 kN to the right and 5,93 kNm to the left.

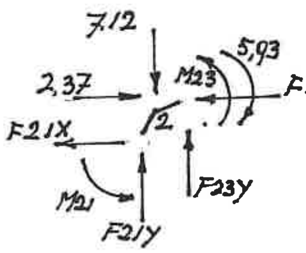


Fig.4.

F12X calculated right bottom. Further with EI omitted.

F12Y= 7,312(0,93)-12,657(1,22)
+0,602(2,30)
= 6,80 -15,44 +0,83= -7,81 kN

M12=-0,603(0,93)-0,362(1,22)
+0,685(2,30)
=-0,56 -0,44 +1,58= 0,58 kNm

F21X=-5,78 kN F21Y= 7,51 kN
M21= 2,15 kNm

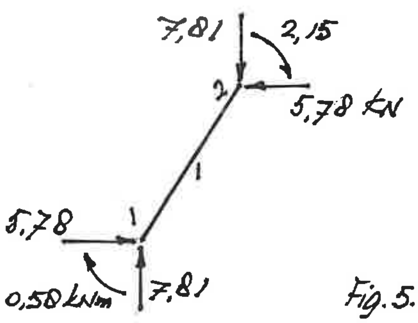


Fig.5.

Forces and moments drawn with their real directions.

Both member matrices S5 form together construction matrix CC. Three joints and each joint with three unknowns is 3 x 3 = 9 equations.

The 'displacements' UX1, UY1, UR1, UX3, UY3 and UR3 are prescribed, known, all zero. The concerning rows and columns of CC are filled with zeros and the elements on the main diagonal are made 1. Three unknowns remain, UX2, UY2 and UR2, three equations to solve.

Underlined pairs of elements of S51 and S52 are added, like
14267 +7871= 19138, -4624 -7312= -11936,
190 -602= -412, etc.

$$\begin{bmatrix} F21X+F23X \\ F21Y+F23Y \\ M12+M23 \end{bmatrix} = \begin{bmatrix} 19138 & -11936 & -413 \\ -11936 & 14580 & 208 \\ -413 & 208 & 2636 \end{bmatrix} \cdot \begin{bmatrix} UX2 \\ UY2 \\ UR2 \end{bmatrix}$$

$\underline{f}$ CC $\underline{u}$

When UX2, UY2 and UR2 are known the elements of $\underline{f}$ can be calculated. UX2, UY2 and UR2 are still unknown.

$$\begin{bmatrix} 19138 & -11936 & -413 \\ -11936 & 14580 & 208 \\ -413 & 208 & 2636 \end{bmatrix} \cdot \begin{bmatrix} UX2 \\ UY2 \\ UR2 \end{bmatrix} = \begin{bmatrix} 2,37 \\ 7,12 \\ 5,93 \end{bmatrix}$$

$\underline{\text{x EI/1000}}$ $\underline{u}$ $\underline{f}$

The calculation of the elements of $\underline{f}$, the joint loads as follows.

Σ hor. joint 2 =0
F21X+F23X -2,37 =0 F21X+F23X= 2,37 kN
Σ vert. joint 2 =0
F21Y+F23Y -7,12 =0 F21Y+F23Y= 7,12 kN
Σ mom. joint 2 =0
M21+M23 -5,93 =0 M21+M23= 5,93 kNm

The equations written out, without EI.

19,141*UX2 -11,942*UY2 -0,413*UR2 = 2,37

-11,936*UX2 +14.580*UY2 +0,208*UR2 = 7,12

-0,413*UR2 +0,208*UY2 +2,636*UR2 = 5,93

With computer Gauss follow

<u>UX2= 0.93/EI,</u> <u>UY2= 1,22/EI,</u> <u>UR2= 2,30/EI</u>.

Fig. 2 en 5.
Calculation of F12X of member end 1, the second trio elements of the first row of S51 times respectively UX2, UY2 and UR2.

F12X= EI(-4,871*UX2 +7,312*UY2 +0,603*UR2)
= EI(-4,871*0,93+7,312*1,22+0,603*2,30)/EI
= -4,53 +8,92 +1,39 = 5,78 kN

A positve answer, so as assumd directed to the right.

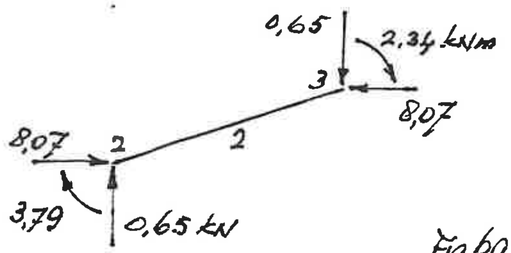


Fig.6a.

+

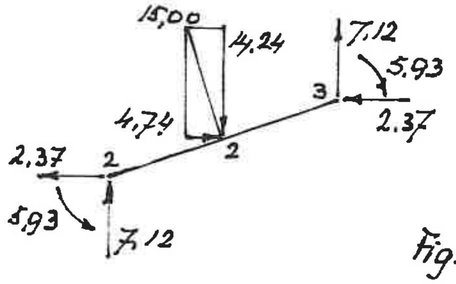


Fig.6b.

=

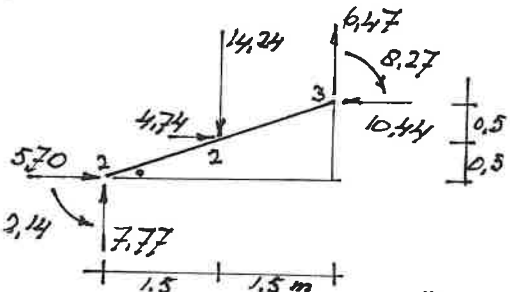


Fig.6c.

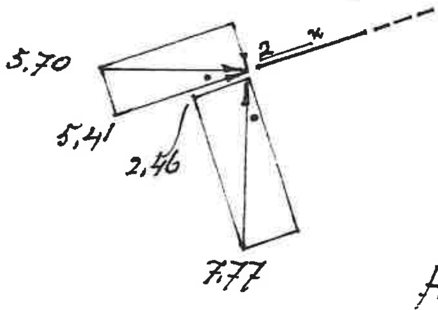


Fig.7.

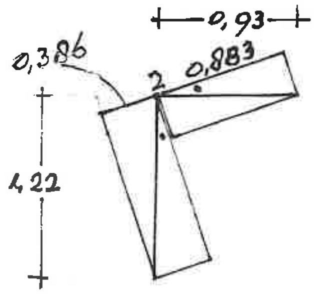


Fig.8.

((0,93/EI)/3,16)*3,00= 0,883/EI and
((1,22/EI)/3,16)*1,00= 0,386/EI.

The member shortens 0,497/EI.

UX2= 0.93/EI, UY2= 1,22/EI, UR2= 2,30/EI.

Fig. 3 and 6a.
The member end forces and moments of member 2 due to the displacements U1X, U1Y, UR1, U2X, U2Y and UR2 alone are calculated here below with help of member stiffness matrix S52, see page .

F23X is the first trio elements of the first row of S52 times resp. U2X, U2Y and UR2.
EI is omitted in the calculation.
F23X= 14,267(0,93) -4,624(1,22) +0,190(2,30)
= 13,27 -5,64 +0,44 = 8,07 kN

F23Y is the first trio elementsnof the second row of S52 times resp. UR2, U2Y and UR2.
F23Y= -4,624(0,93) +1,920(1,22) +0,570(2,30)
= -4,30 +2,34 +1,31 = -0,65 kN

M23 is the first trio nelements of the third row of S52 times resp. U2X, U2Y and UR2.
M23= 0,190(0,93) +0,570(1,22) +1,266(2,30)
= 0,18 +0,70 +2,91 = 3,79 kNm

F32X and F32Y as large as but opposite to resp. F32X= -8,07 kN en F32Y= 0,65 kN.

M32 is the first trio elements of the sixth row of S52 times resp. U2X, U2Y and UR2.
M32= 0,190(0,93) +0,570(1,22) +0,633(2,30)
= 0,18 +0,70 +1,96 = 2,34 kNm

The member end forces and member end moments are drawn with their real directions.

Fig. 2 and 6b.
Member end forces and moments due to the member loads alone. The components of the forces are calculated for figure 2.

Fig. 6c.
The final member end forces and moments as the addition of the figures 6a and 6b.

Fig.7.
Calculation of member end force F23x at member end 2. For that purpose 5,70 kN and 7,77 kN are resolved perpedicular to and alog the member axis.
(5,70/3,16)*3,00= 5,41 kN
(7,77/3,16)*1,00= 2,46 kN

Both directed like the member axis, the forces push on member end 2, 5,41+2,46= 7,87 kN.
The force of 15 kN is perpendicular to the member so that the components of the member end forces at member end 3 give a force as large as opposite directed to the x axis, thus also a compression force. (Is 9,91-2,05= 7,86 kN.)

Fig.8.
The displacements UX2=0,93/EI and UY2=1,22/EI can be resolved perpendicular to and along the member axis. Joint 3 does not displace.
The member shortens 0,497/EI.

ΔL='FL/EA' or
0,497/EI=F*3,16/50EI so that F=7,86 kN like

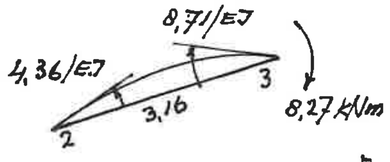


Fig.9a

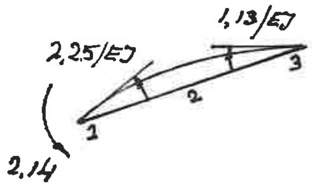


Fig.9b.

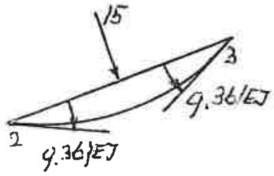


Fig.9c

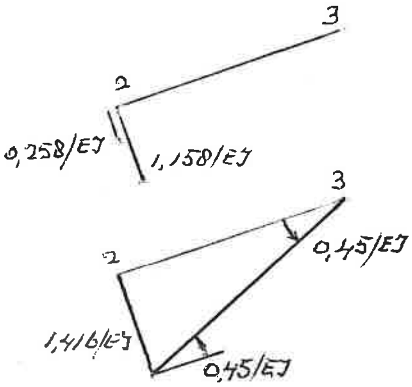


Fig.9d.

Carrying out a check calculation with help of the final results of the calculations. Determination of slope deflections at the member ends with the formulas of page 97.
See fig.6c.

Fig.9a.
Member end 2 with slope deflection
8,27*3,16/6EI= 4,36/EI and
member end 3 with slope deflection
8,27*3,16/3EI= 8,71/EI.

Fig.9b.
Member end 2 with2,14*3,16/3EI= 2,25/EI and
member end 3 with2,14*3,16/6EI= 1,13/EI.

Fig.9c.
Load of 15 kN in the middle, at both ends aris slope deflections
15*(3,16^2)/16EI= 9,36/EI.

Fig.9d.
The slope deflection of both member ends due to de member end displacements perpendicular to the member axis alone. At member end 3 zero.

The displacement components of U2X= 0,93/EI and U2Y= 1,22/EI are, see fig.8,

((0,93/EI)/3,16)*1,00= 0,258/EI and
((1,22/EI)/3,16)*3,00= 1,158/EI.

Both with same direction, so
0,258/EI +1,158/EI= 1,416/EI.

The slope deflection is
(1,416/EI)/3,16= 0,45/EI.

The slope deflection, or member end rotation, of member end 3 must be zero, elements times 1/EI, 8,71 +1,13 -9,36 -0,45= 0,03 is 0! OK

Is the angle at member end 2 equal UR2=2,30/EI to the right?
9,36 -4,36 -2,25 -0,45= 9,36 -6,95= 2,30 ! OK

S5 and horizontal and vertical members.

A1	A2	A3	-A1	-A2	A3
A2	A4	A5	-A2	-A4	A5
A3	A5	D	-A3	-A5	E
-A1	-A2	-A3	A1	A2	-A3
-A2	-A4	-A5	A2	A4	-A5
A3	A5	E	-A3	-A5	D

Fig.1.

A1	*	*	-A1	*	*
*	A4	A5	*	-A4	A5
*	A5	D	*	-A5	E
-A1	*	*	A1	*	*
*	-A4	-A5	*	A4	-A5
*	A5	E	*	-A5	D

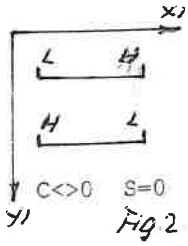


Fig.2

A1	*	A3	-A1	*	A3
*	A4	*	*	-A4	*
A3	*	D	-A3	*	E
-A1	*	-A3	A1	*	-A3
*	-A4	*	*	A4	*
A3	*	E	-A3	*	D

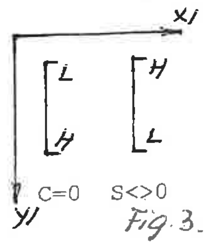


Fig.3.

Fig.1, 2 and 3.
The member ends are rigidly connected with the joints, the member ends are joints.

A1= R*C^2 +A*S^2, A2= R*S*C -A*S*C,

A3= -B*S, A4= R*S^2 +A*C^2, A5= B*C.

Horizontal members.

D1=X1(H)-X1(L)<>0
D2=Y1(H)-Y1(L) =0

C=D1/L1<>0
S=D2/L1 =0

Vertical members.

D1=X1(H)-X1(L)= 0
D2=Y1(H)-Y1(L)<>0

C=D1/L1= 0
S=D2/L1<>0

With help of the formulas one finds the elements of S5 which are zero.

(Members with a hinge, then more (sometimes coinciding) zeros, see page 54,58.)

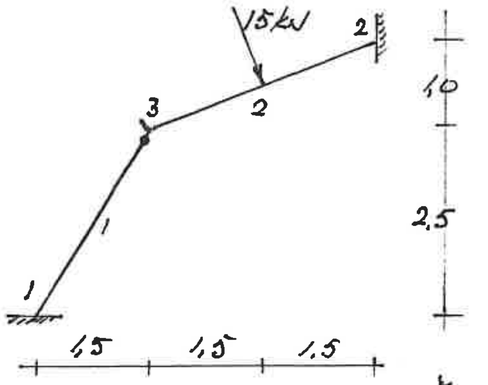


Fig.1.

Member 1.

R='EA/L'= EA/2,92= 0,342 EA
R=(0,342)50EI= 17,1 EI

Member end 1 is a hinge.
A= 3*EI/L^3= 3*EI/(2,92^3)= 0,120 *EI
B= 3*EI/L^2= 3*EI/(2,92^2)= 0,352 *EI

D= 3*EI/L= 3*EI/2,92= 1,027 *EI

Member 2.

D1=X1(3)-X1(2)= 1,5-4,5=-3,0 m
D2=Y1(3)=Y1(2)= 1,0-0,0= 1,0 m
L1=Sqr(-3,0^2+1,0^2)= 3,16 m

R='EA/L'= EA/3,16= 0,316 EA   EA=50EI
R=(0,316)50EI= 15,8 EI

A=12*EI/L^3= 12*EI/(3,16^3)= 0,380 *EI
B= 6*EI/L^2= 6*EI/(3,16^2)= 0,601 *EI

D= 4*EI/L= 4*EI/3,16= 1,266 *EI
E= 2*EI/L= 2*EI/3,16= 0,633 *EI

With the formulas for A1, A2, A3, A4 and A5 follow
A1= 14,267 *EI,   A2= 2,624 *EI,
A3= 0,190 *EI,   A4= 1,920 *EI and
A5= 0,570 *EI which fill S52.

Member 2 like on page with both member ends rigidly connected with the joints. The joint load forces and moments due to 15 kN are like calculated there, now on joint 3 instead of joint 2, 7,12 kN downward, 2,37 kN to the right.

member 1			member 2		
F13X		U1X	F23X		U2X
F13Y		U1Y	F23Y		U2Y
M13	= S51	UR1	M23	= S52	UR2
F31X		U3X	F32X		U3X
F31Y		U3Y	F32Y		U3Y
M31		UR3	M32		UR3

Because of the different joint numbering, 1-3-2 i.s.o. 1-2-3, f = S5 u looks different than on page 61.

Example.

Fig.1.
Construction size like on page 61, now with an internal hinge. Both members same beding stiffness EI and strain stiffness EA=50EI.
Irregular joint numbering... deliberate.
Member 1 with L=1 and H=3, member 2 L=2, H=3.
Member end 3 of member 1 is a hinge,
member end 3 of member 2 is a 'real'joint.

X1(1)= 0,0   X1(2)= 4,5   X1(3)= 1,5 m
Y1(1)= 3,5   Y1(2)= 0,0   Y1(3)= 1,0 m

Member 1.   D1=X1(3)-X1(1)= 1,5-0,0= 1,5 m
            D2=Y1(3)-Y1(1)= 1,0-3,5=-2,5 m
L1=Sqr(1,5^2+(-2,5^2))= 2,92 m

C=D1/L1= 1,5/2,92= 0,514
S=D2/L1=-2,5/2,92=-0,856

A1= R*C^2 +A*S^2
  =17,1EI*(0,514^2) +0,120EI*(-0,856^2)
  = 4,518EI +0,088EI= 4,606 *EI
A2= R*S*C -A*S*C
  =17,1EI(-0,856)(0,514)-0,120EI(-0,856)(0,514)
  =-7,524EI +0,053EI= -7,471 *EI

A3=-B*S =-0,352EI(-0,856)= 0,301 *EI
A4= R*S^2 +A*C^2
  =17,1EI*(-0,856^2) +0,120EI*(0,514^2)
  =12,530EI =0,032EI= 12,562 *EI
A5= B*C= 0,352EI(0,514)= 0,181 *EI

At the highest member end number H=3 a hinge, then sixth row and sixth column of S5 filled with zeros, see page .

	1	2	3	7	8	9
1	4606	-7471	301	-4606	7471	0
2	-7471	12562	181	-7471	-12562	0
3	301	181	1027	-301	-181	0
7	-4606	7471	-301	4606	-7471	0
8	7471	-12562	-181	-7471	12562	0
9	0	0	0	0	0	0

x EI/1000   S51

	4	5	6	7	8	9
4	14267	-4624	-190	-14267	4624	-190
5	-4624	1920	-570	4624	-1920	-570
6	-190	-570	1266	190	570	633
7	-14267	4624	190	14267	-4624	190
8	4624	-1920	570	-4624	1920	570
9	-190	-570	633	190	570	1266

x EI/1000   S52

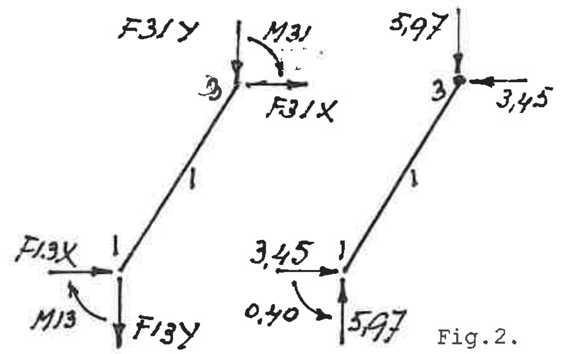


Fig.2.

The member end forces and moments of member 1 with help of S51 of the prese-ding page, EI omitted.

```
F13X=-4,606*UX3    +7,471*UY3
    =-4,606(0,63) +7,471(0,85)
    =-2,90 + 6,35 =  3,45 kN
F13Y= 7,471(0,63) -12,562(0,85)
    = 4,71 - 10,68= -5,97 kN
 M13=-0,301(0,83) -0,181(0,85)
    =-0,25 - 0,15 = -0,40 kNm
```

Member 1 without member loads, then are F31X and F31Y as large as but opposite directed.
F31X=-3,45 kN en F31Y= 5,97 kN.
M31= 0(0,63) +0(0,85)= 0 kNm

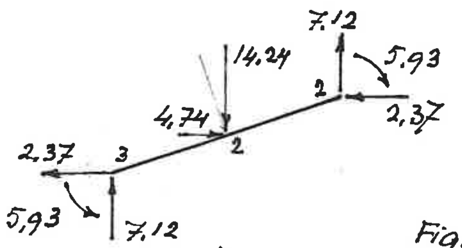


Fig.3a.

+

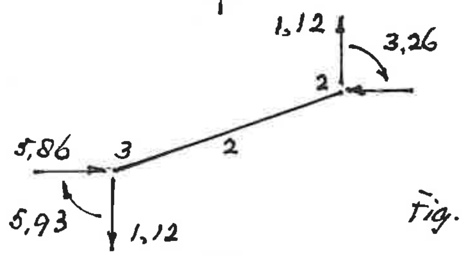


Fig.3b.

=

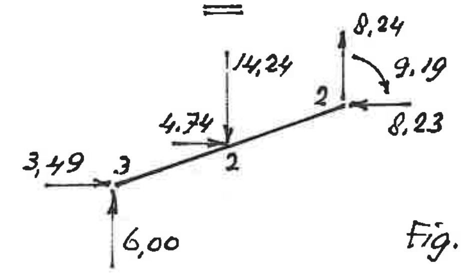


Fig.3c.

Fig.3c is the addition of figure 3a and 3b. The member end forces and moments are the final fortces and moments. The moment at member end 3 is 0 kNm, joint 3 is a hinge.

UX1, UY1, UR1, UX2, UY2 and UR2 are known now, atre all zero, UX3, UY3 and UR3 are calcula-ted.
The in constructionmatrix CC coinciding ele-ments of S51 and S52 are added. For exaxmple the concerning elements of the third row.

```
4606 +14267= 18873    -7471 -4624= -12095
  0  +190  =   190    Etc.
```

$$\begin{bmatrix} F31X+F32X \\ F31Y+F32Y \\ M31+M32 \end{bmatrix} = \begin{bmatrix} 18873 & -12095 & 190 \\ -12095 & 14482 & 570 \\ 190 & 570 & 1266 \end{bmatrix} \cdot \begin{bmatrix} UX3 \\ UY3 \\ UR3 \end{bmatrix}$$

$\underline{f}$ CC $\underline{u}$

There are three equations to solve.

$$\begin{bmatrix} 18873 & -12095 & 190 \\ -12095 & 14482 & 570 \\ 190 & 570 & 1266 \end{bmatrix} \cdot \begin{bmatrix} UX3 \\ UY3 \\ UR3 \end{bmatrix} = \begin{bmatrix} 2,37 \\ 7,12 \\ 5,93 \end{bmatrix}$$

$\underline{\text{x EI/1000}}$ $\underline{u}$ $\underline{f}$

The elements of $\underline{f}$ follow with the equilibrium equations for joint 3 like on page 62 .

The equations written out, without EI.

```
 19,138*UX3 -12,095*UY3 +0,190*UR3= 2,37

-12,095*UX3 +14,482*UY3 +0,570*UR3= 7,12

  0,190*UX3  +0,570*UY3 +1,266*UR3= 5,93
```

With computer Gauss follow

<u>UX3= 0,63/EI</u>, <u>UY3= 0,85/EI</u>, <u>UR3= 4,21/EI</u>.

Fig.3a.
The member end forces and moments due to the member load of 15 kN like fig.6b of page 63 .

Fig.3b
The member end forces and moments of member 2 due to the displacements <u>alone</u> with help of ma-trix S52.

```
F23X= -14,267(0,63) +4,624(0,85) -0,190(4,21)
    = -8,99 +3,93 -0,80= -5,86 kN

F23Y=   4,624(0,63) -1,920(0,85) -0,570(4,21)
    =  2,91 -1,63 -2,40= -1,12 kN

 M23=   0,190(0,63) +0,570(0,85) +0,633(4,21)
    =  0,12 +0,48 +2,66= 3,26 kNm

F32X= -F23X= 5,86 kN   F32Y= -F23Y= 1,12 kN

 M32=   0,190(0,63) +0,570(0,85) +1,266(4,21)
    =  0,12 +0,48 +5,33= 5,93 kNm
```

Forces and moments are drawn with their real directions.

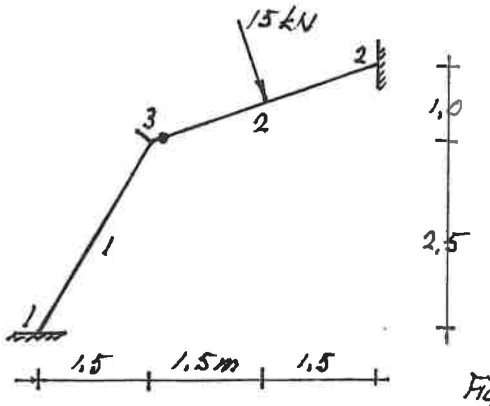


Fig.1.

Member 1.

R='EA/L'= EA/2,92= 0,342 EA
R=(0,342)50EI= 17,1 EI

A=12*EI/L^3= 12*EI/(2,92^3)= 0,482 *EI
B= 6*EI/L^2= 6*EI/(2,92^2)= 0,704 *EI
D= 4*EI/L= 4*EI/2,92= 1,370 *EI
E= 2*EI/L= 2*EI/2,92= 0,685 *EI

Member 2.

D1=X1(3)-X1(2)= 1,5-4,5=-3,0 m
D2=Y1(3)=Y1(2)= 1,0-0,0= 1,0 m
L1=Sqr(-3,0^2+1,0^2)= 3,16 m

R='EA/L'= EA/3,16= 0,316 EA EA=50EI
R=(0,316)50EI= 15,8 EI

C=D1/L1=-3,0/3,16=-0,949
S=D2/L1= 1,0/3,16= 0,316

Member end 3 is a hinge..
A= 3*EI/L^3= 3*EI/(3,16^3)= 0,095 *EI
B= 3*EI/L^2= 3*EI/(3,16^2)= 0,300 *EI

D= 3*EI/L= 3*EI/3,16= 0,949 *EI

With the formulas for A1, A2, A3, A4 and A5 follow calculation,

with joint load forces due to 15 kN.
Member end 3 of member 2 is a hinge!

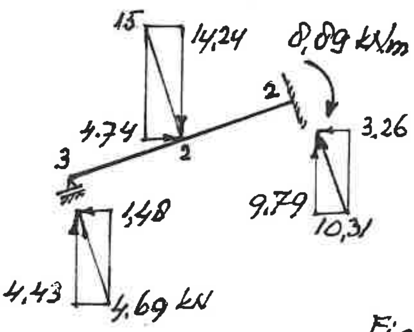


Fig.2.

See formulas page 97.
At member end 3 (5/16)*15= 4,69 kN,
at member end 2 (11/16)*15= 10,31 kN
and moment (3/16)*15*3,16= 8,89 kNm.

Example.

Fig.1.
One more time the same construction like on the previous pages, now with member end 3 of member 2 a hinge. Both members with same bending stiffness EI and strain stiffness EA=50EI.
Irregular joint humbering.
Member 1 with L=1 and H=3, member 2 L=2 H=3.

X1(1)= 0,0 X1(2)= 4,5 X1(3)= 1,5 m
Y1(1)= 3,5 Y1(2)= 0,0 Y1(3)= 1,0 m

Member 1. D1=X1(3)-X1(1)= 1,5-0,0= 1,5 m
D2=Y1(3)-Y1(1)= 1,0-3,5=-2,5 m
L1=Sqr(1,5^2+(-2,5^2))= 2,92 m
C=D1/L1= 1,5/2,92= 0,514
S=D2/L1=-2,5/2,92=-0,856

A1= R*C^2 +A*S^2=
= 17,1EI*(0,514^2) +0,482EI*(-0,856^2)
= 4,518EI +0,353EI= 4,871 *EI
A2= R*S*C-A*S*C=
= 17,1EI(-0,856)(0,514)-0,482EI(-0,856)(0,514)
=-7,524EI +0,212EI= -7,312 *EI

A3=-B*S =-0,704EI*(-0,856)= 0,603 *EI
A4= R*S^2 +A*C^2=
= 17,1EI*(-0,856^2) +0,482EI*(0,514^2)
= 12,530EI +0,127EI= 12,657 *EI
A5= B*C= 0,704EI*(0,514)= 0,362 *EI

Value the same like on page 61.

	1	2	3	7	8	9
1	4871	-7312	603	-4871	7312	603
2	-7312	12657	362	7312	-12657	362
3	603	362	1370	-603	-362	685
7	-4871	7312	-603	4871	-7312	-603
8	7312	-12657	-362	-7312	12660	-362
9	603	362	685	-603	-362	1370

x EI/1000 S51

S52 of member 2, with L=1 and H=3, member end 3 a hinge, then sixth row and sixth column filled with zeros, see page 65.

	4	5	6	7	8	9
4	14238	-4710	95	-14238	4710	0
5	-4710	1664	-285	4710	-1664	0
6	95	-285	949	-95	285	0
7	-14238	4710	-95	14238	-4710	0
8	4710	-1664	285	-4710	1664	0
9	0	0	0	0	0	0

x EI/1000 S52

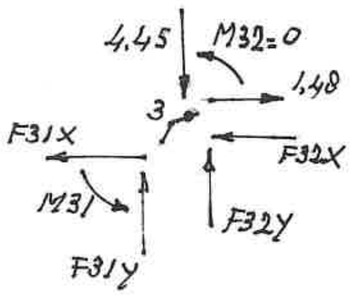


Fig.3.

Like on page 66 follow three equations to solve UX3, UY3 and UR3.

$$\underbrace{\begin{bmatrix} F31X+F32X \\ F31Y+F32Y \\ M31+M32 \end{bmatrix}}_{f} = \underbrace{\begin{bmatrix} 19109 & -12022 & -603 \\ -12022 & 14324 & -362 \\ -603 & -362 & 1370 \end{bmatrix}}_{CC} \cdot \underbrace{\begin{bmatrix} UX3 \\ UY3 \\ UR3 \end{bmatrix}}_{u}$$

$$\underbrace{\begin{bmatrix} 19109 & -12022 & -603 \\ -12022 & 14324 & -362 \\ -603 & -362 & 1370 \end{bmatrix}}_{x\ EI/1000} \cdot \underbrace{\begin{bmatrix} UX3 \\ UY3 \\ UR3 \end{bmatrix}}_{u} = \underbrace{\begin{bmatrix} 1,48 \\ 4,45 \\ 0,00 \end{bmatrix}}_{f}$$

Fig.3 en 2.
The elements of f follow with the equilibrium equations for joint 3 like on page .

The equations written out without EI.

19,109*UX3 -12,022*UY3 -0,603*UR3= 1,48

-12,022*UX3 +14,324*UY3 -0,362*UR3= 4,45

-0,603*UX3 -0,362*UY3 +1,370*UR3= 0,00

With computer Gauss page 95 follow
UX3= 0,63/EI, UY3= 0,85/EI, UR3= 0,50/EI.

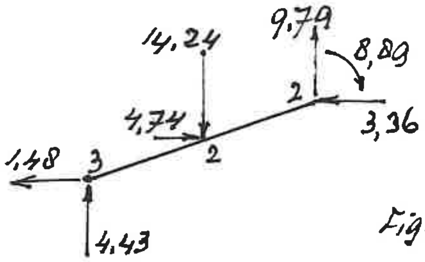


Fig.4a

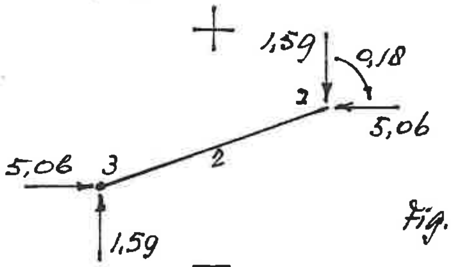


Fig.4b.

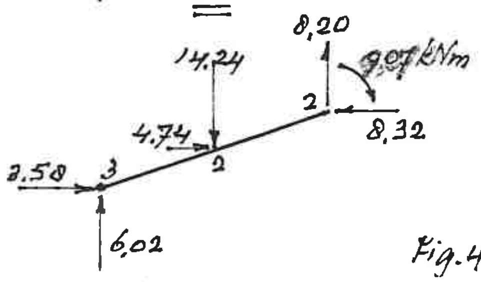


Fig.4c.

Fig.4a.
Member end forces and moments due to the member load force of 15 kN.
Fig.4
The member end forces and moments of member 2 due to the displacements alone with help of member matrix S52.
Fig.4c.
Addition of figire a and b gives the final member end forces and moments, M32=0 kNm.

The separately to calculate angle H23.
Fig.4c divided into three cases.

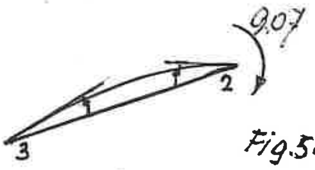


Fig.5a.

Fig.5a.
Due to 9,07 kNm at member end 2 arises (9,07(3,16))/6EI= 4,78/EI to the left..

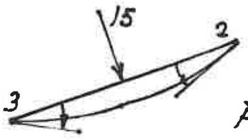


Fig.5b.

Fig.5b.
Due to 15 kN arises
(15(3,16^2))/16EI= 9,36/EI to the right.

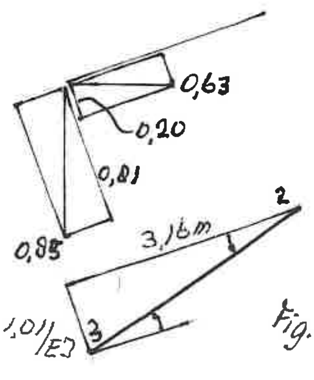


Fig.5c.

Fig.5c.
The displacements of member end 3 perpendicular to the member axis become
(0,85/EI)*3,0/3,16= 0,81/EI and
(0,63/EI)*1,0/3,16= 0,20/EI. Together 1,01 EI, gives (1,01/EI)/3,16= 0,32/EI.

H32= 9,36/EI -4,78/EI -0,32/EI= 4,26/EI, that is ... like joint roatation UR3=4,21/EI of page 66 .
At member end 2, for the clamp addition of teh angles 9,55/EI -9,36/EI -0,32/EI= -0,13/EI, not bad... almost 0... is zero.

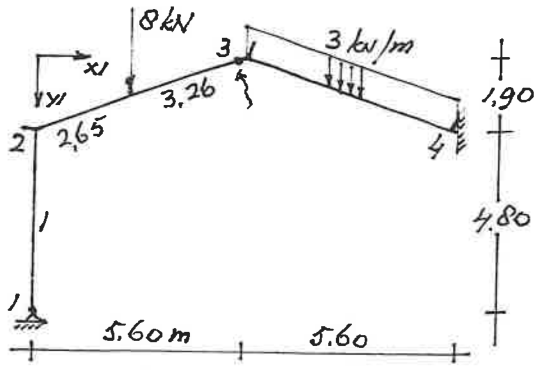


Fig 1.

Member 1. L=1 and H=2.

At L=1 a hinge, third row and third co-column filled with zeros, page 59, indicated with dots. A verticale member, concerning elements indicated with zeros.

R='EA/L'= EA/4,80= 0,208 EA   EA=60EI

A= 3EI/L^3= 3EI/(4,80^3)= 0,027 EI
B= 3EI/L^2= 3EI/(4,80^2)= 0,130 EI
D= 3EI/L= 3EI/4,80 = 0,625 EI

Member 2. L=2 and H=3.

At H=3 a hinge, sixth row and sixth column filled with zeros.

R='EA/L'= EA/5,91= 0,169 EA   EA=60EI

A= 3EI/L^3= 3EI/(5,91^3)= 0,015 EI
B= 3EI/L^2= 3EI/(5,19^2)= 0,086 EI
D= 3EI/L= 3EI/5,91 = 0,508 EI

member 1			member 2		
F12X		U1X	F23X		U2X
F12Y		U1Y	F23Y		U2Y
M12		UR1	M23		UR2
	=S51			=S52	
F21X		U2X	F32X		U3X
F21Y		U2Y	F32Y		U3Y
M21		UR2	M32		UR3

Member 3. L=3 and H=4.

Both member ends a real oint.

R='EA/L'= EA/5,91= 0,169 EA   EA=60EI

A=12EI/L^3= 12EI/(5,91^3)= 0,058 EI
B= 6EI/L^2= 6EI/(5,91^2)= 0,172 EI
D= 4EI/L= 4EI/5,91= 0,677 EI
E= 2EI/L= 2EI/5,91= 0,338 EI

member 3			
F34X		U3X	A1= R*C^2 +A*S^2
F34Y		U3Y	
M34		UR3	A2= R*S*C -A*S*C
	=S53		
F43X		U4X	A3= -B*S
F43Y		U4Y	
M43		UR4	A4= R*S^2 +A*C^2
			A5= B*C

Example.

Fig.1.
Three members and four joints, binding stiffness EI and strain stiffness EA=60EI.

X1(1)= 0,00 m   Y1(1)= 6,70 m
X1(2)= 0,00 m   Y1(2)= 1,90 m
X1(3)= 5,60 m   Y1(3)= 0,00 m   C=D1/L1
X1(4)= 11,20 m   Y1(4)= 1,90 m   S=D2/L1

	1	2	3	4	5	6
1	27	0	.	-27	0	130
2	0	12480	0	0	-12480	0
3	.	0	.	.	0	.
4	-27	0	.	27	0	-130
5	0	-12480	0	0	12480	0
6	130	0	.	-130	0	625

x EI/1000   S51   C=0   S=1

A1= 0,027 EI   A2= 0 EI
A3= 0,130 EI   A4= 12,480 EI
A5= 0 EI

	4	5	6	7	8	9
4	9115	-3081	28	-9115	3081	.
5	-3081	1058	88	3081	-1058	.
6	28	82	508	-28	-82	.
7	-9115	3081	-28	9115	-3081	.
8	3081	-1058	-82	-3081	1058	.
9	.	.	.	.	.	.

x EI/1000   S52   C=0,948   S=-0,321

A1= 9,115 EI   A2= -3,081 EI
A3= 0,028 EI   A4= 1,058 EI
A5= 0,082 EI

	7	8	9	10	11	12
7	9119	3068	-55	-9119	-3068	-55
8	3068	1097	163	-3068	-1097	163
9	-55	163	677	55	-163	338
10	-9119	-3068	55	9119	3068	55
11	-3068	-1097	-163	3068	1097	-163
12	-55	163	338	55	-163	677

x EI/1000   S53   C=0,948   S= 0,321

A1= 9,115 EI   A2= 3,068 EI
A3=-0,055 EI   A4= 1,097 EI
A5= 0,163 EI

Joint load forces and moments due to member load force of 8 kN.

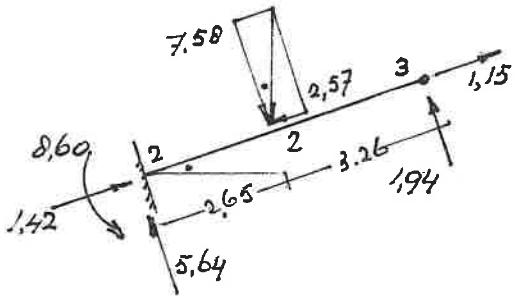


Fig.2a.

Fig.2a.
The components of 8 kN are

(8,00/5,91)*5,60= 7,58 kN and

(8,00/5,91)*1,90= 2,57 kN.

With the formulas page follow the reactions.

(7,58*3,26(3*5,91^2-3,26^2))/(2*5,91^3) =

(24,71(104,78-10,63))/412,85=

2326,45/412,85= 5,64 kN at member end 2 2.

7,58 -5,64= 1,94 kN at member end 3.

The reaction moment at member end 2 with

(7,58*3,26^2(5,91^2-3,26^2)/(2*5,91^2) =

(24,71(34,93-10,63)/69,86=

(600,45/69,86= 8,60 kNm at member end 2.

Member end 3 a hinge, moment zero.

Due to 2,57 kN along the member axise arise

(3,26/5,91)*2,57= 1,42 kN at member end 2 and

(2,65/5,91)*2,57= 1,15 kN at member end 3.

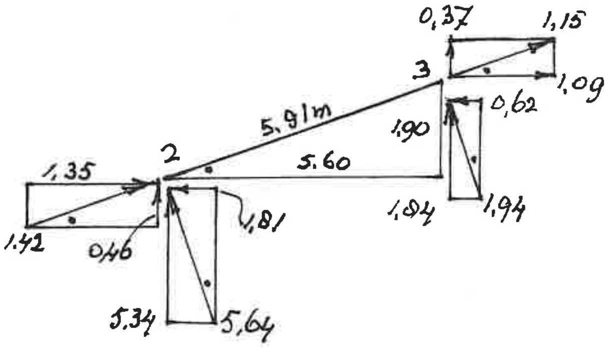


Fig.2b.

Fig.2b en 2c.
The horizontal and vertical components are
(1,42/5,91)*5,60= 1,35 kN,
(1,42/5,91)*1,90= 0,46 kN,

(5,64/5,91)*1,90= 1,81 kN and
(5,64/5,91)*5,60= 5,34 kN at member end 2.

(1,15/5,91)*5,60= 1,09 kN,
(1,15/5,91)*1,90= 0,37 kN,

(1,94/5,91)*1,90= 0,62 kN and
(1,94/5,91)*5,60= 1,84 kN at member end 3.

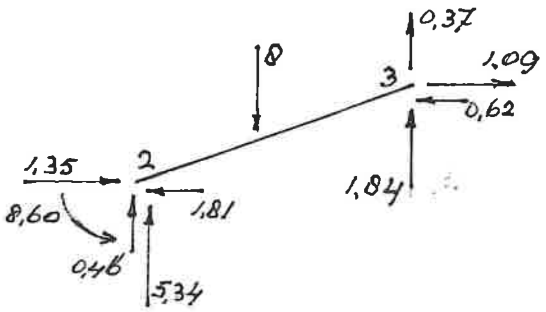


Fig.2c.

Fig.2c en 2d.
On the joints act member end forces and moments with assumed directions like on page to the right, upward and to the right.

On joint 2 act forces and moments as large as but opposite directed.

Elements of $\underline{f}$ in CC $\underline{u}$ = $\underline{f}$ follow with equilibrium of the joint.

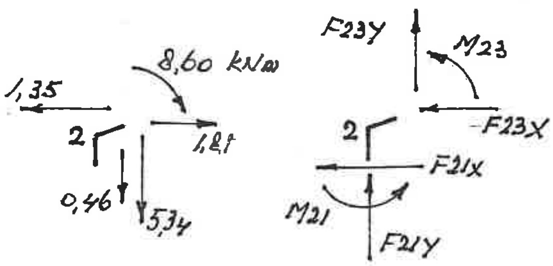


Fig.2d.

Σ hor. joint 2 =0
F21X+F23X +1,35 -1,81= 0 F21X+F23X= 0,46 kN

Σ vert. joint 2= 0
F21Y+F23Y -0,46 -5,34= 0 F21Y+F23Y= 5,80 kN

Σ mom. joint 2= 0
M21+M23 -8.60=0 M21+M23= 8,60 kNm

Joint load forces and joint load moments due to the uniformly distributed load of 3 kN/m.

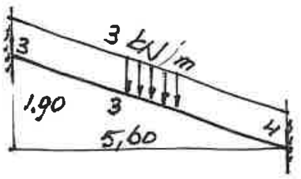


Fig. 3a.

Fig.3a en 3b.
The components of 3 kN/m are

(3,00/5,91)*5,60= 2,84 kN/m and

(3,00/5,91)*1,90= 0,96 kN/m.

With the formale page follow the reactions of the on both ends clamped member.

(2,84(5,91)/2)= 8,39 kN,

(1/12)(2,84)(5,91^2)= 8,27 kNm and

(2,84*5,91)/2= 2,84 kN.

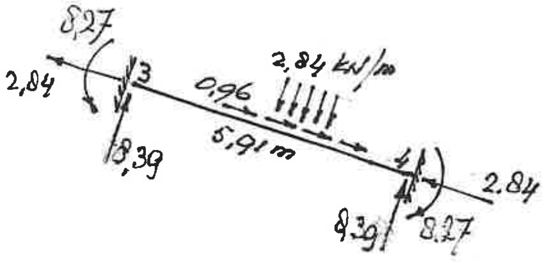


Fig. 3b.

Fig.3c.
The horizontal and vertical components of the member end forces calculated like on the preceding page 70.

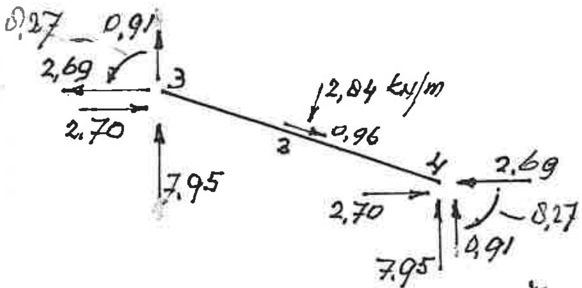


Fig. 3c.

Fig.3d.
On the separated joint 3 act forces as large as these member end forces but opposite directed, of member end 3 of member 2 of fig.2c, and of member end 3 of member 3 of fig.3c. Further the unknown member end forces F32X, F32Y, F34X and F34Y, and the member end moments M32 and M34.

Σ hor. joint 3 =0
F32X+F34X +1,09 -0,62 +2,70 -2,69= 0
F32X+F34X= -0,48 kN

Σ vert. joint 3= 0
F32Y+F34Y -0,37 -1,84 -0,91 -7,95= 0
F32Y+F34Y= 11,07 kN

Σ mom. joint 3= 0
M32+M34 -8.27=0 M32+M34= 8,27 kNm

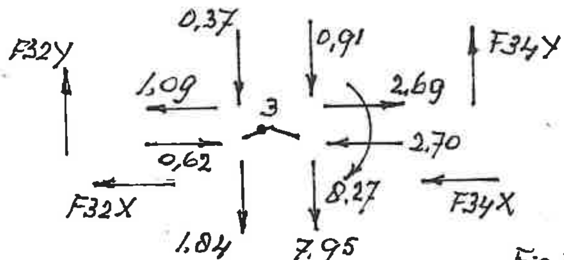


Fig. 3d.

Fig.4.
On joint 4 act the member end forces of member end 4 of member 3. Reaction forces and reaction moment 'not yet there'.
The joint load forces and joint load moment follow again with equilibrium of the joint without the reactions at the clamp.

Σ hor. joint 4 =0
F43X +2,70 -2,69= 0 F43X= -0,01 kN

Σ vert. joint 4= 0
F43Y -7,95 -0,91= 0 F43Y= 8,86 kN

Σ mom. joint 4= 0
M43 +8,27= 0 M43= -8,27 kNm

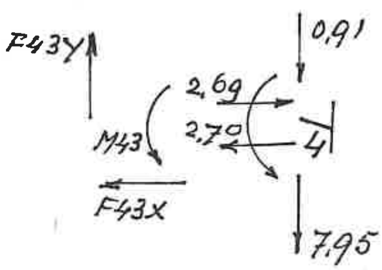


Fig. 4.

May be better as follows?
Suppose joint load force F4X like earlier assumed direction to the right,
F4X= 2,69 -2,70= -0,01 kN,
F4Y assumed downward,
F4Y= 7,95 +0,91= 8,86 kN,
M4 assumed to the right, M4= -8,27 kNm.

	1	2	3	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	10	11	12	
1	27	0	.	-27	0	130							U1X
2	0	12480	.	0	-12480	0							U1Y
3	.	.	.	.	.	.							UR1
<u>4</u>	-27	0	.	9142	-3081	-102	-9115	3081	.				<u>U2X</u>
<u>5</u>	0	-12480	.	-3081	13538	82	3081	-1058	.				<u>U2Y</u>
<u>6</u>	130	.	.	-102	82	1133	-28	-82	.				<u>UR2</u>
<u>7</u>				-9115	3081	-28	18234	-13	-55	-9119	-3068	-55	<u>U3X</u>
<u>8</u>				3081	-1058	-82	-13	2155	163	-3068	-1097	163	<u>U3Y</u>
<u>9</u>				.	.	.	-55	163	677	55	-163	338	<u>UR3</u>
10							-9119	-3068	55	9119	3068	55	U4X
11							-3068	-1097	-163	3068	1097	-163	U4Y
12							55	163	338	55	-163	677	UR4

x EI/1000 CC

The known displacements are U1X=0, U1Y=0, U4X=0, U4Y=0 and UR4=0, the equations 1, 2, 3 and 10, 11 and 12 fall off. Since member end 1 is a hinge the elements of the third row and third column here above are zero indicated with dots. More dots because of member end 3 of member 2 being a hinge. Equations 4 to 9 with unknowns U2X, U2Y, UR2, U3X, U3Y and UR3 will be solved. HE12 and HE32 are separately calculated.

1	9,142	-3,081	-0,102	-9,115	3,081	0		U2X		0,46	<u>4</u>	F21X+F23X
2	-3,081	13,538	0,088	3,081	-1,058	0		U2Y		5,80	<u>5</u>	F21Y+F23Y
3	-0,102	0,082	1,133	-0,028	-0,082	0	.	UR2	=	8,60	<u>6</u>	M21+M23
4	-9,115	3,081	-0,028	18,230	-0,013	-0,055		U3X		-0,48	<u>7</u>	F32X+F34X
5	3,081	-1,058	-0,082	-0,013	2,155	0,163		U3Y		11,07	<u>8</u>	F32Y+F34Y
6	0	0	0	-0,055	0,163	0,677		UR3		8,27	<u>9</u>	M32+M34

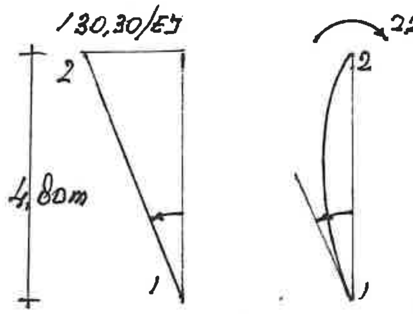


Fig.5.

See matrix S51 page for the calculation of member end moment M21.

0,130EI(U2X) +0(U2Y) +0,126(UR2)=

0,130EI(-130,30/EI)+0+0,126EI(8,29/EI)=

16,94 +0 +5,18= 22,12 kNm, positive

answer, as assumed to the right.

The six equations result with computer Gauss in the following answers.

<u>U2X=-130,30/EI</u> <u>U2Y= 0,80/EI</u> <u>UR2= 8,29/EI</u>

<u>U3X= -65,25/EI</u> <u>U3Y=194,76/EI</u> <u>UR3=-39,98/EI</u>

Fig.5.
Slope deflection HE12 at member end 1.

Due to the displacement 130,30/EI of joint 2 to the left arises
(130,30/EI)/4,80= 27,15/EI to the left.

Due to member end moment 22,12 kNm at member end 2 to the right arises a slope deflection at member end 1,
(22,12*4,80)/(6EI)= 17,70/EI to the left.

Together HE12= 27,15/EI +17,70/EI= <u>44,45/EI</u>, to the left.
See page 77 with UR1= -44,02/EI

Fig.6a t/m 6d.
The separately to be calculated slope deflection H32.

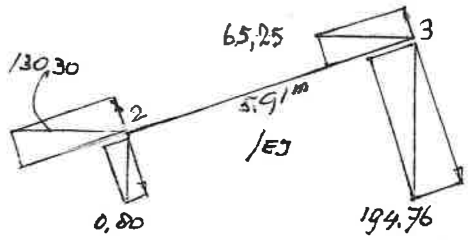


Fig.6a.

Fig.6a.
At the member ends the horizontal and vertical displacements are sketched, not drawn on scale. U2X= -130,30/EI, negative answer, not as assumed to the right but to the left. Etc.

The displacements are resolved perpendicular to the member at member end 2 and 3, drawn with their real directions.

((130,30/EI)/5,91))*1,90= 41,89/EI
((0,80/EI)/5,91))*5,60= 0,76/EI

((65,25/EI)/5,91))*1,90= 20,98/EI
((194,76/EI)/5,91))*5,60= 184,54/EI

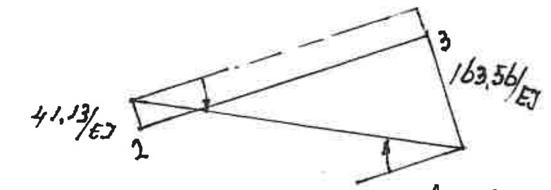


Fig.6b.

Fig.6b.
At member end 2 41,89 -0,76= 41,13 /EI and at member end 3 184,54-20,98= 163,56 /EI.
Due to the displacements alone arises a slope deflection H1 to the right.
H1=(41,13+163,56)/5,91= 34,63 /EI

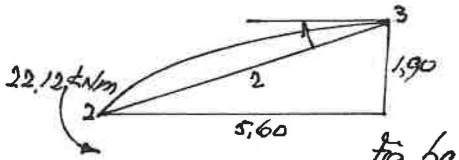


Fig.6c

Fig.6c.
The moment at member end 2 of member 2 is as large as the moment at member end 2 of member 1 but opposite directed, thus 22,12 kNm to the left.
At member end 3 arises due to this moment alone angle H2 to the right,
H2=(22,12*5,91)/6EI= 21,79 /EI

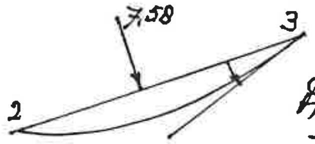


Fig.6d

Fig.6d.
Due to the load alone arises at member end 3 a slope deflection H3 to the left,

H3= (7,58(2,65)(3,26)(5,91+2,65))/6*5,91EI

=(65,48(8,56))/35,46EI= 15,81 /EI

Together to the right H1+H2+H3=
H32= (34,63+21,79-15,81)/EI= 40,61/EI.

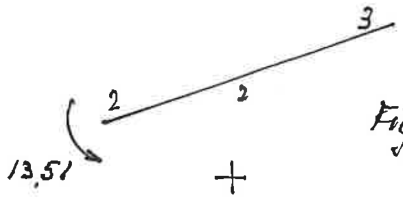


Fig.7a.

+

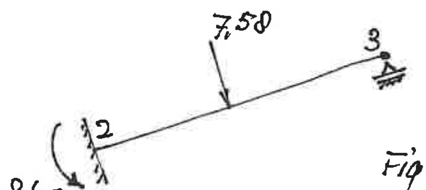


Fig.7b.

=

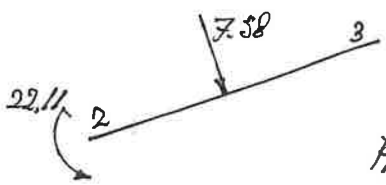


Fig.7c.

Fig.7a, 7b en 7c.
At member end 2 arises 22,12 kNm to the left, see fig.6c. Or to be calculated as follows.

Fig.7a.
Due to the displacements U2X, U2Y, UR2, U3X and U3Y (not drawn) alone with help of S52 page EI omitted,

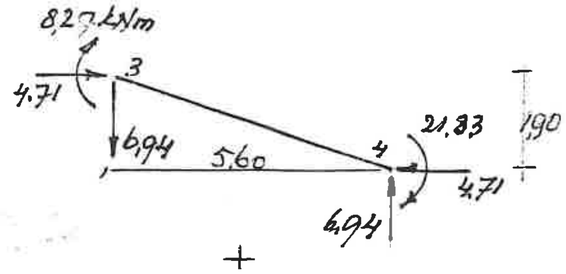
0,028(U2X) +0,082(U2Y) +0,508(UR2)
-0,028(U3X) -0,082(U3Y) + 0(UR3)=

0,028(-130,30) +0,082(0,80) +0,508(8,29)
-0,028(-65,25) -0,082(194,76)=

-3,65 +0,07 +4,21 +1,83 -15,97 +0= -13,51 kNm, negative answer, so not to the right as assumed but to the left.

Fig.7b. Fig.2a page 70.
Due to the load alone 8,60 kNm to the left.

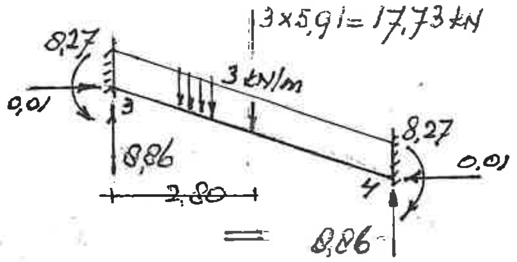
Together 13,51+8,60= 22,11 kNm to the left, OK.



Σ hor.=0 and Σ vert=0. Σ mom.=0?

Σ mom. member end 3= 0?
8,67+21,83+4,71(1,90)-6,94(5,69)=
30,50 +8,95 -38,86= 0,59 ≈ 0

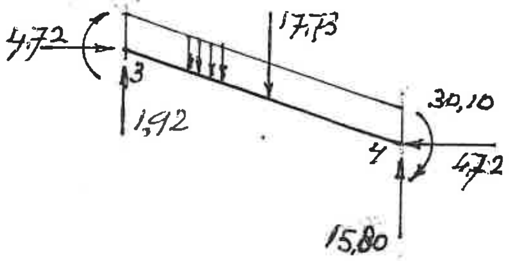
Fig.8a.



Σ hor.=0
Σ vert.=0 8,86+8,86-17,73=-0,01 ≈ 0

Σ mom. member end 3 3=0 ?
17,73(2,80)+0,01(1,90)-8,86(5,60)
+8,27-8,27=
49,64+0,02-49,62 +0= 0,04 ≈ 0

Fig. 8b.



Σ hor.=0
Σ vert.=0 1,92+15,80-17,73= -0,01 ≈ 0

Σ mom. member end 3 =0 ?
17,73(2,80)+4,72(1,90)+30,10
+0,40(?)-15,80(5,60)=
49,64+8,97+30,10+0,40(?)-88,48=0,63 ≈ 0

Fig.8c.

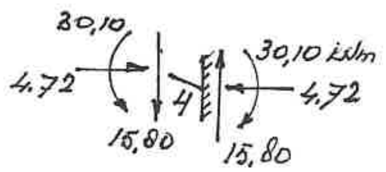


Fig.9.

Fig.8a to 8c.
Calculation of member end forces of member 3.

F34X		9119	3068	-55	-	-	-		U3X
F34Y		3068	1097	163	-	-	-		U3Y
M23		-55	163	677	-	-	-		UR3
F43X	=	-9119	-3068	55	-	-	-	.	U4X
F43Y		-3068	-1097	-163	-	-	-		U4Y
M43		-55	163	338	-	-	-		UR4

x EI/1000 S53 page 69.

U3X=-65,25/EI U3Y=194,76/EI UR3=-39,98/EI

Since U4X=0, U4Y=0 and UR4=0 the 4th, 5th and 6th column of matrix S5 here above are omitted. The concerning elements multiplied by U4X, U4Y and UR4 are zero, deliver no contribution to the member ned forces and member end moments.

Fig.8a. EI omitted.

F34X= 9,119(U3X) +3,068(U3Y) -0,055(-39,98)

= 9,119(-65,25)+3,068(194,76)-0,055(-39,98)

=-595,01 +597,52 +2,20= 4,71 kN

F34Y= 3,068(-65,25)+1,097(194,76)+0,163(-39,98)

=-200,19 +213,65 -6,52= 6,94 kN

M34=-0,055(-65,25)+0,163(194,76)+<u>0,677</u>(-39,98)

= 3,59 +31,75 -27,07= 8,27 kNm to the right

F43X= -4,71 kN en F43Y= -6,94 kN.

M43=-0,055(-65,25)+0,163(194,76)+<u>0,338</u>(-39,98)

= 3,59 +31,75 -13,51= 21,83 kNm

Fig.8b.
Forces and moments due to member loads <u>alone</u>.
See figure 3c of page 71 .

Fig.8c.
The final member end forces and moments due to joint displacements and member loads as addition of figure 8a and 8b.

Ofcourse M43=0 because of the hinge.
In the figures member end forces and member end moments are drawn with their real directions.

Fig.9.
The clamp reactions at support 4.
On joint 4 act forces and moments as large as but opposite directed.

With the three equilibrium equations then follow the support reactions as shown in the second figure.

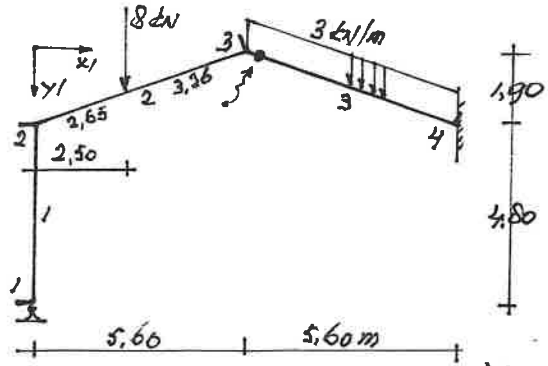


Fig.1.

Member 1. L=1 and H=2.

At L=1 now a 'real joint', the short stripe. For the vertical member the zeros of S51 like on page .
R='EA/L'= EA/4,80= 0,208 EA EA=60EI

A=12EI/L^3= 12EI/(4,80^3)= 0,109 EI
B= 6EI/L^2= 6EI/(4,80^2)= 0,260 EI
D= 4EI/L= 4EI/4,80= 0,833 EI
E= 2EI/L= 2EI/4,80= 0,417 EI

Member 2. L=2 and H=3.

At member end 3 a 'real joint'.
R='EA/L'= EA/5,91= 0,169 EA EA=60EI

A=12EI/L^3= 12EI/(5,91^3)= 0,058 EI
B= 6EI/L^2= 6EI/(5,91^2)= 0,172 EI
D= 4EI/L= 4EI/5,91= 0,677 EI
E= 2EI/L= 2EI/5,91= 0,338 EI

[F12X F12Y M12 F21X F21Y M21] =S51 [U1X U1Y UR1 U2X U2Y UR2]

member 1

[F23X F23Y M23 F32X F32Y M32] =S52 [U2X U2Y UR2 U3X U3Y UR3]

member 2

Member 3. L=3 and H=4.

At member end L=3 a hinge, third row and third column filled with zeros, see page 59.
R='EA/L'= EA/5,91= 0,169 EA EA=60EI

A= 3EI/L^3= 3EI/(5,91^3)= 0,015 EI
B= 3EI/L^2= 3EI/(5,19^2)= 0,086 EI
D= 3EI/L= 3EI/5,91 = 0,508 EI

[F34X F34Y M34 F43X F43Y M43] =S53 [U3X U3Y UR3 U4X U4Y UR4]

member 3

A1= R*C^2 +A*S^2

A2= R*S*C -A*S*C

A3= -B*S

A4= R*S^2 +A*C^2

A5= B*C

Example.

Fig.1.
Three members and four joints, bending stiffness EI and strain stiffness EA=60EI.

X1(1)= 0,00 m Y1(1)= 6,70 m
X1(2)= 0,00 m Y1(2)= 1,90 m
X1(3)= 5,60 m Y1(3)= 0,00 m C=D1/L1
X1(4)= 11,20 m Y1(4)= 1,90 m S=D2/L1

	1	2	3	4	5	6
1	109	0	260	-109	0	260
2	0	12480	0	0	-12480	0
3	260	0	833	-260	0	417
4	-109	0	-260	109	0	-260
5	0	-12480	0	0	12480	0
6	260	0	417	-260	0	833

x EI/1000 S51 C=0 S=-1

A1= 0,109 EI A2= 0 EI
A3= 0,260 EI A4= 12,480 EI
A5= 0 EI

	4	5	6	7	8	9
4	9119	-3068	55	-9119	3068	-55
5	-3068	1097	163	3068	-1097	163
6	55	163	677	-55	-163	338
7	-9119	3068	55	9119	-3068	-55
8	3068	-1097	-163	-3068	-1097	-163
9	55	163	338	-55	-163	677

x EI/1000 S52 C=0,948 S=-0,321

A1= 9,119 EI A2= -3,068 EI
A3= 0,055 EI A4= 1,097 EI
A5= 0,163 EI

	7	8	9	10	11	12
7	9115	3081	.	-9115	-3081	-28
8	3081	1058	.	-3081	-1058	82
9	.	.	.	.	.	.
10	-9115	-3081	.	9115	3081	28
11	-3081	-1058	.	3081	1058	-82
12	-28	82	.	28	-82	508

x EI/1000 S53 C=0,948 S= 0,321

A1= 9,115 EI A2= 3,081 EI
A3=-0,028 EI A4= 1,058 EI
A5= 0,082 EI

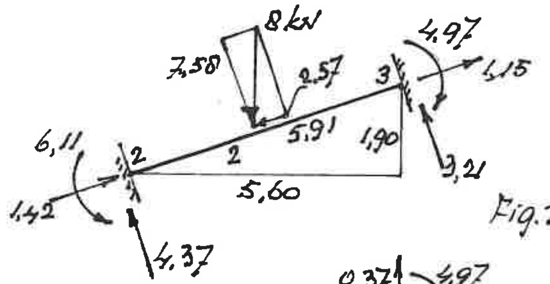


Fig.2a.

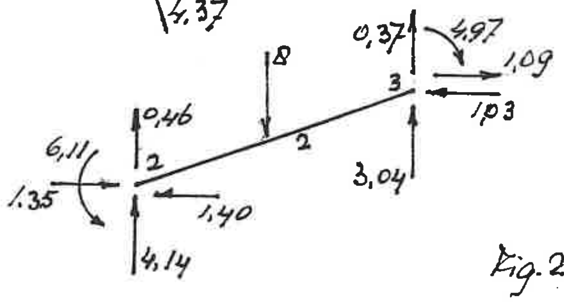


Fig.2b.

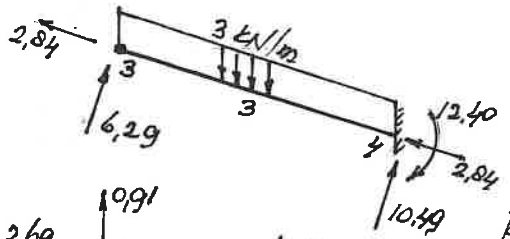


Fig.2c.

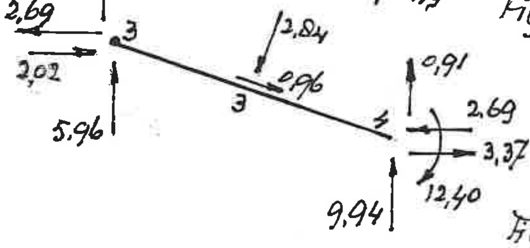


Fig.2d.

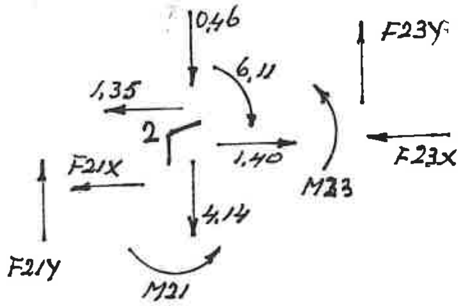


Fig.3.

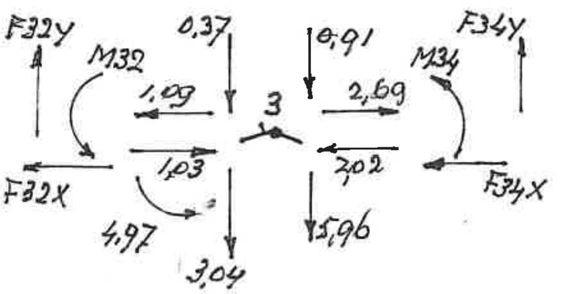


Fig.4.

Fig. 2a.
Member end forces and moments due to the member load force of 8 kN of member 2.
The components of 8 kN are.

7,58 kN and 2,57 kN.

Both member ends are rigidly connected with the joints. The member is clamped at both ends. The reactions follow with the formula paper.

Av= (7,58*(3,26^2))*(3*2,65+3,26)/(5,91^3)=

= (80,56*11,21)/206,43= 4,37 kN

Bv= 7,58 -4,37= 3,21 kN

MA= (7,58*2,65*(3,26^2))/(5,91^2)=

= 213,48/34,93= 6,11 kNm

MB= (7,58*(2,65^2)*3,26)/(5,91^2)=

= 173,53/34,93= 4,97 kNm

At member end 2 and 3 along the member axis like on page
1,42 kN and 1,15 kN.

Fig.2b.
The horizontal and vertical components of the just calculated member end forces are calculated like on page 70.

Fig.2c.
The member end forces and moments due the member load of 3 kN/m of member 3.
The components of 3 kN/m are 0,69 kN/m and 2,84 kN/m. With the formula page follow the reactions due to the load alone.

Av=(3/8)(2,84*5,91)= 6,29 kN

Bv=(5/8)(2,84*5,91)= 10,49 kN

MA=0 kNm and MB=(1/8)(2,84*5,91^2)= 12,40 kNm.

Fig.2d.
Member 3 with the horizontal and vertical components of the member end forces.

Joint load forces of joint 2.
Fig.3 and 2b.

For alone member end 2 of member 2.
F21X+F23X= 0,05 kN   F21Y+F23Y= 4,60 kN
M21+M23= 6,11 kN

Joint load forces of joint 3.
Fig.4, 2b and 2d.

Of the member ends 3 ofmember 2 and member 3.
F32X+F34X= 0,61 kN   F32Y+F34Y= 10,28 kN
M32+M34=-4,97 kNm

Also joint 4, with
F43X=-0,68 kN   F43Y= 10,85 kN   M43= 12,40 kNm

Joint 1 without joint load forces.

	1	2	3	4	5	6	7	8	9	10	11	12		
1	109	0	260	-109	0	260	.	.	.	.	.	.		U1X
2	0	-12480	0	0	-12480	0	.	.	.	.	.	.		U1Y
3	260	0	833	-260	0	417	.	.	.	.	.	.		UR1
4	-109	0	-260	9228	-3068	-205	-9117	3068	55	.	.	.		U2X
5	0	-12480	0	-3068	13577	163	3068	-1097	163	.	.	.		U2Y
6	260	0	417	-205	163	1510	-55	-163	338	.	.	.	·	UR2
7	.	.	.	-9119	3068	-55	18234	13	-55	-9115	-3081	-28		U3X
8	.	.	.	3068	-1097	-163	13	2155	-163	-3081	-1058	82		U3Y
9	.	.	.	55	163	338	-55	-163	677	0	0	0		UR3
10	.	.	.	.	.	.	-9115	-3081	0	9115	3081	28		U4X
11	.	.	.	.	.	.	-3081	-1058	0	3081	1058	-82		U4Y
12	.	.	.	.	.	.	-28	82	0	55	-82	508		UR4

x EI/1000 CC

Prescribed displacements U1X=0, U1Y=0, U4X=0, U4Y=0 and UR4=0, the concerning equations 1, 2, 10, 11 and 12 can be omitted. The third equation is not omitted because of the unknown joint rotation UR1, see page 75. Thus remain seven equations to solve with the unknowns UR1 of joint 1, U2X, U2Y and UR2 of joint 2 and U3X, U3Y and UR3 of joint 3.

1	0,833	-0,260	0	0,417	0	0	0		UR1		0	M2
2	-0.260	9.228	-3,068	-0,205	-9,117	3,068	0,055		U2X		0,05	F2X
3	0	-3,068	13,577	0,163	3,068	-1,097	0,163		U2Y		4,60	F2Y
4	0,417	-0,205	0,163	1,510	-0,055	-0,163	0,338	·	UR2	=	6,11	M3
5	0	-9,119	3,068	-0,055	18,234	0,013	-0,055		U3X		0,61	F3X
6	0	3,068	-1,097	-0,163	0,013	2,155	-0,163		U3Y		10,28	F3Y
7	0	0,055	0,163	0,338	-0,055	-0,163	0,677		UR3		-4,97	M3

Here below on the left the results for the next construction of page 79, on the right of a computer program.

U2X=-128,94/EI

U2Y= 0,79/EI

UR2= 8,28/EI

U3X= -64,58/EI

U3Y= 192,75/EI

Solution of the seven equations with computer Gauss the following results

UR1= -44,02/EI

U2X=-127,87/EI U2Y= 0,79/EI UR2= 8,20/EI

U3X= -64,04/EI U3Y= 191,22/EI UR3= 39,60/EI

On page 72 was found

HE12=-44,85/EI (ca. 2% difference with -44,02)

U2X=-130,30/EI U2Y= 0,80/EI UR2= 8,29/EI

U3X= -65,25/EI U3Y= 194,76/EI UR3=-39,98/EI

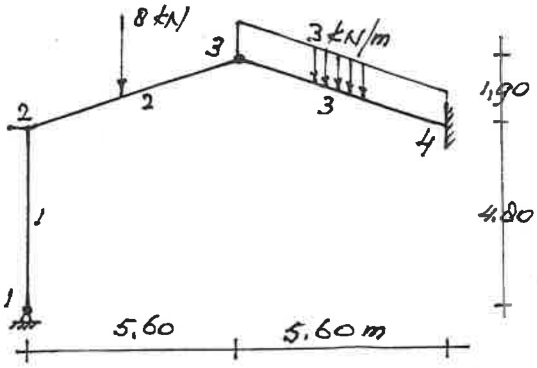


Fig.1.

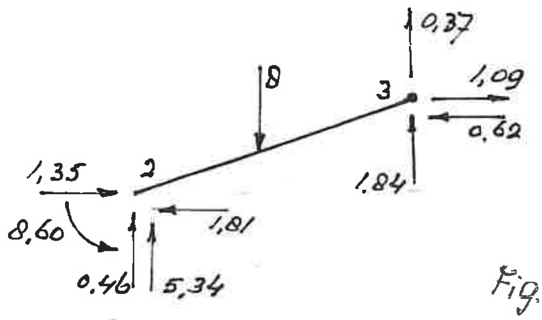


Fig.2.

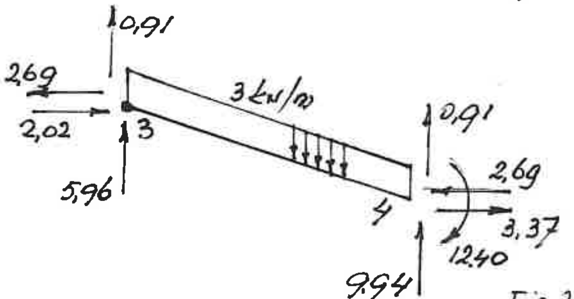


Fig.3.

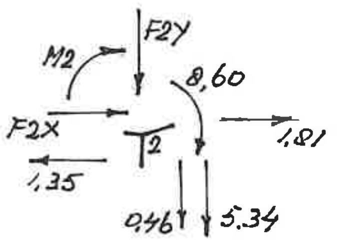


Fig.4.

Fig.4.
F2X= 1,81-1,35= 0,46 kN
F2Y= 0,46+5,34= 5,80 kN
M2= 8,60 kNm

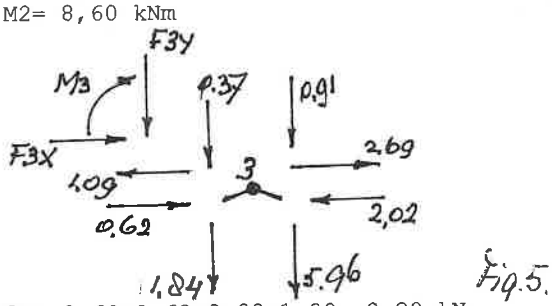


Fig.5.

F3X= 2,69+0,62-2,02-1,09= 0,20 kN
F3Y= 0,37+1,84+0,91+5,96= 9,08 kN
(M3= 0 kNm)

Example.

Fig.1.
Three members and four joints, bending stiffness EI, strain stiffness EA=60EI.

```
X1(1)=  0,00 m   Y1(1)= 6,70 m
X1(2)=  0,00 m   Y1(2)= 1,90 m
X1(3)=  5,60 m   Y1(3)= 0,00 m       C=D1/L1
X1(4)= 11,20 m   Y1(4)= 1,90 m       S=D2/L1
```

See the member stiffness matrices on page 75 to compare them.

	1	2	3	4	5	6
1	27	0	.	-27	0	130
2	0	12480	0	0	-12480	0
3	.	0	.	.	0	.
4	-27	0	.	27	0	-130
5	0	-12480	0	0	12480	0
6	130	0	.	-130	0	625

Member 1 x EI/1000 S51 C=0 S=1

	4	5	6	7	8	9
4	9115	-3081	28	-9115	3081	.
5	-3081	1058	88	3081	-1058	.
6	28	82	508	-28	-82	.
7	-9115	3081	-28	9115	-3081	.
8	3081	-1058	-82	-3081	1058	.
9	.	.	.	.	.	.

Member 2 x EI/1000 S52 C=0,948 S=-0,321

	7	8	9	10	11	12
7	9115	3081	.	-9115	-3081	-28
8	3081	1058	.	-3081	-1058	82
9	.	.	.	.	.	.
10	-9115	-3081	.	9115	3081	28
11	-3081	-1058	.	3081	1058	-82
12	-28	82	.	28	-82	508

Member 3 x EI/1000 S53 C=0,948 S= 0,321

The assumed joint loads with their assumed directions are
F2X, F2Y and M2 for joint 2 and
F3X, F3Y and M3 for joint 3.
They are the resultants of the on the joints acting member end forces like represented in the figures.

	1	2	3	4	5	6	7	8	9	10	11	12		
1	27	0	.	-27	0	130								U1X
2	0	12480	.	0	-12480	0								U1Y
3	.	0	.	.	0	.								UR1
4	-27	0	.	9142	-3081	-102	-9115	3081	.					U2X
5	0	-12480	0	-3081	13538	82	3081	-1058	.					U2Y
6	130	0	.	-102	82	1133	-28	-82	.					UR2
7				-9115	3081	-28	18230	.	.	-9115	-3081	-28	·	U3X
8				3081	-1058	-82	.	2116	.	3068	-1058	28		U3Y
9				.	.	.	.	.	.	.	.	.		UR3
10							-9115	-3081	.	9115	3081	28		U4X
11							-3081	-1058	.	3081	1058	-82		U4Y
12							-28	82	.	28	-82	508		UR4

x EI/1000 CC

The known displacements are U1X=0, U1Y=0, U4X=0, U4Y=0 and UR4=0, the concerning equations 1, 2, 10, 11 and 12 fall off. Joint rotations UR1 and UR3 are not calculatednso that also the equations 3 and 9 fall off. Five equations remain to calculate the unknown U2X, U2Y, UR2, U3X and U3Y.
On the preceding page the joint load forces F2X, F2Y and M2 of joint 2, figure 4, and F3X and F3Y of joint 3, figure 5, calculated as resultants,
F2X= 1,81-1,35= 0,46 kN i.s.o. F21X+F23X +1,35-1,81=0 so that F2X+D23X= 0,46 kN, etc.

1	9,142	-3,081	-0,102	-9,115	3,081		U2X		0,46	4	F21X+F23X		F2X
2	-3,081	13,538	0,088	3,081	-1,058		U2Y		5,80	5	F21Y+F23Y		F2Y
3	-0,102	0,082	1,133	-0,028	-0,082	·	UR2	=	8,60	6	M21+M2		M2
4	-9,115	3,081	-0,028	18,230	0		U3X		0,20	7	F32X+F34X		F3X
5	3,081	-1,058	-0,082	0	2,116		U3Y		9,08	8	F32Y+F34Y		F3Y

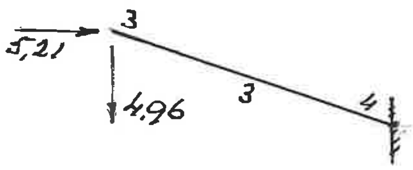


Fig.6a.

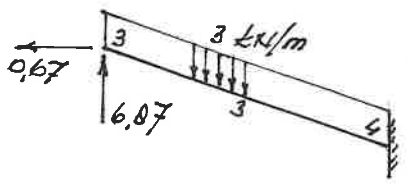


Fig.6b.

See figure 3.
At member end 3 of member 3
2,69-2,02= 0,67 kN to the left and
5,96+0,91= 6,87 kN upward.

With computer Gauss follows the solution of the five equations.

U2X=-128,94/EI U2Y= 0,79/EI UR2= 8,28/EI

U3X= -64,58/EI U3Y= 192,75/EI

Fig.6a.
The separarted member 3, a clamped member with member and member end loads.
With help of S53 follow due to the displacements alone
F34X= 9,115(-64,58) +3,081(192,75)=
= -588,65 +593,86= 5,21 kN and
F34Y= 3,081(-64,58) +1,058(192,75)=
= -198,97 +203,93= 4,96 kN. M34=0 kNm
Fig.6b.
To be added with the member end forces due to the loads alone, of figure 3.

Fig.6c and 6d.
The addition of figure 6a and 6b with the member end forces and member load. It is the clamped member of which the member end displacements can be calculated with the 'forget-me-nots'.

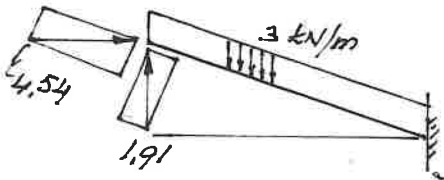


Fig 6c.

Therefore the member end forces and member load are resolve dperpedicular to and along the member axis.

Perpendicular to the member,

(4,54/5,91)*1,90= 1,46 kN and
(1,91/5,91)*5,60= 1,80 kN in same direction
together 3,26 kN.

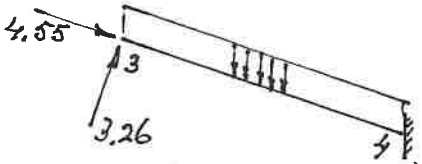


Fig.6d.

Along the member axis,

(5,54/5,91)*5,60= 5,16 kN and
(1,91/5,91)*1,90= 0,61 kN not in the same direction, 4,55 kN direction joint 4.

Fig.7a and 7b.
Due to 3,26 kN displaces member end 3 perpendicular to the member,

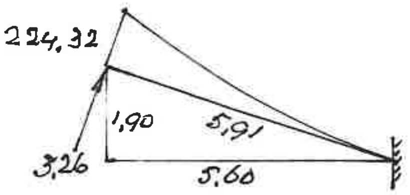


Fig.7a.

F*L^3/3EI= (3,26*5,91^3)/3EI
= 672,95/3EI = 224,32 /EI and

due to the distributed load of 2,84 kN/m
Q*L^4/8EI= (2,84*5,91^4)/8EI
= 3464,72/8EI= 433,09 /EI.

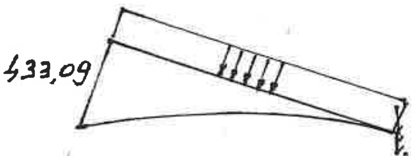


Fig.7b.

Member end 3 displaces
433,09/EI-224,32/EI= 208,77 /EI 'downward'.

Fig.7c and 7d.
Due to the axial force of 4,55 kN displaces member end 3 direction joint 4 along the member

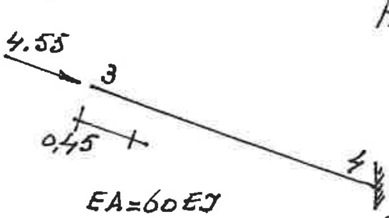


Fig.7c.

F*L/EA= (4,55*5,91)/EA= 26,89/EA, page 78 gives strain stiffness EA=60EI,
so 26,89/60EI=0,45/EI and
due to the distributed load of 0,96 kN/m
(Q*L^2/2)/EA= (0,96*5,91^2)/EA= 33,53/EA and
with EA=60EI follows 33,53/60EI= 0,59 /EI.

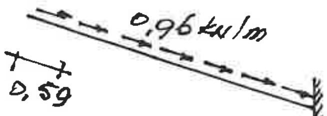


Fig.7d.

Together (0,45+0,59)/EI= 1,04 /EI direction joint 4.

Fig.8.
Next the calculation of the horizontal and vertical components of these displacements (not drawn on scale!)

(208,77/5,91)*1,90= 67,12 /EI to the left and
(1,04/5,91)*5,60= 0,99 /EI to the right.

That's 67,12-0,99= 66,13 /EI to the left, found on the preceding page U3X=-64,58 /EI.

(208,77/5,91)*5,60= 197,82 /EI downward and
(1,04/5,91)*1,90= 0,32 /EI downward.

That's 197,62+0,32= 197,94 /EI downward and U3Y= 192,75 /EI found on the preceding page..

The differences are 2-3 percent, OK.

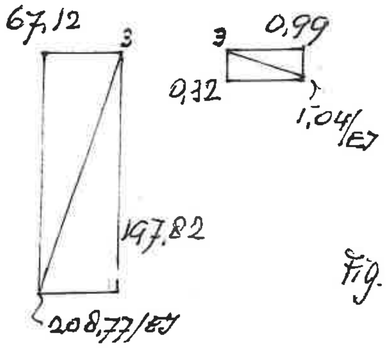


Fig.8.

7. Beam/member grids. (not yet checked)

7.1. The relation between member end forces FAB and FBA and joint displacements UA and UB.

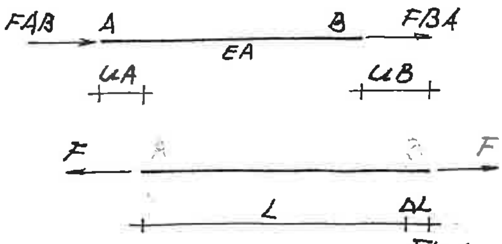


Fig.1.

$$\begin{bmatrix} FAB \\ FBA \end{bmatrix} = \begin{bmatrix} R & -R \\ -R & R \end{bmatrix} \cdot \begin{bmatrix} UA \\ UB \end{bmatrix} \qquad R=EA/L$$

$\underline{f}$ S5 $\underline{u}$

Fig.1.
Earlier dealt with.
Suppose displacement UB is larger than UA then the member lengthens ΔL=UB-UA and is a tension member. Then on the member ends act forces F as drawn.
With Hooke's law is ΔL=FL/EA, then follows F=(EA/L)*ΔL.
With stiffness factor R=EA/L is F=R*ΔL.
With ΔL=UB-UA
follows F=R*(UB-UA) oar F=R*(-UA+UB).

Now is FAB=-F or FAB=R*(UA-UB) 1)
and is FBA= F or FBA=R*(-UA+UB) 2)
Both equation are represented on the left in matrix form.

7.2. The relation between member end torsion moments MTAB and MTBA and member end rotations UTA and UTB.

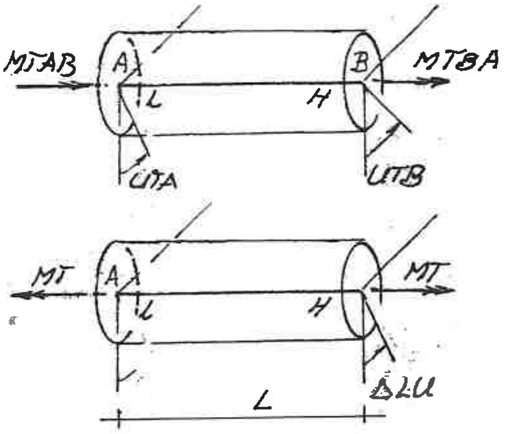


Fig.2.

$$\begin{bmatrix} MTAB \\ MTBA \end{bmatrix} = \begin{bmatrix} T & -T \\ -T & T \end{bmatrix} \cdot \begin{bmatrix} UTA \\ UTB \end{bmatrix} \qquad T=G*IP/L$$

$\underline{f}$ S5 $\underline{u}$

Fig.2.
The torsion moments MTAB and MTBA represented with a double arrow are assumed directing to the right and cause member end rotations UTA and UTB. With assumption UTB larger than UTA is the torsion over length L ΔLU=UTB-UTA to the left seen from B to A or H to L.
With member ends with a torsion moment MT it is ΔLU at end B w.r.t. end A.

With Hooke's lae is ΔLU=(MT*L)/(GIp) rad.
G is the shearing modulus of elasticity
Ip is the polar mpment of inertia so that
MT=(GIp/L)*ΔLU

With stiffness factor T=GIp/L (R=EA/L) follows
MT=T*ΔLU, and with ΔLU=UTB-UTA is

MT=T*(UTB-UTA) or MT= T*(-UTA+UTB).

Member end moments MTAB and MT at member end A are te same moment, then is MTAB=-MT so that
MTAB=-T*(-UTA+UTB) or MTAB= T*(UTA-UTB). 1)

And is MTBA=MT so that MTBA= T*(-UTA+UTB). 2)

On the left the two equations are given in matrix form.

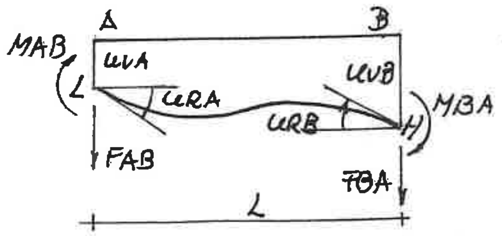


Fig.3.

$$\begin{bmatrix} FAB \\ MAB \\ FBA \\ MBA \end{bmatrix} = \begin{bmatrix} A & B & -A & B \\ B & D & -B & E \\ -A & -B & A & -B \\ B & E & -B & D \end{bmatrix} \cdot \begin{bmatrix} UVA \\ URA \\ UVB \\ URB \end{bmatrix}$$

$\underline{f}$ S5 $\underline{u}$

A=12*EI/L^3 B=6*EI/L^2

D=4*EI/L E=2*EI/L

Fig.3.
Starting point for a beam/member like explained for a continuous beam with joint/support displacements.
In that case is no torsion, the beams are only vertically loaded.
On the left matrix representation $\underline{f}$ = S5 $\underline{u}$ with the belonging member stiffness matrix.

Grids in the horizontal plane deform by bending and torsion.
The matrices of figure 1 and 2 are independent from each other, will be put together by some matrix manipulations.

$$
\begin{bmatrix} FLHy \\ MLHx \\ LLHz \\ FHLy \\ MHLx \\ MHLz \end{bmatrix}
=
\begin{bmatrix}
A & 0 & B & -A & 0 & B \\
0 & T & 0 & 0 & -T & 0 \\
B & 0 & D & -B & 0 & E \\
-A & 0 & -B & A & 0 & -B \\
0 & -T & 0 & 0 & T & 0 \\
B & 0 & E & -B & 0 & D
\end{bmatrix}
\cdot
\begin{bmatrix} UVLy \\ URLx \\ URLz \\ UVHy \\ URHx \\ URHz \end{bmatrix}
$$

$\underline{f}'$ S $\underline{u}'$

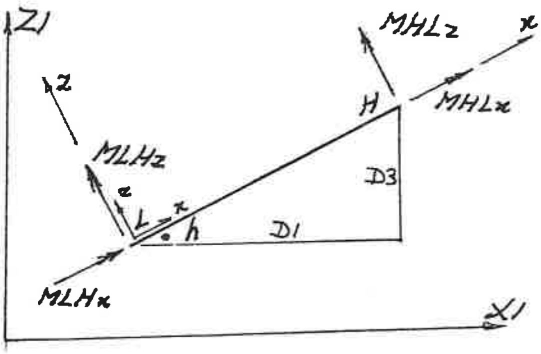


Fig.4a.

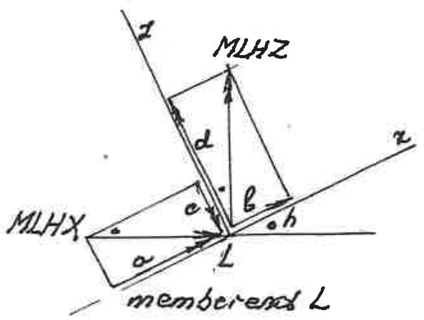


Fig.4b.

Joint load moments MLHZ and MLHX with their assumed directions.

$$
\begin{bmatrix} FLHy \\ MLHx \\ LLHz \\ FHLy \\ MHLx \\ MHLz \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & C & S & 0 & 0 & 0 \\
0 & -S & C & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & C & S \\
0 & 0 & 0 & 0 & -S & C
\end{bmatrix}
\begin{bmatrix} FLHY \\ MLHX \\ MLHZ \\ FHLY \\ MHLX \\ MHLZ \end{bmatrix}
$$

$\underline{f}'$ T $\underline{f}$

7.3. Combination of bending and torsion.

Fig.4a.
Now with member ends L and H i.s.o. A and B. The member axis system is x-y at the member end with the lowest member end number L.
Looking from above on the horizontal plane. Perpendicular on that plane act downward directed force FLHy and FHLy (like FAB and FBA), the vertical member end displacements are UVLy and UHLy.

MLHz and MHLz are the bending moments at the member ends, named MAB and MBA in figure 3. These moments are indicated with double arrow points.
The belonging slope deflections, joint rotations, are URLz and URHz.
The at the member ends acting torsion moments are MLHx and MHLx with double arrow points. Their rotations are URLx and URHx. Given on the left in matrix form.
Matrix S is the combination of the matrices of figure 1 and figure 3, preceding page.

Fig.4b.
These firces and moments (vectors) can be resolved into forces and moments w.r.t. the construction axis system X1-Y1-Z1, indicated with capitals X, Y and Z as follows,
FLH**Y**, MLH**X**, MLH**Z** with assumed directions like X1-, Y1- and Z1-axis.

For member end L. Perpendicular to the horizontal plane, FLH**y**= FLHY. 1)

Torsion moment MLHx consists of the components a of MLHX and b of MLHZ. (X is X1 and Z=Z1)
Cos(h)= a/MLHX or a= Cos(h)*MLHX and
Sin(h)= b/MLHZ or b= Sin(h)*MLHZ from which

MLH**x**= Cos(h)*MLHX +Sin(h)*MLHZ. 2)

Bending memnet MLHZ consists of c and d.
Sin(h)= c/MLHX or c= Sin(h)*MLHX and
Cos(h)= d/MLHZ or d= Cos(h)*MLHZ from which

MLH**z**= -Sin(h)*MLHX +Cos(h)*MLHZ. 3)

Likewise for member end H, now HL i.s.o. LH.
FHLy= FHLY 4)
MHL**x**= Cos(h)*MHLX +Sin(h)*MHLZ 5)
MHL**z**= -Sin(h)*MHLX +Cos(h)*MHLZ 6)

Here below the first three equations written out in which all six variables.

```
FLHy=  1*FLHY        +0*MLHX        +0*MLHZ
      +1*FHLY        +0*MHLX        +0*MHLZ    1)
MLHx=  0*FLHY +Cos(h)*MLHX +Sin(h)*MLHZ
      +0*FHLY        +0*MHLX        +0*MHLZ    2)
MLHz=  0*FLHY -Sin(h)*MHLX +Cos(h)*MLHZ
      +0*FHLY        +0*MHLX        +0*MHLZ    3)
```

The second three written out in similar way. On the left represented in matrix form with C for Cos(h) and S for Sin(h).
Matrix T is the socalled transformation matrix.

u'		T							u
UVLy		1	0	0	0	0	0		FLHY
URLx		0	C	S	0	0	0		MLHX
URLz		0	-S	C	0	0	0		MLHZ
UVHy	=	0	0	0	1	0	0	.	FHLY
URHx		0	0	0	0	C	S		MHLX
URHz		0	0	0	0	-S	C		MHLZ

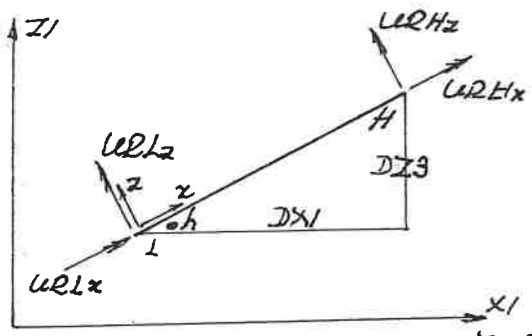


Fig.5a.

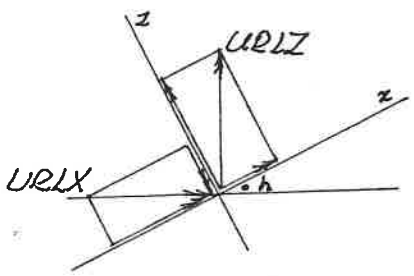


Fig.5b.

Fig.5a en 5b.
The displacements w.r.t. the x-y-z axis system can be drawn like moment vectors with the same assumed directions. They can be resolved into components w.r.t. the construction axis system X1-Y1-Z1, or X-Y-Z. On the left represented in matrix form u' = T u.

Axis system x-y-z, u' with
UVLy, URLx, URLz and UVHy, URHx, URHz.

Axis sytem X1-Y1-Z1, written X-Y-Z, with
UVLY, URLX, URLZ and UVHY, URHX, URHZ.

7.4. Determination of member stiffness matrix S5 with help of matrix manipulation.

On the preceding page was found f' = S u'. With

f' = T f and u' =T u follows T f = S T u.

On the left and on the right of the = sign is multiplied by the inverse T   of T, one gets

T T f = T S T u.

Further is T T = I (unity matrix) and

I f = f in (1) finally gives

f = T S T u or f = S5 u with S5 = T S T

Next siffness matrix S5 is found by two matrix multiplications.

First S is multiplied by T so that
P = S T (S times T, not T times S).
Element P(I,J)= row I of S times column J of T.
In matrix T is S=Sin(h) and C=Cos(h).

P(2,3) is row 2 of S times colum 3 of T.

P(2,3)= S(2,1)*T(1,3) +S(2,2)*T(2,3)
+S(2,3)*T(3,3) +S(2,4)*T(4,3)
+S(2,5)*T(5,3) +S(2,6)*T(6,3)

= 0*0 +T*S +0*T +0*0 +(-T)*0 +0*0= T*S

P(4,6)= S(4,1)*T(1,6) +S(4,2)*T(2,6)
+S(4,3)*T(3,6) +S(4,4)*T(4,6)
+S(4,5)*T(5,6) +S(4,6)*T(6,6)

= (-A)*0 +0*0 +(-B)*0 +A*0 +0*S +(-B)*C= -B*C

T

1	0	0	0	0	0
0	C	-S	0	0	0
0	S	C	0	0	0
0	0	0	1	0	0
0	0	0	0	C	-S
0	0	0	0	S	C

S

A	0	B	-A	0	B
0	T	0	0	-T	0
B	0	D	-B	0	E
-A	0	-B	A	0	-B
0	-T	0	0	T	0
B	0	E	-B	0	D

· T

1	0	0	0	0	0
0	C	S	0	0	0
0	-S	C	0	0	0
0	0	0	1	0	0
0	0	0	0	C	S
0	0	0	0	-S	C

= P

A	-B*S	B*C	-A	-B*S	B*C
0	T*C	T*S	0	-T*C	-T*S
B	-D*S	D*C	-B	E*S	E*C
-A	B*S	-B*C	A	B*S	-B*C
0	-T*C	-T*S	0	T*C	T*S
B	-E*S	E*C	-B	-D*S	D*C

On the preceding page was found
f = S5 u with S5 = T S T.

With P = S T is S5 = T P the second
matrix multiplied shown here below.

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \underline{C} & \underline{-S} & 0 & 0 & 0 \\ 0 & \underline{S} & \underline{C} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \underline{C} & \underline{-S} \\ 0 & 0 & 0 & 0 & \underline{S} & \underline{C} \end{bmatrix}}_{T} \underbrace{\begin{bmatrix} A & -B*S & B*C & -A & -B*S & B*C \\ 0 & T*C & \underline{T*S} & 0 & -T*C & -T*S \\ B & -D*S & D*C & -B & E*S & E*C \\ -A & B*S & -B*C & A & B*S & -\underline{B*C} \\ 0 & -T*C & -T*S & 0 & T*C & T*S \\ B & -E*S & E*C & -B & -D*S & D*C \end{bmatrix}}_{P} = Q$$

A= 12*EI/L^3

B= 6*EI/L^2

D= 4*EI/L

E= 2*EI/L

T= G*Ip/L

C= Cos(h) S= Sin(h)

$$\underbrace{\begin{bmatrix} FLHY \\ MLHX \\ MLHZ \\ FHLY \\ MHLX \\ MHLZ \end{bmatrix}}_{\underline{f}} = \underbrace{\begin{bmatrix} A & -B*S & B*C & -A & -B*S & B*C \\ -B*S & T*C^2+D*S^2 & T*S*C-D*S*C & B*S & -T*C^2+E*S^2 & -T*S*C-E*S*C \\ B*C & T*S*C-D*S*C & T*S^2+D*C^2 & -B*C & -T*S*C-E*S*C & -T*S^2+E*C^2 \\ -A & B*S & -B*C & A & B*S & -B*C \\ -B*S & -T*C^2+E*S^2 & -T*S*C-E*S*C & B*S & T*C^2+D*S^2 & T*S*C-D*S*C \\ B*C & -T*S*C-E*S*C & -T*S^2+E*C^2 & -B*C & T*S*C-D*S*C & T*S^2+D*C^2 \end{bmatrix}}_{S5 \text{ is } Q} \cdot \underbrace{\begin{bmatrix} UVLY \\ URLX \\ URLZ \\ UVHY \\ URHX \\ URHZ \end{bmatrix}}_{\underline{u}}$$

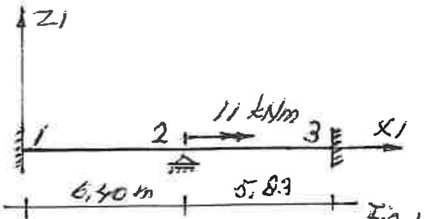


Fig. 1.

$$\begin{bmatrix} FLHX \\ MLHX \\ MLHZ \\ \\ FHLY \\ MHLX \\ MHLZ \end{bmatrix} = \begin{bmatrix} A & W1 & W2 & -A & W1 & W2 \\ . & W3 & W4 & -W1 & W5 & W6 \\ . & . & W7 & -W2 & W6 & W8 \\ \\ . & . & . & A & -W1 & -W2 \\ . & . & . & . & W3 & W4 \\ . & . & . & . & . & W7 \end{bmatrix} \cdot \begin{bmatrix} UVL \\ URLX \\ URLZ \\ \\ UVH \\ URHX \\ URHZ \end{bmatrix}$$

Member 1.

With EI=1 and GIp=1, so GIp= 1*EI.
All rotations expressed in EI.
L1=6,40 m   C=1   S=0   C^2=1   S^2=0

A=12EI/L^3= 12EI/6,40^3= 0,046 *EI
B= 6EI/L^3= 6EI/6,40^2 = 0,146 *EI
D= 4EI/L = 4EI/6,40 = 0,625 *EI
E= 2EI/L = 2EI/6,40 = 0,313 *Ei
T= GIp/L= 1*EI/6,40= 0,156 *EI

W1= -B*S= -0,146(0)= 0
W2= B*C= 0,146(1)= 0,146 *EI

W3= T*C^2+D*S^2= 0,156(1)+0= 0,156 *EI
W4= T*S*C-D*S*C= 0,156(0)+0= 0 *EI
W5=-T*C^2+E*S^2=-0,156(1)+0=-0,156 *EI

W6=-T*S*C-E*S*C=-0,156(0)-0= 0 *EI
W7= T*S^2+D*C^2=
  0,156(0)+0,625(1)= 0,625 *EI
W8=-T*S^2+E*C^2=
  -0,156(0)+0,313(1)= 0,313 *EI

Member 2.

With EI=1 and GIP=1, so GIP= 1*EI.
L1=5.83 m   C=1   S=0   C^2=1   S^2=0

A= 0,061 B=0,177 D=0,686 *EI
E= 0,343
T=GIp/L= 1*EI/5,83= 0,172 *EI

W1= 0   W2= 0,177   W3= 0,172 *EI
W4= 0   W5=-0,172   W6= 0 *EI
W7= 0,686   W8= 0,313

$$\begin{bmatrix} F12Y \\ M12X \\ M12Z \\ \\ F21Y \\ M21X \\ M21Z \end{bmatrix} = S51 \begin{bmatrix} UV1 \\ UR1X \\ UR1Z \\ \\ UV2 \\ UR2X \\ UR2Z \end{bmatrix} \qquad \begin{bmatrix} F23Y \\ M23X \\ M23Z \\ \\ F32Y \\ M32X \\ M32Z \end{bmatrix} = S52 \begin{bmatrix} UV2 \\ UR2X \\ UR2Z \\ \\ UV3 \\ UR3X \\ UR3Z \end{bmatrix}$$

Example.

Fig.1.
Two coinciding members with both bending stiffness EI and torsion stiffness GIp=1.

X1(1)= 0 : Z1(1)= 0   X1(2)= 6,40   Z1(2)= 0
  X1(3)= 12,23   Z1(3)= 0 m
Member 1. D1=X1(2)-X1(1)= 6,40 -0,00= 6,40
  D3=Z1(2)-Z1(1)= 0,00 -0,00= 0,00 m

L1= 6,40 m   C=D1/L1= 6,40/6,40= 1
  S=D3/L1= 0,00/6,40= 0

$$\begin{bmatrix} F12Y \\ M12X \\ M12Z \\ \\ F21Y \\ M21X \\ M21Z \end{bmatrix} = \begin{bmatrix} 46 & 0 & 146 & -46 & 0 & 146 \\ 0 & 156 & 0 & 0 & -156 & 0 \\ 146 & 0 & 625 & -146 & 0 & 313 \\ \\ -46 & 0 & -146 & \underline{46} & \underline{0} & \underline{-146} \\ 0 & -156 & 0 & \underline{0} & \underline{156} & \underline{0} \\ 146 & 0 & 313 & \underline{-146} & \underline{0} & \underline{625} \end{bmatrix} \begin{bmatrix} UV1 \\ URX1 \\ URZ1 \\ \\ UV2 \\ URX2 \\ URZ2 \end{bmatrix}$$

x EI/1000   S51

$$\begin{bmatrix} F23Y \\ M23X \\ M23Z \\ \\ F32Y \\ M32X \\ M32Z \end{bmatrix} = \begin{bmatrix} \underline{61} & \underline{0} & \underline{177} & -61 & 0 & 177 \\ \underline{0} & \underline{172} & \underline{0} & 0 & -172 & 0 \\ \underline{177} & \underline{0} & \underline{686} & -177 & 0 & 343 \\ \\ -61 & 0 & -177 & 61 & 0 & -177 \\ 0 & -172 & 0 & 0 & 172 & 0 \\ 177 & 0 & 343 & -177 & 0 & 686 \end{bmatrix} \begin{bmatrix} UV2 \\ URX2 \\ URZ2 \\ \\ UV3 \\ URX3 \\ URZ3 \end{bmatrix}$$

x EI/1000   S52

Prescribed are vertical joint displacement UV1=0, UV2=0 and UV3=0, and joint rotation URX1=0, URZ1=0, URX3=0 and URZ3=0.
Unknown are URX2 and URZ2.
On joint 2 acts a torsion moment of 11 kNm to the right.
There are no member loads.
Finally one equation remains with the unknown rotation URX2.

With a part of construction matrix CC follows, with addition of the underlined elements of the submatrices S51 and S52,

$$\begin{bmatrix} 107 & 0 & 31 \\ 0 & 328 & 0 \\ 31 & 0 & 1311 \end{bmatrix} \begin{bmatrix} UV2 \\ UR2X \\ UR2Z \end{bmatrix} \quad \begin{bmatrix} 0 \\ 11 \\ 0 \end{bmatrix}$$

x EI/1000

But UV2=0 and UR2Z=0, remains equation
0,328*UR2X= 11 from which UR2X= 33,54 /EI.

With S51 and S52 follow, EI omitted,
M21X= 0,156(33,54)= 5,23 kNm and
M23X= 0,172(33,54)= 5,77 kNm.

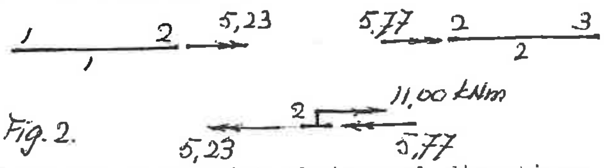


Fig. 2.

The moments drawn with their real directions shows that joint 2 is in equililibrium.

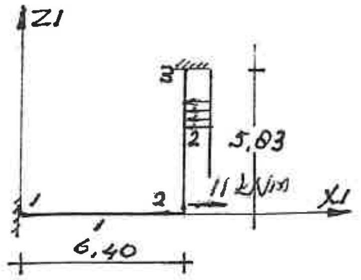


Fig.1.

Member 2.

With EI=1 and GIp=1, so GIp= 1*EI.
All rotations expressed in EI.

X1(2)= 6,40 Z1(2)= 0 m
X1(3)= 6,40 Z1(3)= 5,83 m
D1=X1(3)-X1(2)= 6,40-6,40= 0 m
D3=Z1(3)-Z1(2)= 5,83-0,00= 5,83 m

L1=5,83 m
C=D1/L1= 0,00/5,83= 0 G^2=0
S=D3/L1= 5,83/5,83= 1 S^2=1

A= 0,061 B=0,177 D=0,686 *EI
E= 0,343 T=GIP/L=1*EI/5,83= 0,172 *EI

W1= -B*S= -0,177(1)= -0,177 *EI
W2= B*C= 0,177(0)= 0

W3= T*C^2+D*S^2= 0+0,686(1)= 0,686 *EI
W4= T*S*C-D*S*C= 0 -0 = 0 *EI
W5=-T*C^2+E*S^2= 0+0,343(1)= 0,343 *EI

W6=-T*S*C-E*S*C= 0 -0 = 0 *EI
W7= T*S^2+D*C^2= 0,172(1)+0= 0,172 *EI
W8=-T*S^2+E*C^2=-0,172(1)+0=-0,172 *EI

Member end forces and member end moments unequal zero. EI omitted.

Member 1 without member load.

M12X= -0,156(2,97)= -0,46 kNm
M21X= 0,156(2,97)= 0,46 kNm
Only a torsion moment 0,47 kNm.

Member 2 with member load 3 kN/m.

F23Y= -0,177(2,97)= -0,53 kN
M23X= 0,686(2,97)= 2,04 kNm
F32Y= 0,177(2,97)= 0,53 kN
M32X= 0,343(2,97)= 1,02 kNm
These are forces and moments due to the displacenments alone. Those due to the member load will be added.

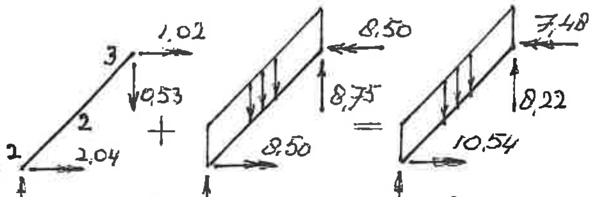


Fig.3

The member is loaded by bending, not by torsion.

Example.

Fig.1.
Two not coinciding members, both with bending stiffness EI and torsion stiffness GIp=1*EI. Member ends 1 and 3 are clamped, joint 2 is vertically supported.

Member 1 like on the preceding page.

F12Y	=	46	0	146	-46	0	146	·	UV1
M12X		0	156	0	0	-156	0		UR1X
M12Z		146	0	625	-146	0	343		UR1Z
F21Y		-46	0	-146	46	0	-146		UV2
M21X		0	-156	0	0	156	0		UR2X
M21Z		146	0	343	-146	0	625		UR2Z

x EI/1000 S51

F23Y	=	61	-177	0	-61	-177	0	·	UV2
M23X		-177	686	0	177	343	0		UR2X
M23Z		0	0	172	0	0	-172		UR2Z
F32Y		-61	177	0	61	177	0		UV3
M32X		-177	343	0	177	686	0		UR3X
M32Z		0	0	-172	0	0	172		UR3Z

x EI/1000 S52

Fig.2.
Member 2 with a vertical uniformly distributed load of 3 kN/m.

The clamp moments are (1/12)*3*5,83^2= 8,50 kNm represented with moment vectors.
Further reaction forces (1/2)*3*5,83= 8,75 kN.

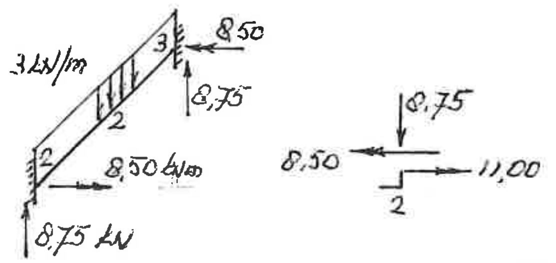


Fig 2

Joint displacement UV1=0, UV2=0 and UV3=0.
Joint rotation UR1X=0, UR1Z=0, UR3X=0, UR3Z=0, concerning equations fall off.

Joint rotation UR2X and UR2Z are unknown.
Here below a part matrix of construction matrix CC as addition of the underlined elements of the part matrices of S51 and S52.
F2Y= -8,75 kN, M2X= 11,00-8,50= 2,50 kNm.

107	-177	-146	UV2	-8,75
-177	842	0	UR2X	2,50
-146	0	797	UR2Z	0,00

x EI/1000

UV2=0, the equation falls off..

0,842*UR2X +0*UR2Z= 2,50 UR2X= 2,97/EI [1]

0*UR2X +0,797*UR2Z= 0,00 UR2Z= 0,00/EI

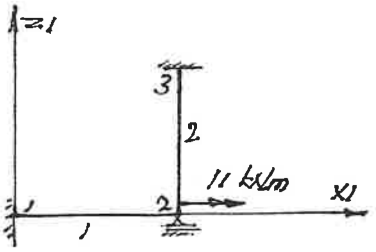


Fig.4.

Alone joint load moment 11 kNm. [2]

F12Y=	0	F23Y=	-2,31 kN
M12X=	-2,04 kNm	M23X=	8,96 kNm
M12Z=	0	M23Z=	0
F21Y=	0	F32Y=	2,31 kN
M21X=	2,04 kNm	M32X=	4,48 kNm
M21Z=	0	M32Z=	0

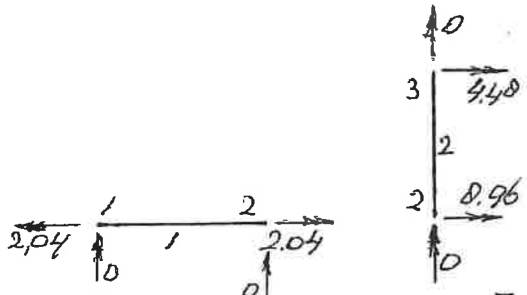


Fig.5.

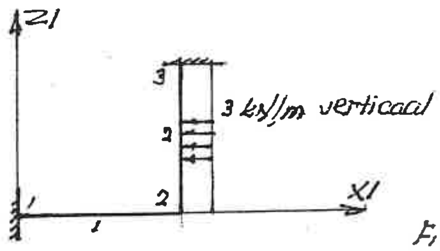


Fig.6.

Alone q-load 3 kN/m. [3]

F12Y= 0
M12X= 1,58 kNm
M12Z= 0
F21Y= 0
M21X= -1,58 kNm
M21Z= 0

F23Y= 1,79 +8,75= 10,54 kN
M23X= -6,93 +8,50= 1,57 kNm
M23Z= 0
F32Y= -1,79 +8,75= 6,96 kN
M32X= -3,46 -8,50= -11,96 kNm
M32Z= 0

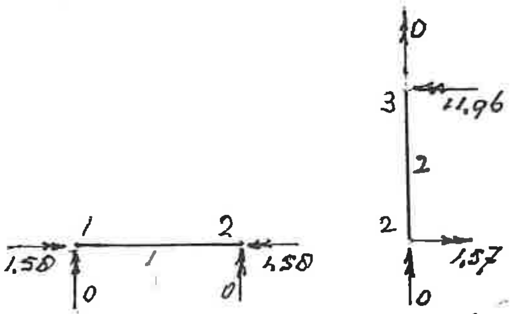


Fig.7.

Fig.4.
Like on the preceding page. No vertical distributed load on member 2, joint 2 is loaded with the moment of 11 kNm.
Two not coinciding members, both with bending stiffness EI=1 and torsion stiffness GIp=1, GIp=1*EI.
Member ends 1 and 3 are clamped, joint 2 is vertically supported with a possibility of horizontal displacement according X1- and Y1 axis in the horizontal plane.
The relation between member end forces and joint displacements like on the preceding page.

Now alone M2X= 11,00 kNm.
Joint rotation UR2X and UR2Z are unknown.Here below the part matrix of construction matrix CC as addition of the underlined elements of the part matrices of S51 and S52.

$$\begin{bmatrix} 107 & -177 & -146 \\ -177 & 842 & 0 \\ -146 & 0 & 797 \end{bmatrix} \begin{bmatrix} UV2 \\ UR2X \\ UR2Z \end{bmatrix} \begin{bmatrix} 0,00 \\ 11,00 \\ 0,00 \end{bmatrix}$$

x EI/1000

0,842*UR2X = 11,00 UR2X= 13,06/EI [2]

Fig.5.
The separated members with the member end moments. Member 1 loaded with torsion, with member end rotation
(2,04*6,40)/GIP is 13,06/EI rad.
Member 2 only loaded by bending.
The rotation of member end 2 (formula page) is

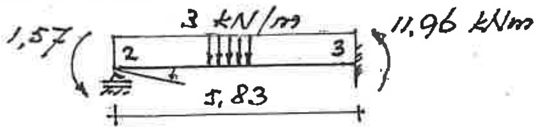
(8,96*5,83)/4EI= 13,06/EI is UR2X.

Fig.6.
Next the same construction with only the vertical distributed load of 3 kN/m for which is found on the preceding page the primary load for joint 2 8,50 kNm and 8,75 kN.

$$\begin{bmatrix} 107 & -177 & -146 \\ -177 & 842 & 0 \\ -146 & 0 & 797 \end{bmatrix} \begin{bmatrix} UV2 \\ UR2X \\ UR2Z \end{bmatrix} \begin{bmatrix} -8,75 \\ -8,50 \\ 0,00 \end{bmatrix}$$

0,842*UR2X = -8,50 UR2X= -10,10/EI [3]

Fig.7.
Member 1 alone loaded with torsion with member end rotation (1,58*6,40)/GIP is 10,11/EI rad.
Member 2 alone loaded with bending..



(1,57*5,83)/4EI -(3*(5,83^3))/48EI=

1,57/EI -12,38/EI= -10,09/EI is 'to the right'!

With F23Y= 10,54 kN and F32Y= 6,96 kN is the separated member in equilibrium.

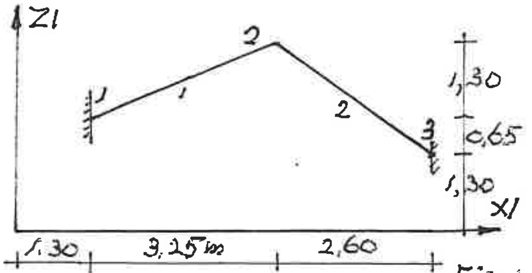


Fig.1.

Member 1. L1=3,50 m C=0,929 S=0,371

T=GIP/3,50= 0,77EI/3,50= 0,220 *EI

A=0,280 B=0,490 D=1,143 E=0,571

W1=0,182 W2=0,455 W3=0,347 W4=-0,318
W5=-0,111 W6=-0,273 W7=1,014 W8=0,462

Member 2. L2=3,25 m C=0,800 S=-0,600

T=GIp/3.25= 0,77EI/3,25= 0,237 *EI

A=0,350 B=0,568 D=1,231 E=0,615

W1=0,341 W2=0,454 W3=0,595 W4=0,477
W5=0,069 W6=0,409 W7=0,873 W8=0,309

The here below drawn member loads in reality vertically directed, perpendiculare to the plane of drawing. The lttle arcs as well.

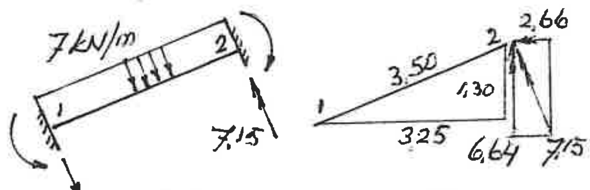


Fig.2.

M12=(1/12)*7*3,50^2= 7,15 kNm
M21= 7,15 kNm

The components of M21.
(7,15/3,50)*1,30= 2,66 kNm
(7,15/3,50)*3,25= 6,64 kNm

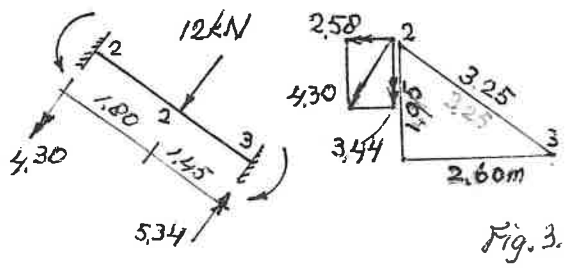


Fig.3.

M23=(12*1,45^2*1,80)/(3,25^2)= 4,30 kNm
M32=(12*1,45*1,80^2)/(3,25^2)= 5,34 kNm

The components of M23.
(4,30/3,25)*1,95= 2,58 kNm
(4,30/3,25)*2,60= 3,44 kNm

Example.

Fig.1.
Two members clamped at the ends 1 and 3 'twofold' against bending and torsion. The members are tubes with same cross-section.
Moments of inertia Ix=Iy, is I, the polar moment of inertia Ip= 2I.
Modulus of elaticity E=210000 N/mm^2
Shear m. of elaticity G= 81000 N/mm^2
Bending stiffness EI= 210000*I Nmm^2
Torsion stiffness GIp= 81000*2I Nmm^2

(GIp/EI)= (162000*I)/(210000*I) or

GIP/EI= 0,77 so that GIp= 0,77EI.

$$\begin{bmatrix} F12Y \\ M12X \\ M12Z \\ F21Y \\ M21X \\ M21Z \end{bmatrix} = \left[\begin{array}{ccc|ccc} 280 & -182 & 455 & -280 & -182 & 455 \\ -182 & 347 & -318 & 182 & -111 & -273 \\ 455 & -318 & 1014 & -455 & -273 & 462 \\ \hline -280 & 182 & -455 & 280 & 182 & -455 \\ -182 & -111 & -273 & 182 & \underline{347} & \underline{-318} \\ 455 & -273 & 462 & -455 & \underline{-318} & \underline{1014} \end{array}\right] \cdot \begin{bmatrix} UV1 \\ URX1 \\ URZ1 \\ UV2 \\ \underline{URX2} \\ \underline{URZ2} \end{bmatrix}$$

x EI/1000 S51

$$\begin{bmatrix} F23Y \\ M23X \\ M23Z \\ F32Y \\ M32X \\ M32Z \end{bmatrix} = \left[\begin{array}{ccc|ccc} 350 & 341 & 454 & -350 & 341 & 454 \\ 341 & \underline{595} & \underline{477} & -341 & 69 & 409 \\ 454 & \underline{477} & \underline{873} & -454 & 409 & 309 \\ \hline -350 & -341 & -454 & 350 & -341 & -454 \\ 341 & 69 & 409 & -341 & 595 & 477 \\ 454 & 409 & 309 & -454 & 477 & 873 \end{array}\right] \cdot \begin{bmatrix} UV2 \\ \underline{URX2} \\ \underline{URZ2} \\ UV3 \\ URX3 \\ URZ3 \end{bmatrix}$$

x EI/1000 S52

Rotations URX2 and URZ2 of joint 2 are unequal zero, all others are zero.
After composing S51 and S52 to CC two equations remain to solve.
Addition of the concerning underlined elements of S51 and S52 follow with
347+595= 942 and -318+477= 159,
-318+477= 159 and 1014+873= 1887.

$$\begin{bmatrix} 942 & 159 \\ 159 & 1887 \end{bmatrix} \begin{bmatrix} UR2X \\ UR2Z \end{bmatrix} \begin{bmatrix} 5,24 \\ -3,20 \end{bmatrix}$$

(CC) $\underline{u}$ $\underline{f}$

Fig.2, 3 and 4.
The elements of $\underline{f}$ are the joint load moments of joint 2. Needed therefore are the components of the (primary) member end moments. On joint 2 acting as large as but opposite directed.

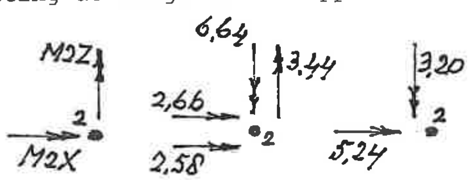
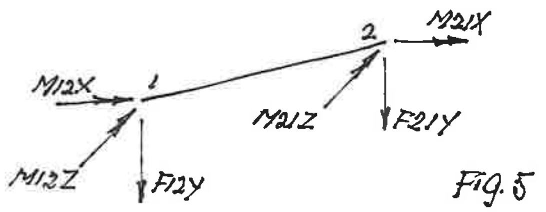


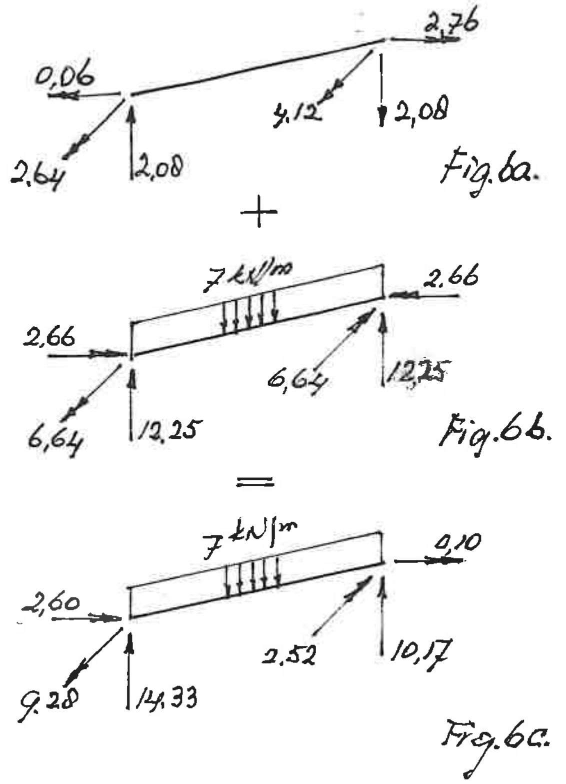
Fig.4.

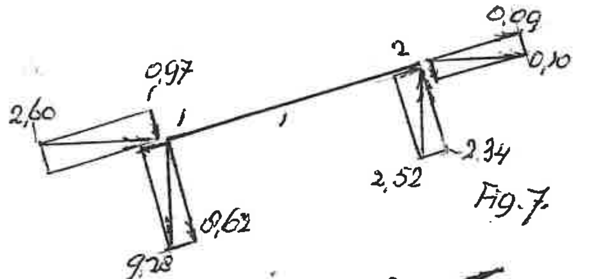
M2X=2,66+2,58=5,24 kNm M2Z=3,44-6,64=-3,20 kNm

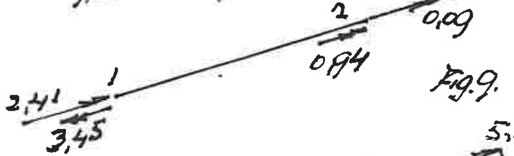
0,942*UR2X +0,159*UR2Z= 5,24
0,159*UR2X +1,887*UR2Z= -3,20
Solution with computer Gauss gives

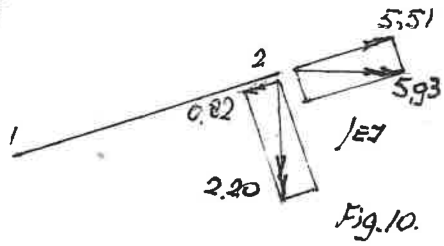
UR2X= 5,93/EI and UR2Z= -2,20/EI.











(5,93/3,50)*3,25= 5,51 /EI
(2,20/3,50)*1,30= <u>0,82 /EI</u>
4,69 /EI

Fig.5.
Assumed directions of member end forces and moments.

Fig.6a.
Calculation of member end forces and moments due to the displacements <u>alone</u>.

F12Y= -0,182(5,93) +0,455(-2,20)= -2,08 kN
M12X= -0,111(5,93) -0,273(-2,20)= -0,06 kNm
M12Z= -0,273(5,93) +0,462(-2,20)= -2,64 kNm

F21Y= 0,182(5,93) -0,455(-2,20)= 2,08 kN
M21X= 0,347(5,93) -0,318(-2,20)= 2,76 kNm
M21Z= -0,318(5,93) =1,014(-2,20)= -4,12 kNm

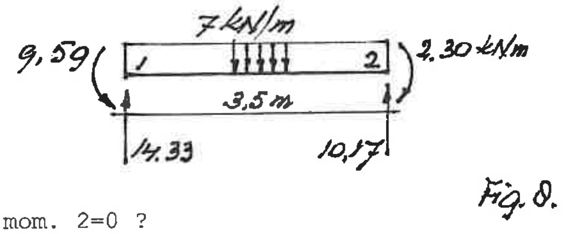
Forces and moments in the figures drawn with their real directions.

Fig.6b.
The member end forces and moments due to the loads <u>alone</u>. Values earlier found on the preceding page. The vertical reaction forces are zijn (7*3,50)/2= 12,25 kN.

Fig.6c is 6a+6b.
The final member end forces and member end moments. The moment vectors of figure 5c are not perpendicular to or along the member axis.

Fig.7.
The components perpendicular on the member, see figure 5c. Not sketched on scale...
(2,60/3,50)*1,30= 0,97 (0,10/3,50)*1,30= 0,04
(9,28/3,50)*3,25= 8,62 (2,52/3,50)*3,25= 2,34
Follows perpendicular on the member
at member end 1 0,97+8,62= 9,59 kNm and
at member end 2 2,34-0,04= 2,30 kNm.

Fig.8.
In the figure are the moments with little arcs drawn with real direction.



Σ mom. 2=0 ?
24,5*1,75+2,30-9,59-10,17*3,5=
42,88+2,30-9,59-35,60= 45,18-45,19= -0,01 OK

Fig.9.
The components along the member. See fig.7.
(2,60/3,50)*3,25= 2,41 (0,10/3,50)*3,25= 0,09
(9,28/3,50)*1,30= <u>3,45</u> (2,52/3,50)*1,30= <u>0,94</u>
Difference 1,04 and 1,03

Torsion stiffness GIp= 0,77EI, see page 88.
The rotation w.r.t. torsion at member end 2 is (1,03*3,50)/GIP= 3,61/0,77EI= <u>4,69/EI</u>.

Fig.10.
Calculated UR2X= 5,93/EI and UR2Z= -2,20/EI, drawn as moment vectors with their real directions. Their components alog the member axis together 5,51/EI-0,82/EI= <u>4,69/EI</u>.

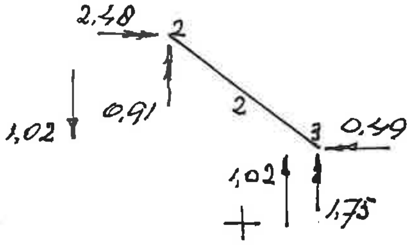


Fig. 11a.

Fig.11a.
Calculation of the member end forces and moments due to the displacements alone.

F23Y= 0,341(5,93) +0,454(-2,20)= 1,02 kN
M23X= 0,595(5,93) +0,477(-2,20)= 2,48 kNm
M23Z= 0,477(5,93) +0,873(-2,20)= 0,91 kNm

F32Y= -0,341(5,93) -0,454(-2,20)= -1,02 kN
M32X= 0,069(5,93) +0,409(-2,20)= -0,49 kNm
M32Z= 0,409(5,93) +0,309(-2,20)= 1,75 kNm

In the figure drawn with their real direction. Th forces F23Y and F32Y are the vertical support reactions. (Y1-as page .)

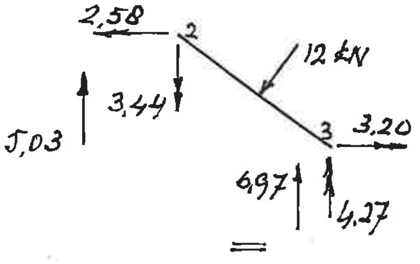


Fig. 11b.

Fig.11b.
The member end forces and moments due to the loads alone. See page with the values for member end 2. For member end 3 is
M32=(12*1,45*1,89^2)/(3,25^2)= 5,34 kNm
with components
(5,34/3,25)*1,95= 3,20 kNm and
(5,34/3,25)*2,60= 4,27 kNm.
Fig.11c is 11a+11b.
The final member end forces and member end moments due to displacements and loads.

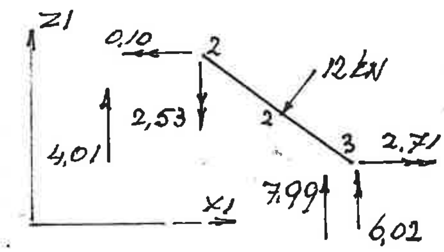


Fig. 11c.

Fig.12.
(x/2,60)=(1,45/3,25) x=2,60(1,45,3,25)= 1,16 m

(y/1,95)=(1,80/3,25) y=1,95(1,80/3,25)= 1,08 m

For lengths of line pieces of figure 13.

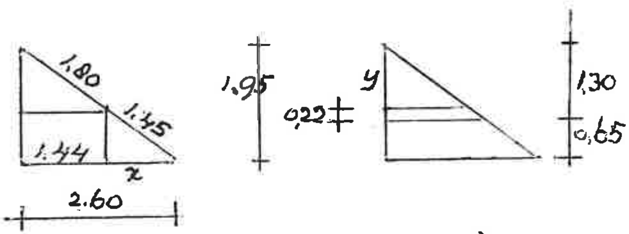


Fig. 12.

Fig.13.
The construction with loads and clamp reactions see fig.6c page and fig.11c.
De constructie met belastingen en inklemmingsreacties, zie fig.6c blz. en fig.11c.
The clamp moments
2,60 kNm and 9,28 kNm at member end 1 and
2,71 kNm and 6,02 kNm at member end 3.

The loads are perpendicular to the plane of drawing, vertical support reactions as well
17,33 kN of fig.6c page ,
7,99 kN of fig.11c and
14,18 kN is 10,17 kN fig.6c + 4,01 kN fig.11c.

Σ mom. w.r.t. clamp 1 (Z1)= 0?
Moments 'about an axle through 1' parallel to Z1.

7*3,5*(3,25/2) +12*4,69 -14,18*3,25 -7,99*5,85=

39,81 +56,28 -46,09 -46,74 = 3,26 kNm.

The vertical moment vectors 9,28 and 6,02 kNm are added, aware of the directions follows

3,26 -9,28 +6,02= 0 !! Equilibrium

Σ mom. w.r.t. clamp 1 (X1)= 0?
7*3,5*0,65 +12*0,22 -14,18*1,30 +7,99*0,65=

15,93 +2,64 -18,43 +5,19 = 5,33 kNm
plus the moment vectors at 1 and 3,
5,33 -2,60 -2,71= 0,02 ... Equilibrium.

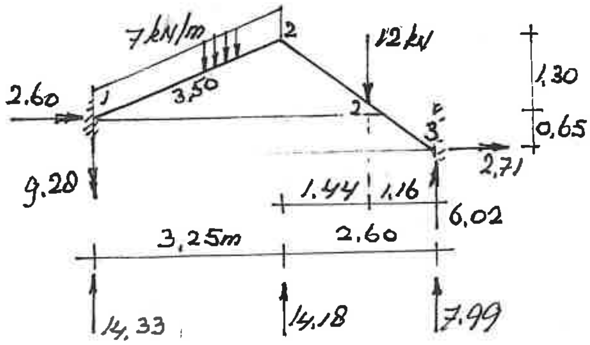


Fig. 13.

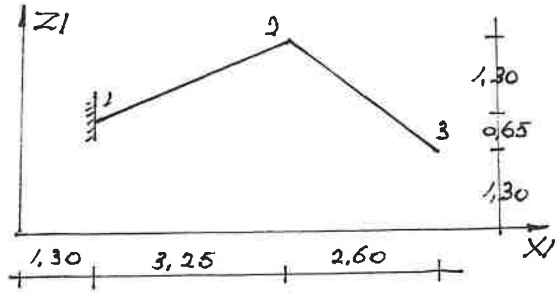


Fig.14.

Fig.14.
The same construction but now member end 3 is not clamped (not twofold clamped, beding and torsion). Now arise member end rotations UR3X and UR3Z, the member end is like a hinge. There is however no stiffness matrix derived with a member end as a hinge. Therefore the member end is regarded as a 'real hinge' like done for on bending loaded members, page . So that the same member stiffness matrices of page can be applied which put together form the construction matrix CC shown here below.

			1	2	3	4	5	6	7	8	9			
1	F12Y		280	-182	455	-280	-182	455					UV1	-
2	M12X		-182	347	-318	182	-111	-273					UR1X	-
3	M12Z		455	-318	1014	-455	-273	462					UR1Z	-
4	F21Y+F23Y		-280	182	-455	630	523	-1	-350	341	454		UV2	-
5	M21X+M23X	=	-182	-111	-273	523	942	159	-341	69	409	·	UR2X	5,24
6	M21Z+M23Z		455	-273	462	-1	159	1887	-454	409	309		UR2Z	-3,20
7	F32Y					-350	-341	-454	350	-341	-454		UV3	-
8	M32X					341	69	409	-341	595	477		UR3X	-3,20
9	M32Z					454	409	309	-454	477	873		UR3Z	-4,27

x EI/1000

Prescribed are the displacements UV1=0, UR1X=0, UR1Z=0, UV2=0 and UV3=0. The unknown displacements, rotations, to be calculated are UR2X, UR2Z, UR3X and UR3Z.
The elements of force vector f are the joint load moments of fugure 2 and 3 page .

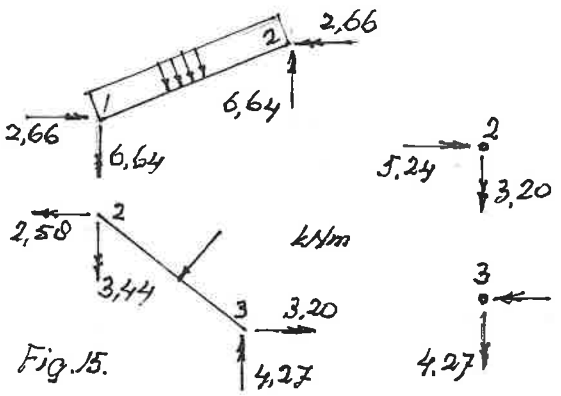


Fig.15.

Fig.15 see page .
The joint load moments are M2X and M3X assumed directed to the right, like X1 axis, M2Z and M3Z assumed directed upward like Z1 axis.

M2X= 2,66+2,58= 5,24 kNm
M2Z= 3,44-6,64= -3,20 kNm

M3X= -3,20 kNm and M3Z= -4,27 kNm.

The other elements of f are given no value because the concerning displacements are known, all zero.

The known displacements are U1X=0, U1Y=0, U4X=0, U4Y=0 and UR4=0. Four equations remain with the unknowns UR2X, UR2Z, UR3X and UR3Z. With computer Gauss the here below underlined values are found. In the equations 0,942/EI and 0,159/EI etc. EI in the equations here below omitted.

1)	0,942*UR2X	+0,159*UR2Z	+0,069*UR3X	+0,409*UR3Z= 5,24	UR2X= 10,34/EI
2)	0,159*YR2X	+1,887*UR2Z	+0,409*UR3X	+0,309*UR3Z= -3,20	UR2Z= -1,47/EI
3)	0,069*UR2X	+0,409*UR2Z	+0,595*UR3X	+0,477*UR3Z= -3,24	UR3X= 3,24/EI
4)	0,409*UR2X	+0,209*UR2Z	+0,477*UR3X	+0,873*UR3Z= -4,27	UR3Z= -10,99/EI

Member 1.

F12Y=
(-0,182EI)(10,34/EI)+0,455EI(-1,47/EI)=
-1,88 -0,67 = -2,55 kN
M12X=
-0,111EI(10,34/EI) -0,273EI(-1,47/EI)=
-1,15 +0,40 = -0,75 kNm
M12Z=
-0,273EI(10,34/EI) +0,462EI(-1,47/EI)=
-2,82 -0,68 = -3,50 kNm

Similar way, F21Y= 2,55 kN
M21X= 4,06 kNm and M21Z= -4,78 kNm.

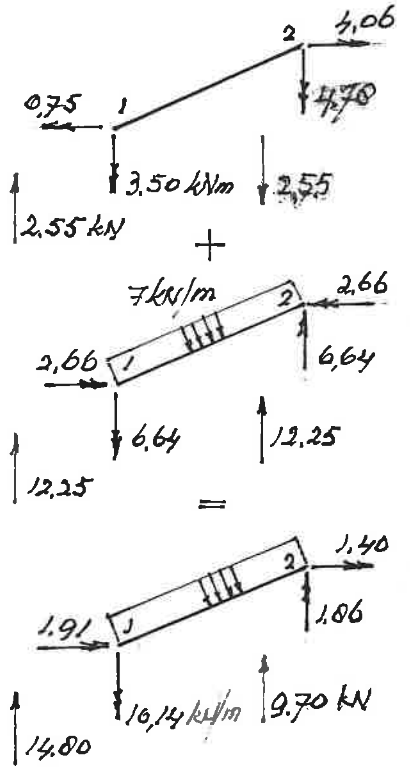


Fig.16a. Fig.16b. Fig.16c

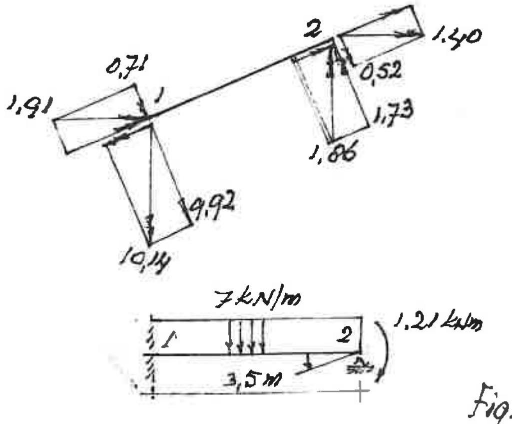


Fig.17.

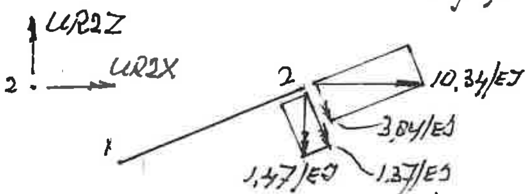


Fig.18.

Fig.16a.
The member end forces and moments for member 1 due to the displacements/rotations UV1, UR1X, UR1Z, UV2, UR2X and UR2Z alone.
Orescribed UV1=0, UR1X=0, UR1Z=0 and UV2=0.
Further UR2X= 10,34/EI and UR2Z=-1,47/EI found on the preceding page.
With member stiffness matrix S51 of member 1 of page are written out on the left the first three equations, EI not omitted.
For member 1 appear in displacement vector u alone UR2X and UR2Z unequal zero.

See fig.5 page with the assumptions for forces and moments.
Drawn are the member end moments with their real directions. The vertical reachtions are here drawn in the horizontal X1-Z1 plane.
F12Y= -2,55 kN, negative answer, not downward like the Y1 axis as assumed but upward.
F21Y= 2,55 kN, positive answer, as assumed downward.

Fig.16b.
See fig. . Moments and forces due to the load alone.

Fig.16c.
The final member end forces and moments due to displacements and load.

Fig.17 en 16c.
Not properly on scale... it's about the values of the components of the moments.
Perpendicular to the member axis, in kNm.

(1,91/3,50)*1,30= 0,71	(1,40/3,50)*1,30=0,52
(10,14/3,50)*3,25= 9,92	(1,86/3,50)*3,25=1,73
together 10,13	together 1,21

Also drawn in the vertical plane. With the given vertical load now the slope deflection/ member end rotation due to bending can be calculated at member end 2, see formuls page .
The member clamped on the left.

Due to 1,21 kNm (1,21*3,50)/(4*EI)= 1,06/EI
Due to 7 kN/m (7*3,50^3)/48*EI)= 6,25/EI
Answer 6,25-1,06= 5,19/EI to the left.

Fig.18.
Was found UR2X= 10,34/EI and UR2Z= -1,47/EI, in the figure the vectors are drawn with their real directions. They can be resolved perpendicular to the member axis.

((10,34/EI)/3,50)*1,30= 3,84/EI
((1,47/EI)/3,50)*3,25= 1,37/EI
together 5,21/EI to the left

like found in figure 17, 5,21/EI ≈ 5,19/EI OK.

The components at 2 along the member axis give a 'torsion angle' 9,15/EI.
9,15/EI= (M*3,50/GIp)= (M*3,50/0,77EI) page from which torsion moment M=(9,15/3,50)*0,77= 2,01 kNm. The components along the member axis of the moments at member end 2, figure 16c, deliver together 1,99 kNm, OK.

Member 2.

It now concerns the four member end rotations UR2X= 10,34/EI, UR2Z= -1,47/EI, UR3X= 3,24/EI and UR3Z= -10,99/EI. With member stiffness matrix S5 of page   the first three equations are written out. EI is omitted.

F23Y= 0,341*UR2X +0,454*UR2Z +0,341*UR3X +0,454*UR3Z =

F23Y= 0,341(10,34) +0,454(-1,47) +0,341(3,24) +0,454(-10,99)=
3,53   -0,67   +1,10   -4,99   = 4,63 -5,66 = -1,03 kN

M23X= 0,595(10,34) +0,477(-1,47) +0,069(3,24) +0,409(-10,99)=
6,15   -0,70   +0,22   -4,49   = 6,37 -4,68 = 1,18 kNm

M23Z= 0,477(10,34) +0,873(-1,47) +0,409(3,24) +0,309(-10,99)= 6,26 -4,68 = 1,58 kNm

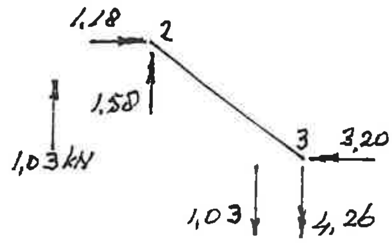


Fig. 19a.

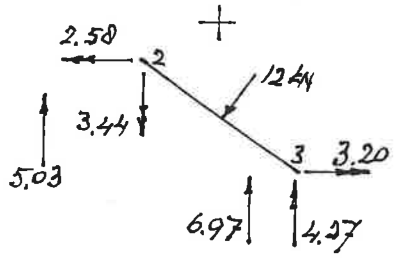


Fig. 19b.

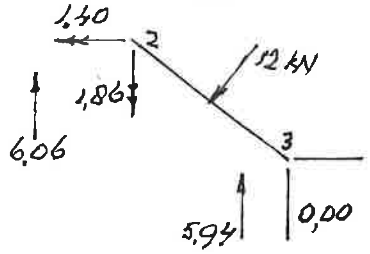


Fig. 19c.

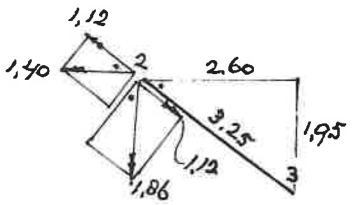


Fig. 20.

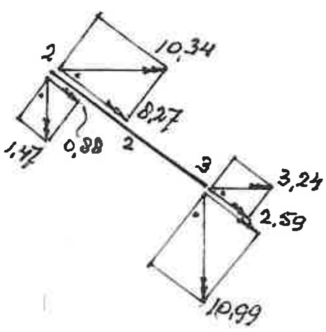


Fig. 21.

Fig.19a.
Due to the displacements UR2X, UR2Z, UR3X and UR3Z alone, UV2=0 and UV3=0.

Fig.19b.
Due to the load, see figure 3 page   .
The load force of 12 kN is in reality perpendicular to the plane of drawing, the drawn forces as well, shear forces 5,03 kN and 6,97 kN. Then is 5,03+6,97= 12,00 kN, equilibrium.

Fig.19c is 19a + 19b.
The final member end forces and moments also now drawn with their real direction. At member end 2 no moments, no clamp there.

Fig.20.
At member end 3 no clamp, no torsion moment, no torsion in membner 2.
Components of the moments along the member axis at member end 2. Member length 3,25 m.

(1,40/3,25)*2,60= 1,12 kNm
(1,86/3,25)*1,95= 1,12 kNm Indeed, both as large as but opposite directed, zero, no torsion moment at member end 2.

Fig.21.
Components of the member end rotaties, or joint rotations, UR2X and UR2Z along the member axis.

((10,34/EI)/3,25)*2,60= 8,27/EI
((1,47/EI)/3,25)*1,95= 0,88/EI
together 9,15/EI

If member 2 is not loaded by torsion, then the components of UR3X and UR3Z must be at member end 3 also together 9,15/EI.

((3,24/EI)/3,25)*2,60= 2,59/EI
((10,99/EI)/3,25)*1,95= 6,59/EI
together 9,18/EI ≈ 9,15/EI OK!!

At member end 3 of member 2 no moments.
But there is a bending moment at member end 2, the sum of the components of 1,40 and 1,86 kNm perpendicular to the member axis.

(1,40/3,25)*1,95= 0,84 kNm
(1,86/3,25)*2,60= 1,49 kNm
together 2,33 kNm

5.1. Solution of N equations with the elimination method of GAUSS.

The given set of three equations with three unknowns will be converted into an upper triangle set.
One may read this way, A11X1 as A11 times X1, 2X1 as 2 times X1, etc.

```
A11X1 +A12X2 +A13X3= B1     1)
A21X1 +A22X2 +A23X3= B2     2)
A31X1 +A32X2 +A33X3= B3
```

```
 2X1 +  3X2 +  4X3=  8     1)
 5X1 + 10X2 + 15X3= 30     2)
 3X1 +  6X2 +  5X3= -2     3)
```

Elimination of X1 from eq. 2) and 3).

X1 from eq. 2).
Equation 1) is divided by A11 is 2, and multiplied by A21 is 5,
(A21/A11) is 5/2, times eq. 1).

(5/2)(2X1 + 3X2 + 4X3=8) gives

5X1 +7,5X2 + 10X3= 20. This equation is subtracted from eq. 2). One finds eq. 2'). X1 is eliminated from eq. 2), A21 has become zero.

```
           5X1 + 10X2 + 15X3= 30     2)
(5/2)*1)   5X1 +7,5X2 + 10X3= 20 -
                2,5X2 +  5X3= 10     2')
```

X1 from eq. 3).
Equation 1) is divided by A11 is 2, and multiplied by A31 is 3,
(A31/A11) is 3/2, times eq. 1).

(3/2)(2X1 + 3X2 + 4X3=8) gives

3X1 +4,5X2 + 6X3= 12. This equation is subtracted from eq. 3). One finds eq. 3'). X1 is eliminated from eq. 3), A31 has become zero.

```
           3X1 +  6X2 +  5X3= -2     3)
(3/2)*1)   3X1 +4,5X2 +  6X3= 12 -
                1,5X2 -   X3=-14     3')
```

Elimination of X2 from equation 3).

Equation 2') is divided by the 'new' A22 is 2,5 and multiplied by A32 is 1,5 of eq. 3').
(A32/A22) is 1,5/2,5 times eq. 2').

```
 2X1 +  3X2 +  4X3=  8     1)
      2,5X2 +  5X3= 10     2')
      1,5X2 -   X3=-14     3')
```

(1,5/2,5)/(2,5X2 + 5X3= 10) gives

1,5X2 + 3X3= 6. This equation is subtracted from eq. 3'). One finds wq. 3'').
X2 is eliminated from eq. 3', A32 has become zero.

```
                1,5X2 -   X3=-14     3')
(1,5/2,5)*2')   1,5X2 +  3X3=  6 -
                      -  4X3=-20     3'')
```

This way three equations are found from which the three unknowns can be solved by backward substitution. One starts with the last equation and works back to the first equation.

Backward substitution.

From eq. 3'') follows X3=-20/A33.
And with A33=-4 is X3=-20/-4=5.

```
 2X1 +  3X2 +  4X3=  8     1)
      2,5X2 +  5X3= 10     2')
            -  4X3=-20     3'')
```

In the beginning was A33=5, became in eq. 3') A33=-1, and in eq. 3'') A33=-4.

From eq 2') follows X2=(10-5X3)/A22 or
X2=(10-5(5))/2,5=-15/2,5=-6.

```
A11X1 +A12X2 +A13X3=  8
       A22X2 +A23X3= 10
              A33X3=-20
```

From eq.1 follows X1=(8-4X3-3X2)/A11 or
X1=(8-4(5)-3(-6))/2=(8-20+18)/2=6/2=3.

Solution: X1=3   X2=-6   X3=5

```
Private Sub GAUSS()
NNG=0
For K=1 To N-1
If AA(K,K)=0 Then
T=0 : For I=K+1 To N
If AA(I,K)<>0 Then
T=1 : For J=K To N
R=AA(K,J) : AA(K,J)=AA(I,J)
AA(I,J)=R
Next J
R=BB(K) : BB(K)=BB(I) : BB(I)=R
Exit For
End If
Next I
If T=0 Then ... See next page.

End If
```

A11X1 +A12X2 +A13X3 +A14X4= B1 1)

A22X2 +A23X3 +A24X4= B2 2')

A32X2 +A33X3 +A34X4= B3 3')

A42X2 +A43X3 +A44X4= B4 4')

$$\underset{A}{\begin{bmatrix} A11 & A12 & A13 & A14 \\ 0 & A22 & A23 & A24 \\ 0 & A32 & A33 & A34 \\ 0 & A42 & A43 & A44 \end{bmatrix}} \underset{\underline{x}}{\begin{bmatrix} X1 \\ X2 \\ X3 \\ X4 \end{bmatrix}} \underset{\underline{b}}{\begin{bmatrix} B1 \\ B2 \\ B3 \\ B4 \end{bmatrix}}$$

I=3 Suppose AA(I,K)=AA(3,2)=0
Then follows Next I.
I=4 Supp. AA(4,2)<>0 then the elements J=K To N=2 To 4 of row K=2 and row I=4 of matrix AA, are exchanged,
T wordt T=1.

J=2
R=AA(2,2) AA(2,2)=AA(4,2) AA(4,2)=R
AA(2,2) has become <>0.
J=3
R=AA(2,3) AA(2,3)=AA(4,3) AA(4,3)=R
J=4
R=AA(2,4) AA(2,4)=AA(4,4) AA(4,4)=R

R=BB(2) BB(2)=B(4) BB(4)=R

```
'The elimnation process.
For I=K+1 To N

V=AA(I,K)/AA(K,K) : NNG=NNG+1
For J=K To N : NNG=NNG+1
AA(I,J)=AA(I,J)-V*AA(K,J)
Next J
BB(I)=BB(I)-V*BB(K) : NNG=NNG+1
Next I

Next K
```

Private Sub GAUSS()

The solution of N equations with the elimina= tion method of Gauss.

For K=1 To N-1 the elimination process will be carried out in the equations I=K+1 To N.
Suppose N=4 equations, then as follows:
if K=1 then X1 from eq. 2 to 4,
if K=2 then X2 from eq. 3 to 4,
if K=3 then X3 from equation 4.

In the program code the arrays are
AA(,) BB() and XX().
In the elimination process there is divided by the diagonal element AA(K,K), see left below.
So AA(K,K) can not be zero.
Is AA(K,K)=0 then in the column under AA(K,K) shall be searched for an element AA(I,K)<>0. As soon it is found equation K and I will be exchanged;
the elements J=K To N of the rows K and I of matrix AA, and
the elements K and I of vector/column BB.

Suppose the first elimination is done, see on the left, and that AA(K,K)=AA(2,2)=0. Then in the K=2nd column of AA for I=3 To 4 will e searched for an element AA(I,2)<>0.
Before it is assumed with T=0 (third line left above) that such an element is not found.
Is after Next I..........End If still T=0 then a solution is not possible and is the subroutine left with Exit Sub.

Has T become T=1 then an element AA(K,I)<> has been found, and after the exchange, the elimination process is carried out with the 'new' AA(2,2).

K=2 Elimination of X2 from eq. 3' en 4'.

For I=K+1 To N= 3 To 4

X2 from eq. I=3. (that's eq. 3')
The elements AA(K,J), AA(2,2) to AA(2,4) of equation K=2 (is 2'), are divided by AA(K,K) is AA(2,2) and multiplied by AA(I,K) is AA(3,2).

But first V=AA(I,K)/AA(K,K) is AA(3,2)/AA(2,2).

For J=K To N=2 TO 4 the elements AA(2,J) are multiplied by V and subtracted from the elements AA(3,J).

AA(I,J)=AA(I,J)-V*AA(K,J)

J=2 AA(3,2)=AA(3,2)-V*AA(2,2)
AA(3,2) has become zero.
J=3 AA(3,3)=AA(3,3)-V*AA(2,3)
V 'holds' AA(3,2) from before For J= K To N, so, not the AA(3,2) which became just zero.
J=4 AA(3,4)=AA(3,4)-V*AA(2,4) and after
Next J Follows

BB(I)=BB(I)-V*BB(K)
BB(3)=BB(3)-V*BB(2)

Tis way the third equation 3'' has arisen.

See preceding page.

```
If T=0 Then
CurrentX=900 : CurrentY=750
Print "No solution possible."
Exit Sub
End If
```

```
A11X1 +A12X2 +A13X3 +A14X4= B1      1)

      A22X2 +A23X3 +A24X4= B2      2')

             A33X3 +A34X4= B3      3'')

                    A44X4= B4      4''')
```

$$\begin{bmatrix} A11 & A12 & A13 & A14 \\ . & A22 & A23 & A24 \\ . & . & A33 & A34 \\ . & . & . & A44 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \\ X4 \end{bmatrix} \begin{bmatrix} B1 \\ B2 \\ B3 \\ B4 \end{bmatrix}$$

```
If AA(N,N)=0 Then Exit Sub

'Backward substitution.
G=N+1
For I=N To 1 Step-1 : S=0
If I<N THEN
For J=N To G Step-1 : S=0
S=S+AA(I,J)*XX(J)  : NNG=NNG+1
Next J
End If
G=G-1 : NNG=NNG+1
XX(I)=(BB(I)-S)/AA(I,I)
Next I

End Sub
```

X2 from eq. I=4.

V=AA(I,K)/AA(K,K)=AA(4,2)/AA(2,2)
For J=K To N=2 To N the elements AA(2,J) are multiplied by V and subtracted from the elements AA(4,J).

```
J=2   AA(4,2)=AA(4,2)-V*AA(2,2)
      AA(4,2) has become zero.
J=3   AA(4,3)=AA(4,3)-V*AA(2,3)
J=4   AA(4,4)=AA(4,4)-V*AA(2,4)

      BB(4)=BB(4)-V*BB(2)
```

This way arises eq. 4'').

Next follows the elimination of X3 from equation I=4 (is 4'') and arises eq. 4''').

Backward substitution.

After the elimination process N equations are arisen from which the N to 1 (counting back) unknowns can be solved, as is written out below for N=4.

```
I=4   X4={(B4 -(           0           ))}/A44
I=3   X3={(B3 -(A34X4                  ))}/A33
I=2   X2={(B2 -(A24X4 +A23X3           ))}/A22
I=1   X1={(B1 -(A14X4 +A13X3 +A12X2))}/A11
```

But first G=N+1.
And then the calculation of XX(I).

For I=N To 1 Step-1 : S=0
For each I sum S is determined with
For J=N To G Step-1 except when I=N.
If I<N Then .... sum S .... End If
After End If becomes G=G-1.
After that XX(I) is calculated for each I with
XX(I)=(BB(I)-S)/AA(I,I)

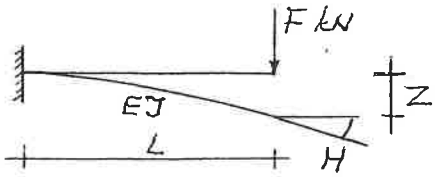
With N=4 is G=N+1=5

```
I=4   I<4?  no  S stays S=0
            XX(4)=(BB(4)-0)/AA(4,4)

I=3   I<4?  yes   J=N To G=4 To 4 Step-1
      J=4   S=0 +AA(3,4)*XX(4)      G=G-1=3

            XX(3)=(BB(3)-S)/AA(3,3)

I=2   I<4?  yes   J=N To G=4 To 3 Step-1
      J=4    S=0 +AA(2,4)*XX(4)
      J=3   S=S +AA(2,3)*XX(3)      G=G-1=2

            XX(2)=(BB(2)-S)/AA(2,2)

I=1   I<4?  yes   J=N To G=4 To 2 Step-1
      J=4   S=0 +AA(1,4)*XX(4)
      J=3   S=S +AA(1,3)*XX(3)
      J=2   S=S +AA(1,2)*XX(2)

            XX(1)=(BB(1)-S)/AA(1,1)
```

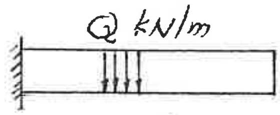
End Sub

Formulas for slope deflections and displacements.

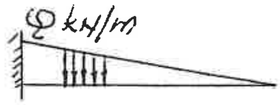
Seven standard formulas and several formulas for simple beams on two supports.
E is modulus of elasticity in kN/m^2
EI is bendimg stiffness, EI is E*I with
I moment of inertia in m^4,
EI is (kN/m^2)*m^4 is kNm^2.
EA is strain stiffness, EA is E*A with
A cross sectional area in m^2,
EA is (kN/m^2)*m^2 is kN.
With the formulas follow Z in m and H in radians.



H= F*L^2/(2*EI)   Z= F*L^3/(3*EI)

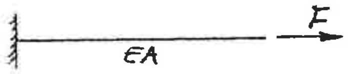


H= Q*L^3/(6*EI)   Z= Q*L^4/(8*EI)

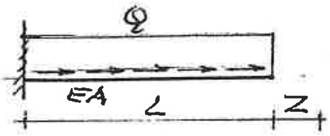


H= Q*L^3/(24*EI)   Z= Q*L^4/(30*EI)

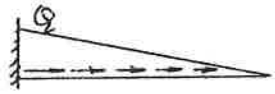
H= M*L/EI   Z= M*L^2/(2*EI)



Z=F*L/EA

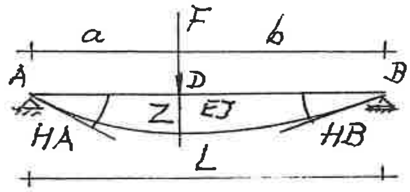


Z=Q*L^2/(2*EA)



Z=Q*L^2/(6*EA)

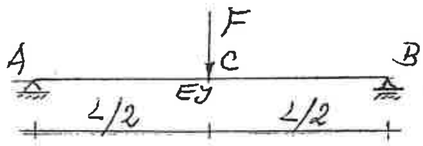
'forget-me-nots'



HA= F*a*b*(L+b)/(6*L*EI)

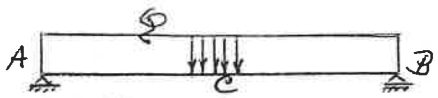
HB= F*a*b*(L+a)/(6*L*EI)

ZD= F*a^2*b^2/(3*L*EI)



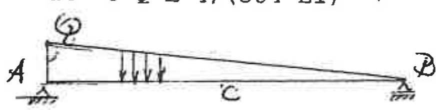
HA= HB= F*L^2/(16*EI)

ZC= F*L^3/(48*EI)



HA= HB= Q*L^3/(24*EI)

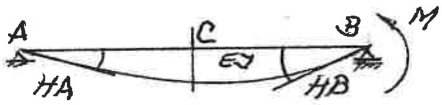
ZC= 5*Q*L^4/(384*EI)



HA= Q*L^3/(45*EI)

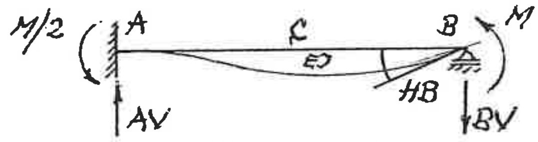
HB= 7*Q*L^3/(360*EI)

ZC= (5*Q*L^4/(384*EI))/2



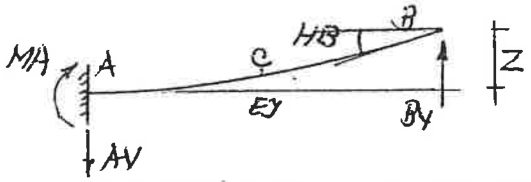
HA= M*L/(6*EI)   HB= M*L/(3*EI)

ZC= M*L^2/(16*EI)



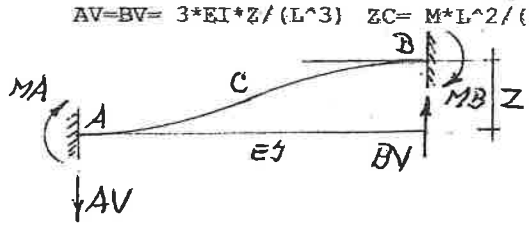
HB= M*L/(4*EI)   ZC= M*L^2/(32*EI)

AV=BV= 3*M/(2*EI)



MA=3*EI*Z/(L^2)   HB= 3*Z/(2*L)

AV=BV= 3*EI*Z/(L^3) ZC= M*L^2/(32*EI)



MA=MB= 6*EI*Z/(L^2)   ZC= Z/2

AV=BV= 12*EI*Z/(L^3)