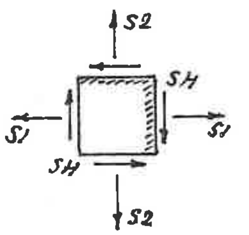
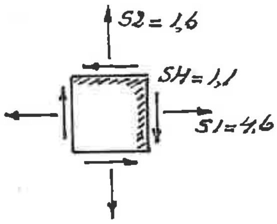
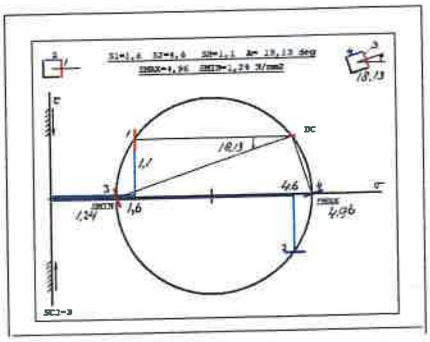
Plane stresses and Mohr's stress circle **without** Sign Conventions.

The assumed directions of normal stresses S1 and S2 and shear stress SH are shown here below. With these assumptions Mohr's stress circle is drawn, equations derived and written. Angle A assumed to the left.

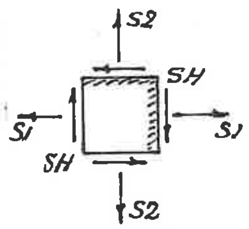


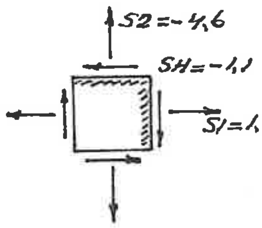


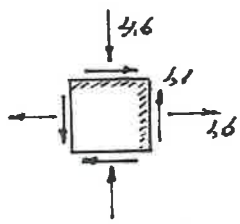


Suppose S1= 4,6 N/mm^2, S2= 1,6 N/mm^ 2 and SH= 1,1 N/mm^2.
The second figure with stresses drawn with their real directions with added values. (No plus signs!) After calculations follow the principal stresses SMAX and SMIN, with angle A=18,13 deg to the left and no shear stresses.

Suppose S1= 1,6 N/mm^2, S2= -4,6 N/mm^2 and SH= -1,1 N/mm^2.
First two figures here below stresses drawn with assumed directions.
Third figure with the stresses drawn with their real directions with added values. Without plus and minus sign!!







Third figure, S1= 1,6N/mm^2, positive value, directed as assumed.
S2= -4,6 N/mm^2, with a minus sign, so not directed as assumed but opposite directed, no minus sign added in this third figure!
SH= -1,1 N/mm^2, with a minus sign, so not directed as assumed but opposite directed, no minus sign added!

Drawing Mohr's stress circle becomes easy when omitting the usual Sign Conventions for Normal and Shear stresses, see page 13.
On following pages all explained in detail, writing computer code with Visual Basic 6.0. as well.

Plane stress state.

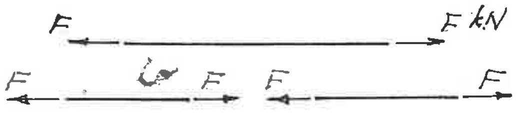


Fig. 1a.

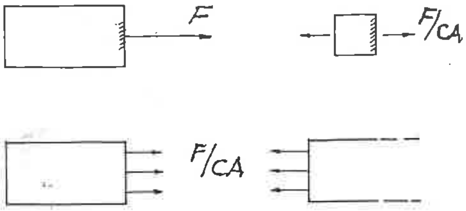
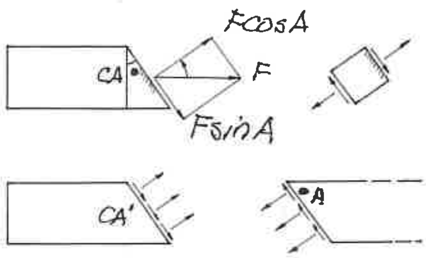


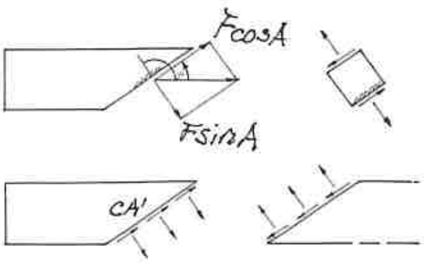
Fig. 1b.



NSA= F*(Cos(A)^2)/CA

SHA= F*(Sin(A)*Cos(A))/CA

Fig. 2a.



NSA90= F*(Sin(A)^2/CA

SHA90= F*(Sin(A)*Cos(A))/CA

Fig. 2b.

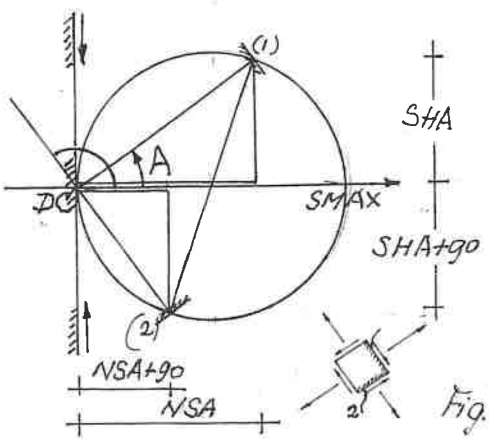


Fig. 3.

Pure tension, special case of plane stress.

Fig.1a.
A member axially loaded with F kN. The member is cut into two parts which are in equilibrium by the drawn forces.

Fig.1b.
Force F uniformly divided over cross section area CA of the left part gives normal stresses F/CA. On the right part as large as but opposite directed. No shear stresses here.
The stress at a point of the cross section of both parts can be represented by the sqaure drawn with the tensile stresses F/CA.

Fig.2a.
A cross section CA' with angle A. Force F is resolved into FcosA and FsinA.
With cosA= CA/CA' follows cross section area CA'= CA/cosA.

These two forces FcosA and FsinA uniformly divided over CA' give normal stresses
NSA= FcosA/CA'= FcosAcosA/CA
and shear stresses
SHA= FsinA/CA'= FsinAcosA/CA.
See the sqaure with normal and shear stress.
On the right part stresses as large as but opposite directed.

Fig.2b.
Next a cross section with angle A+90. Again force F resolved into FcosA and FsinA.
With sinA= CA/CA' follows cross section area CA'= CA/sinA.

Both forces FcosA and FsinA uniformly divided over CA'give normal stresses
NSA90= FsinA/CA'= FsinAsinA/CA and
shear stresses
SHA90= FcosA/CA'= FcosAsinA/CA.
Again, on the right as ;arge as but opposite directed. See the sqaure with normal and shear stresses combined with that of fig.2a.

Note that the shear stresses of fig.2a and 2b are equal, SHA= SHA90= FcosAsinA/CA.
Both cases can be represented by the square with angle A with normal and shear stresses shown on the left.

Fig.3.
One can calculate normal and shear stress for any angle A applying the formulas.
Or drawing the socalled stress circle of Mohr here shown for the case of pure tension, S2=0.
More explanation on following pages.

F/CA fig.1b is a principal stress, the second pricipal stress is zero. Pricipal stresses when shear stresses are zero. Later written as SMAX and SMIN, here SMAX=F/CA and SMIN=0.
DC is the direction centre. From there a line under angle A intersects the circle (1) and gives NSA and SHA. The diameter gives (2) with NSA90 and SHA90.

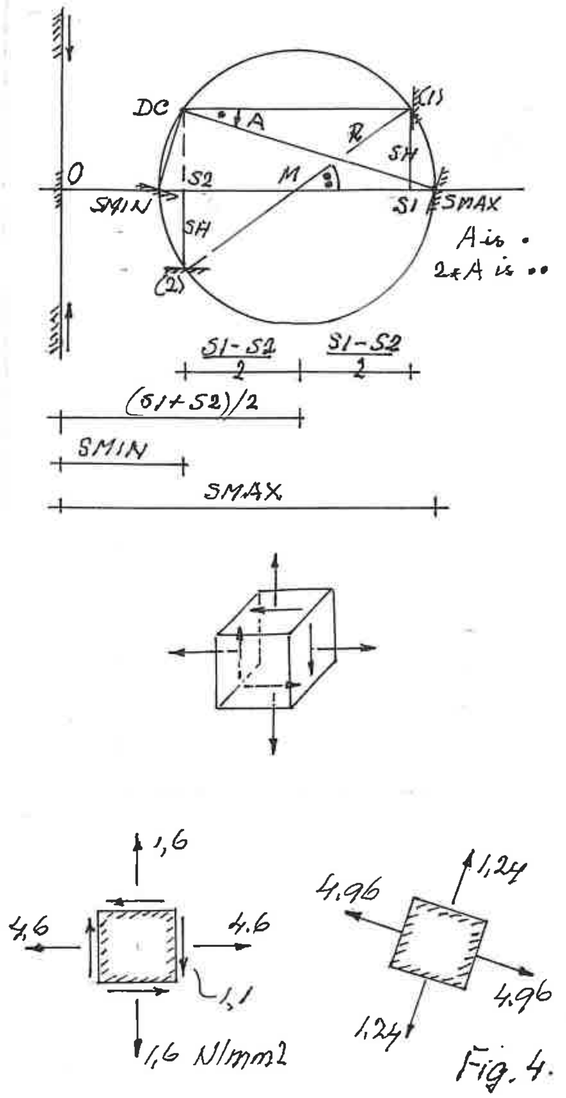


Fig. 4.

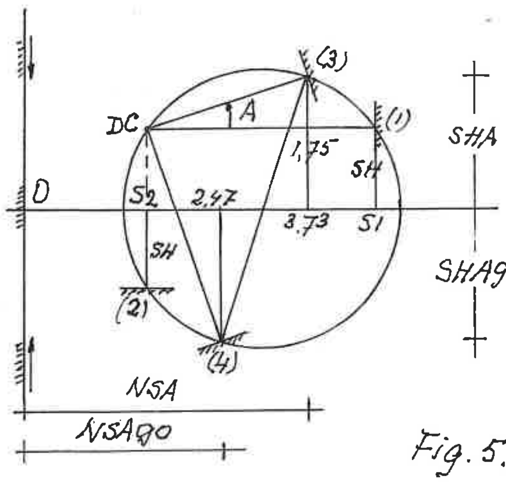


Fig. 5.

SHA= ((S1-S2)/2)*sin2A + SH*cos2A

= 1,5*0,559 + 1,1*0,829
= 0,84 + 0,91= 1,75 N/mm2

HSA90= -HSA. HSA90= -1,75 N/mm2

Fig.4.
Normal stresses S1= 4,6 and S2= 1,6 N/mm2, shear stress SH= 1,1 N/mm2.
Drawing the stress circle of Mohr, note the assumptions to plot shear stresses, ⇅ above and ⇅ below the zero line.
A little stress plane is turned to a position that the material side ⫼ is on the left, next the concerning stresses are plotted. S1 and SH give (1), the vertical plane is drawn in point (1) of the circle.

The horizontal plane is turned as mentioned, then SH is upward ⇈ . S2 and this SH give (2), the horizontal plane is drawn in point (2).
A line (1)-(2) gives the centre of the Mohr circle, the circle can be drawn. The so-called principal stresses SMAX and SMIN are found, their fornulas can be derived from the figure, here written in code.
SMAX= OM + R and SMIN= OM - R, with SMAX the 'largest' and SMIN the 'smallest' value.

R= Sqr(((S1-S2)/2)^2+SH^2)
 = Sqr(((4,6-1,6)/2)^2+1,1^2)= Sqr(3,46)= 1,86

SMAX= (S1+S2)/2 + R = 3,10+1,86= 4,96 N/mm2

SMIN= (S1+S2)/2 - R = 3,10-1,86= 1,24 N/mm2

A horizontal line perpendicular to the vertical plane in (1) and a vertical line perpendicular to the horizontal plane in (2) give intersection point DC, the direction centre.
A line from direction centre DC to principal stress SMAX gives angle A to the right, then A is negativ.
Geometry learns angle (1)-M-SMAX is 2*A, is two times angle (1)-DC-SMAX is A.
2*A to the right, so Tan(-2*A)=SH/((S1-S2)/2).

-2*A= Atn(SH/((S1-S2)/2))
 = Atn(1,1/((4,6-1,6)/2))=Atn(0,7333) gives
-2*A= 36,25 so that angle A=-18,13 degrees.

Fig.5.
A line through DC A=17 degrees to the left.
Like done above arise (3) and (4) determining normal stresses NSA, NSA90, and shear stresses SHA and SHA90 by drawing the concerning lines.
With the Mohr formulas, next page, the values of these stresses are calculated.
With 2A=2*17= 34 degrees.

NSA= (S1+S2)/2 + ((S1-S2)/2)*cos2A - SH*sin2A

 3,10 + 1,5*0,829 - 1,1*0,559
= 3,10 + 1,24 - 0,61 = 3,73 N/mm2

With A+90 is 2*A=34+180=214 degrees,
 cos2A= -0,829 and sin2A= -0,559

NSA90= 3,10 + 1,5*(-0,829) - 1,1*(-0,559)
 = 3,10 - 1,24 + 0,61 = 2,47 N/mm2

See shear stress calculations SHA and SHA90 given on the left.
In point (3) and (4) the stress planes under A=17 and A=17+90 degrees are drawn.

Determining the Mohr formulas for normal stress NSA and shear stress SHA.

Fig.6a.
Like fig.5 of the preceding page with its assumptions, S1>0, S2>0, S1>S2, SH>0. With angle A to the left to find Mohr's formula for NSA.
Fig.6b and 6c.
Triangle (1)-M-S1 of fig. 5a. is turned to the left over 2A degrees. The rectangular triangle gets a new position. Drawing some lines, see here below, and applying some geometry rules the formula for NSA can be written, in code, with OM=(S1+S2)/2

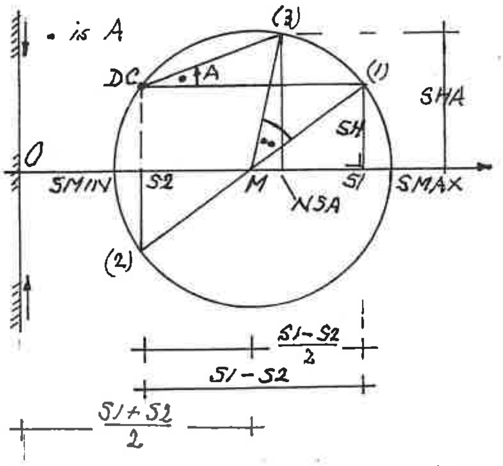


Fig.6a.

NSA=   OM + ((S1-S2)/2)*Cos(2*A) - SH*Sin(2*A).

Same formula for NSA90 with 2*(A+90), or with 2*A, then exchange + and -,

NSA90= OM - ((S1-S2)/2)*Cos(2*A) + SH*Sin(2*A).

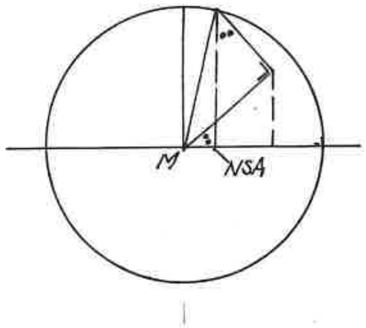


Fig. 6b.

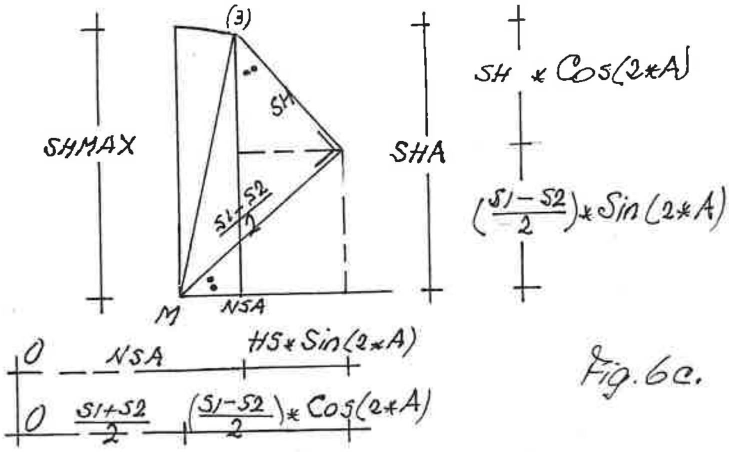


Fig.6c.

The shear stress formula, for SHA ,

SHA   =   ((S1-S2)/2)*Sin(2*A) + SH*Cos(2*A).

Same formula for SHA90 with 2*(A+90), or just SHA90=-SHA.

Fig.7, see fig.5 prec. page.
Here shear stress SH is plotted opposie way, SH of S1 below the zero line and SH of S2 above the zero line. Following the way of drawing the circle etc. direction centre DC is found.
With angle A=17 degrees to the left, the intersection points donot show results like fig.5. NSA and HSA calculated applying the formulas give NSA= 4,95 not 3,73 N/mm2. SHA= -0,07 not 1,75 N/mm2. And the principle/main axes of SMAX and SMIN are wrong, see circle of fig.4.

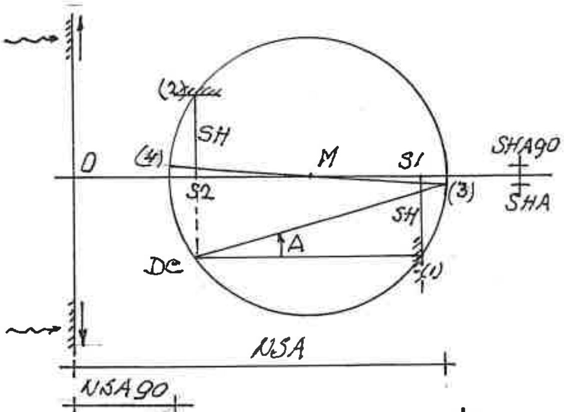


Fig.7.

Example.
S1=70, S2= -15 and SH   = -35 N/mm2.
With angle A=-30 degrees, that's to the right.
NSA=   OM + ((S1-S2)/2)*Cos(2*A) - SH*Sin(2*A).
   =   27,5 + 42,5*0,5 - (-35)*(-0,866)
   =   27,5 + 21,3 -30,3 = 18,5 N/mm2
NSA90= OM - ((S1-S2)/2)*Cos(2*A) + SH*Sin(2*A).
   = 27,5 - 21,3 +30,3 = 36,5 N/mm2
SHA =   ((S1-S2)/2)*Sin(2*A) + SH*Cos(2*A).
   = 42,5*(-0.866)+(-35)*0,5 = -54,3 N/mm2

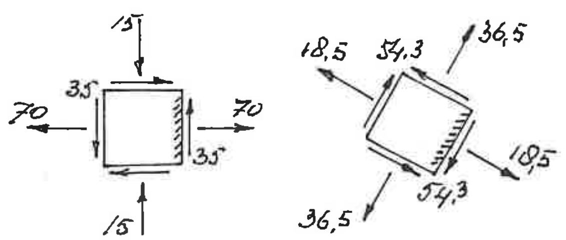
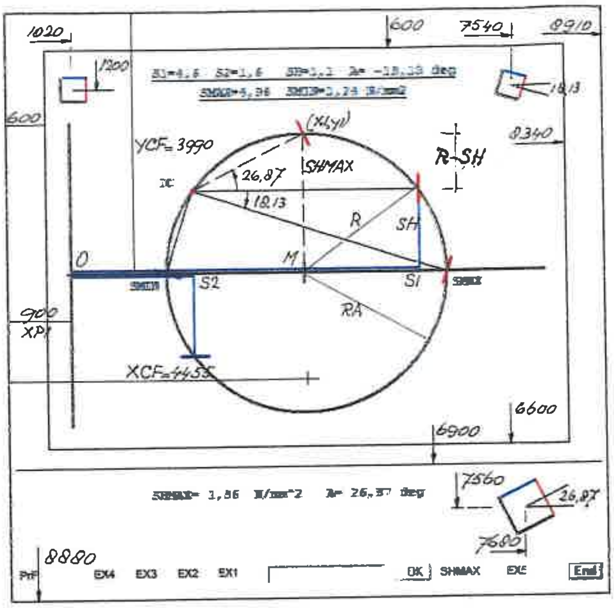


Fig.8.




```
Private Sub Forn_Load()
Height=8940 : Width=8955 : Top=0
Left=(Screen.Width-Width)/2
DRAWLINES                  page
FontName="Courier New"
FontSize=10
End Sub
```

```
Private Sub DRAWLINES()
'large frame
Line (30,30)-(8925,30)
Line (30,8880)-(8925,8880)
Line (30,45)-(30,8880)
Line (8910,45)-(8910,8880)

'single line
Line (120,6900)-(8835,6900)

'rectangle
line (600,600)-(8340,600)
Line (600,6600)-(8340,6600)
Line (600,600)-(8340,6600)
Line (8340,600)-(8340,6615)
End Sub
```

```
Private Sub COK_Click()
A1=0
T1=TSTRING.Text : NC=Len(T1)
TTSTRINGTE

If TE=3 Or TE=4 Then
S1=Val(TTSTRING(1))
S2=Val(TTSTRING(2))
SH=Val(TTSTRING(3))
A1=Val(TTSTRING(4))

ElseIf TE=1 Then
A1=Val(TTSTRING(1))
End If

RA=2100
If S1<>0 Or S2<>0 Or SH<>0 Then
STRESSCIRCLE1
End If
End Sub
```

STRESSCIRCLEMOHR

The results printed (half size) on the left appear for S1, S2, SH in text box TSTRING with 4.6,1.6,1.1 Ebnter. And clicking SHMAX. See page 8 where this circle is explained.

Private Sub COK_Click()

For a given angle A1, at the start A1=0, normal and shear stresses can be calculated, page 3 . Only if one of the three possible stresses is <>0 the the stress circle can be drawn with STRESSCIRCLE1 page 5 .

```
Private Sub LSHMAX_Click()
CALCSHMAX
End Sub
```

```
Private Sub CALCSHMAX()
If (S1-S2)=0 Then Exit Sub
If S1<>0 Or S2<>0 Then
R=Sqr((S1-S2)/2)^2+SH^2) : SHMAX=R
A=Atn(2*(R-SH)/(S1-S2))          'radians
XX=7680 : YY=7560
W=300 : LH=720 : SQUARE2            page 7
X1=XCF : Y1=YCF-RA
DrawWidth=1 : DrawStyle=1
Line (X1,Y1)-(XDC,YDC)
'small short stripes
DrawWidth=3
S=210*Sin(A) : C=210*Cos(A)
X3=X1-S : Y3=Y1-C
X4=X1+S : Y4=Y1+C
Line (X3,Y3)-(X4,Y4),vbRed
DrawWidth=1 : DrawStyle=0

A=A*180/3.14159                   'degrees
CurrentY=CY : CurrentX=2100        page 8
Print "SHMAX= " & _
Format(SHMAX,"0.00") & _
"  N/mm2" & "   A= " & _
Format(A,"0.00") & " deg"
End If
End Sub
```

With CALCSHMAX maximum shear stress SHMAX is calculated, it is R in the stress circle. In the circle drawn it's RA.

Two lines with stripes are drawn, XS3 and YS3 of Direction Centre DC, and X1 and Y1 at the top of the circle, and from top to (XCF,YCF). For the short red and blue stripes, explained later page 6 .
Calculations are carried out with radians, results printed in degrees. PI=3.14159.
2*PI rad is 360 deg, 1 rad is 180/PI deg.

```
Private Sub TTSTRINGTE()
TE=0 : E=""
For K=1 To NC
CR=Mid(T1,K,1)
If CR<>"," Then E=E & CR
If CR="," Or K=NC Then
TE=TE+1 : TTSTRING(TE)=E : E=""
End If
Next K
End Sub
```

```
Private Sub STRESSCIRCLE1()
Cls
DRAWLINES                    page 4
'circle centre point
DrawWidth=2
XCF=4445 : YCF=3990
Line (900,YCF)-(8010,YCF)
Circle (XCF,YCF),RA
Line (XCF,YCF-150)-(XCF,YCF+150)
'
R=Sqr(((S1-S2)/2)^2+SH^2)
If S1=0 And S2=0 And SH<>0 Then
A=3.14159/4     'rad,  is 45 deg
If SH<0 Then A=-A
Else
A=(-Atn(SH/((S1-S2)/2))/2) 'rad
End If

XS=(S1-S2)/2
XS1=XCF+XS*(RA/R)
YS1=YCF-SH*(RA/R)
XS2=XCF-XS*(RA/R)
YS2=YCF+SH*(RA/R)
XDC=XCF-XS*(RA/R)
YDC=YCF-SH*(RA/R)
XS4=XCF+RA : YS4=YCF
XS5=XCF-RA : YS5=YCF

DrawWidth=1
Line (XDC,YDC)-(XS1,YS1)    (3-1)
Line (XDC,YDC)-(XS4,YS4)    (3-4)
Line (XDC,YDC)-(XS5,YS5)    (3-5)

'vertical SH zero line
XP1=XS2-S2*(RA/R)
YP1=YCF-RA-180
XP2=XP1 : YP2=YCF+RA+180

If XP1<900 Or XP1>8010 Then
DrawWidth=1 : DrawStyle=2
If XP1<900 Then _
XP1=900 : XP2=XP1
If XP1>8010 Then _
XP1=8010 : XP2=XP1
Line (XP1,YP1)-(XP2,YP2)
Else
DrawWidth=2
Line (XP1,YP1)-(XP2,YP2)
End If
DrawWidth=2 : DrawStyle=0
'blue lines for S1 and SH
B=45 : If SH<0 Then B=-45
CL=vbBlue          (vert. and hor.)
Line (XS1,YS1)-(XS1,YCF-B),CL    v
Line (XP1,YCF-B)-(XS1,YCF-B),CL  h

Line (XS2,YS2)-(XS2,YCF+B),CL    v
Line (XP1,YCF+B)-(XS2,YCF+B),CL  h

DrawWidth=5
PSet (XDC,YDC),vbRed             (3)
DrawWidth=2

CurrentY=YDC-240
If Abs(XDC)<Abs(XS1) Then
CurrentX=XDC-480 : Print "DC"
Else
CurrentX=XDC+270 : Print "DC"
End If
```

STRESSCIRCLE1

Using explanations of preceding pages. Pressing Enter after input of S1, S2 and SH the stress circle is printed after clearing the form with Cls and drawing lines with DRAWLINES. XCF and YCF are coordinates of the centre of the circle, the horizontal zero line with Line (900,YCF)-(8010,YCF) and the circle with Circle (XCF,YCF),RA and in the centre a small vertical line Line (XCF,YCF+150)-(XCF,YCF-150). RA=2100 twips is 2,54*2100/1440=3,7 cm. Angle A the calculated angle for the directions of the principal/main stresses SMAX and SMIN.

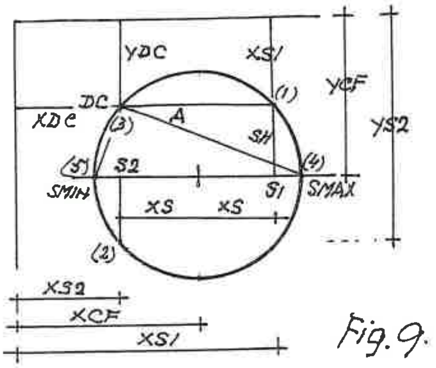
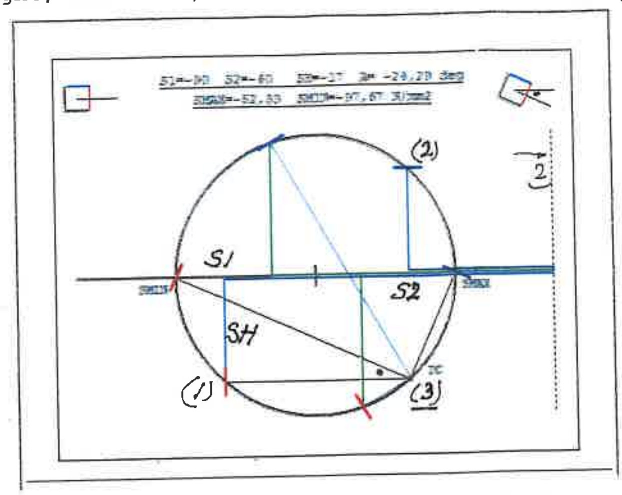


Fig. 9.

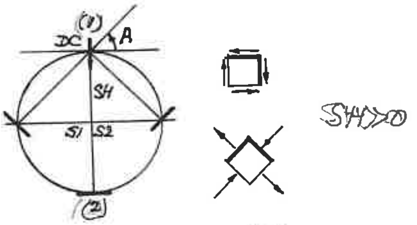
All circles drawn have the same size with radius RA=2100 twips. To draw that size the calculated R of the stress circle will be given the RA value by multiplying the stress values of the stress coircle by (RA/R), thus XS*(RA/R), SH*(RA/R) and S2*((RA/R), etc. Then the coordinates for drawing lines become XS1=XCF+XS*(RA/R), YS1=YCF-SH*(RA/R), etc., but XS4=XCF+RA, YS4=YCF for SMAX, etc. Then lines are drawn with Line (XDC,YDC)-(XS1,YS1) etc.

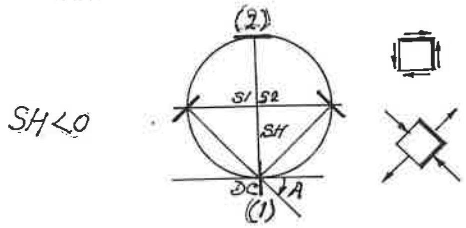
The vertrical zero line can be at any place. XP1 is the distance from the left to that line, XP1=XS2-S2*(RA/R) and adding YP1, XP2 and YP2 the vertical line can be drawn. But it may be too far to the left, XP1<900, or too far to the right, XP1>8010, shown here below, DrawStyle=2.



For S1 and S2 horizontal and vertical blue lines are drawn. At direction centre DC a red dot PSet (XDC,YDC),vbRed and DC printed on the left if Abs(XDC)<Abs(XS1) or on the right if of this dot if Abs(XDC)>Abs(XS1).

```
CurrentX=XCF+RA+90
CurrentY=YCF+60 : Print "SMAX"
CurrentX=XCF-RA-540
CurrentY=YCF+60 : Print "SMIN"
R=Sqr(((S1-S2)/2)^2+SH^2)
SMAX=(S1+S2)/2+R
SMIN=(S1+S2)/2-R          page 2
A=A*(180/3.14159) 'rad to deg

'top text
FontUnderline=2 : FontSize=10
T1="S1=" & S1 & "  S2=" & S2 & _
"   SH=" & SH & "  A= " & _
Format(A,"0.00") & " deg"
TW=TextWidth(T1)
CurrentX=XCF-TW/2 : CXX=CurrentX
CurrentY=840 : Print T1

T1="SMAX=" & _
Format(SMAX,"0.00) & _
"  SMIN=" & _
Format(SMIN,"0.00") & " N/mm2"
TW=TextWidth(T1)
CurrentX=XCF-TW/2 : CX=CurrentX
CurrentY=CurrentY+60 : Print T1
FontUnderline=False

'little stress plane lines
If S1=0 And S2=0 And SH<>0 Then
A=3.14159/4 : If SH<0 Then A=-A
Else
A=(-Atn(SH/((S1-S2)/2)))/2 'rad
End If

DrawWidth=3
Line (XS1,YS1-210)- _
(XS1,YS1+210),vbRed        (1) vert.
Line (XS2-210,YS2)- _
(XS2+210,YS2),vbBlue       (2) hor.
```






```
S=210*Sin(A) : C=210*Cos(A)

XCF+RA : YS6=YCF
If XDC>=XS1 Then XS6=XCF-RA
X1=XS6-S : Y1=YS6-C
X2=XSD+S : Y2=YS6+C
Line (X1,Y1)-(X2,Y2),vbRed

XS6=XCF-RA : YS6=YCF
If XDC>=XS1 Then XS6=XCF+RA
X1=XS6+C : Y1=YS6-S
X2=XSD-C : Y2=YS6+S
Line (X1,Y1)-(X2,Y2),vbBlue
```

The little red and blue stress plane lines.

An exception is made if both normal stresses are zero, S1=0 and S2=0, because in the formula for angle A there will be divided by zero. In this cases A= 3.14159/4 is 45 deg, if SH<0 then A=-A, see the figures shown on the left.

All other cases if XDC<XS1 or if XDC>XS1 with S=210*Sin(A) and C=210*Cos(A).
Calculation gives A<90 and A>-90, Cos(A) always positive, Sin(A) positive or negative.
The length of a short stripe is 2*210=420 tw.
XDC<XS1
Fig.10a with (6) on the right with XS6=XCF+RA : YS6=YCF. Length 210 tw is resolved into S and C. Angle A is to the left, a positive value, so S and C are positive. See the formulas written for X1, Y1, X2 and Y2, the coordinates of the short line ends.

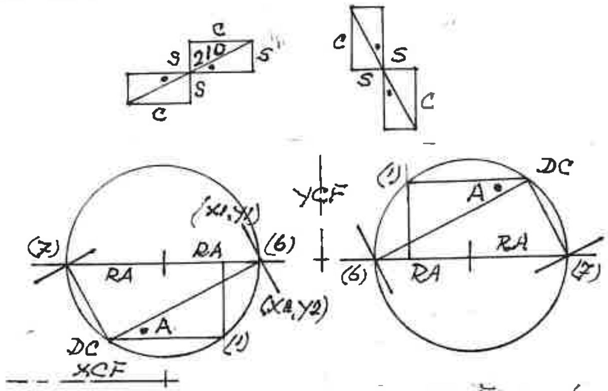


Fig. 10a. Fig. 10b.

X1=XS6-S : Y1=YS6-C : X2=XS7-C : Y2=YS7+S
The red short line can be drawn,
Line (X1,Y1)-(X2,Y2),vbRed.
Same figure with (7) on the left, XS7=XCF-RA : YS7=YCF. Same procedure, the coordinates become
X1=XS7+C : Y1=YS7-S : X2=XS7-C : Y2=YS7+C
Line (X1,Y1)-(X2,Y2),vbBlue
XSD>XS1
Fig.10b, angle A positive as well, now (6) on the left and (7) on the right.

Fig.10c and 10d.
Angle A to the right, then negative, so S is negative and C positive.

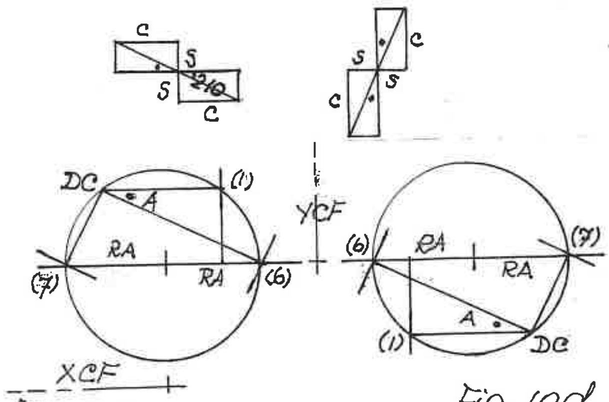
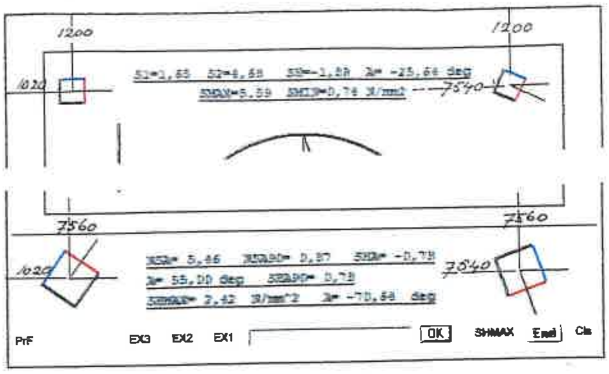


Fig. 10c. Fig. 10d.

When checking the formulas, the code for the coordinates X1, Y1, X2 and Y2 give correct results taking into account the negative S.

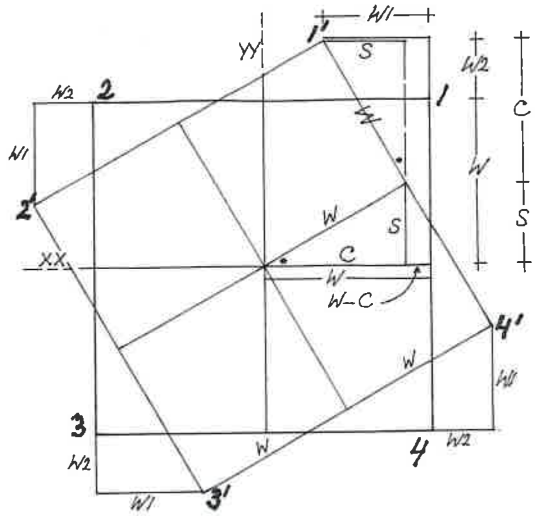
```
W=180 : LH=600 : XX=1020
YY=1200 : SQUARE1

W=180 : LH=600 : XX=7540
YY=1200 : SQUARE2

CY=7200 : ANGLEA1    next page
DrawWidth=1
End Sub

Private Sub SQUARE1()
DrawWidth=2 : W=180
X1=XX-W : X2=XX+W
Y1=YY-W : Y2=YY+W

Line (X2,Y1)-(X2,Y2),vbRed
Line (X1,Y1)-(X2,Y1),vbBlue
Line (X1,Y1)-(X1,Y2)
Line (X1,Y2)-(X2,Y2)
DrawWidth=1
Line (XX,YY)-(XX+LH,YY)        'hor.
End Sub

Private Sub SQUARE2()
DrawWidth=2
S=W*Abs(Sin(A)) : C=W*Abs(Cos(A))
W1=S+W-C
W2=C+S-W

If A>=0 Then
X1=XX+W-W1 : Y1=YY-W-W2          1'
X2=XX-W-W2 : Y2=YY-W+W1          2'
Line (X1,Y1)-(X2,Y2),vbBlue

X1=X2 : Y1=Y2                    2'
X2=XX-W+W1 : Y2=YY+W+W2          3'
Line (X1,Y1)-(X2,Y2)

X1=X2 : Y1=Y2                    3'
X2=XX+W+W2 : Y2=YY+W-W2          4'
Line (X1,Y1)-(X2,Y2)

X1=X2 : Y2=Y2                    4'
X2=XX+W-W1 : Y2=YY-W-W2          1'
Line (X1,Y1)-(X2,Y2),vbRed

ElseIf A<0 Then
X1=XX+W+W2 : Y1=YY-W+W1         1''
X2=XX-W+W1 : Y2=YY-W-W2         2''
Line (X1,Y1)-(X2,Y2),vbBlue

X1=X2 : Y1=Y2                   2''
X2=XX-W-W2 : Y2=YY+W-W1         3''
Line (X1,Y1)-(X2,Y2)

X1=X2 : Y1=Y2                   3''
X2=XX+W-W1 : Y2=YY+W+W2         4''
Line (X1,Y1)-(X2,Y2)

X1=X2 : Y2=Y2                   4''
X2=XX+W+W2 : Y2=YY-W+W1         1''
Line (X1,Y1)-(X2,Y2),vbRed
End If

DrawWidth=1
Line (XX,YY)-(XX+LH,YY)        'hor.
Line (XX,YY)-
(XX+LH*Cos(A),YY-LH*Sin(A)
End Sub
```

After the circle and straight lines squares are drawn. SQUARE1 for left top and SQUARE2 for right top, left bottom and right bottom.



SQUARE1, simple sqaure at left top, start situation, see following pages.

Fig.10 with angle A>0.
SQUARE2, for squares drawn at an angle A. The figure shows the square drawn turned over angle A to the left. Geometry determines the line lengths S and C with
S=W*Abs(Sin(A)) and C=W*Abs(Cos(A)).
The displacements of a corner point are
W1=S+W-C and W2=C+S-W.
XX and YY are coordinate values of the centre.
Lines are drawn from 1' to 2', 2' to 3' etc.

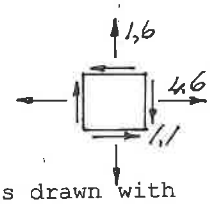


X1=XX+W-W1 : Y1=YY-W-W2 coordinates of 1',
X2=XX-W-W2 : Y2=YY-W+W1 coordinates of 2'.
Line 1' - 2' Line (X1,Y1)-(X2,Y2),vbBlue
The 'second' coordinates X2 and Y2 of 2' become the 'first' coordinates X1 and Y1 of 2',
X1=X2 : Y1=Y2 And the 'second' coordinates, of corner point 3' become
X2=XX-W+W1 : Y2=YY+W+W2 coordinates of 3'.
Line 2' - 3' Line (X1,Y1)-(X2,Y2) etc.
ElseIf A<0 Then
A figure like above but with angle A to the right. Same procedure with almost same formulas for X1, Y1, X2 and Y2, W1 and W2 with opposite signs. +W1 becomes -W2, -W1 becomes +W2, etc.
End If

Next a short horizontal line plus a short line at A degrees w.r.t. that horizontal line shown in the square figures here above.

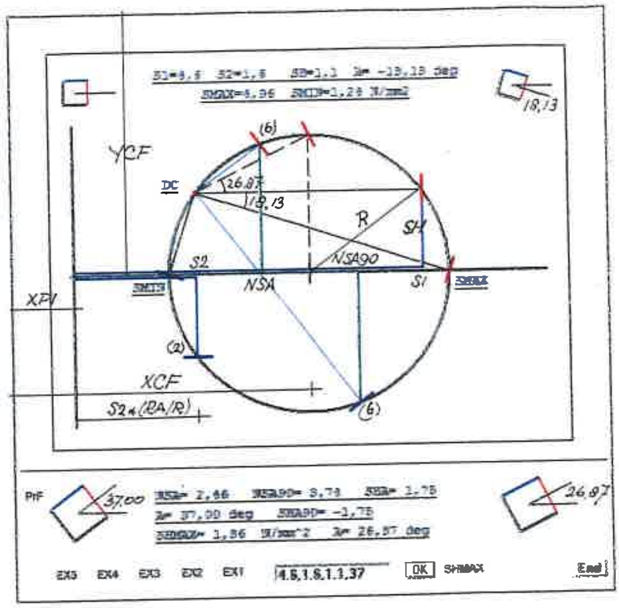
```
Private Sub ANGLEA1()
If A1<>0 Then

A=A1*3.14159/180   'rad
OM=(S1+S2)/2                              *
S=Sin(2*A)  : C=Cos(2*A)                  *
NSA=OM+((S1-S2)/2)*C-SH*S                 *
SHA=((S1-S2)/2)*S+SH*C                    *

XP1=XS2-S2*(RA/R)
XS6=XP1+NSA*(RA/R)
YS6=YCF-SHA*(RA/R)
GREENLINES        (for NSA and SHA)

'short red plane line
S=210*Sin(A)  : C=210*Cos(A)
X1=XS6-S  : Y1=YS6-C
X2=XS6+S  : Y2=YS6+C
DrawWidth=3
Line (X1,Y1)-(X2,Y2),vbRed
'
A=A+3.14159/2   'rad, is +90 deg'
OM=(S1+S2)/2                              *
S=Sin(2*A)  : C=Cos(2*A)                  *
NSA90=OM+((S1-S2)/2)*C-SH*S               *
SHA90=((S1-S2)/2)*S+SH*C                  *

XP1=XS2-S2*(RA/R)
XS6=XP1+NSA90*(RA/R)
YS6=YCF-SHA90*(RA/R)
GREENLINES        (NSA90 and SHA90)
'
'short ble plane line
A=A1   'deg
A=A*3.14159/180   'rad
S=210*Sin(A)  : C=210*Cos(A)
X1=XS6-C  : Y1=YS6+S
X2=XS6+C  : Y2=YS6-S
DrawWidth=3
Line (X1,Y1)-(X2,Y2),vbBlue

'bottom text
FontUnderline=True
CurrentY=7200  : CurrentX=2100
Print "NSA= " & _
Format(NSA,"0.00" & _
"   NSA90= " & _
Format(NSA90,"0.00") & _
"   SHA= " & Format(SHA,"0.00")

CurrentX=2100
CurrentY=CurrentY+60
Print "Angle A= "& _
Format(A1,"0.00") & " deg" & _
"   SHA90=  " & _
Format(SHA90,"0.00")
FontUnderline=False

CY=CurrentY+60           page 4

W=300  : LH=720  : XX=960
YY=7500  : SQUARE2   'left bottom

End If
End Sub

Private Sub TSTRING_DblClick()
TSTRING.Text=""
End Sub
```

Private Sub ANGLEA1()



As example, normal stresses S1=4,6 N/mm2, S2=1,6 N/mm2 and Shear stress SH=1,1 N/mm2 as given in the figure on the right. In text box TSTRING is typed 4.6,1.6,1.1,37 Enter, the circle is drawn with STRESSCIRCLE1 page 5 , see COK page 4 , Direction Centre DC, principle stresses SMAX and SMIN, etc.
The sqaure right top with angle A to the right, A=-18,13 degrees. In TSTRING is typed also 37, see COK page 4 , A1=Val(TTSTRING(4)) for a given angle A1 to calculate normal and shear stresses for that case.

Last part of STRESSCIRCLE1 left top of the preceding page is written CY=7200 : ANGLEA1. Sub ANGLEA1 is carried out if A1<>0. If A1=0 then CY=7200 for printing SHMAX when clicking SHMAX on the right of OK, see page 4 with CurrentY=CY in Private Sub CALCSHMAX(). If A<>0 then after printing NSA, NSA90 etc. with CY=CurrentY+60, see on the left.



After calculating NSA and SHA coordinates XP1, follow XS6 and YS6 used for drawing two green lines with GREENLINES, a vertical line and a horizontal line with DrawWidth=2.
And a blue line DC-(6).

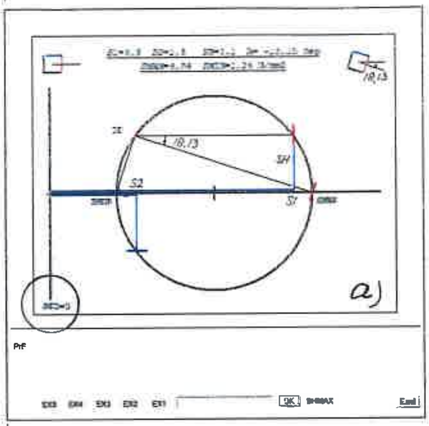
```
Private Sub GREENLINES()
If XP1>900 Then XP1=900
If XP1>8010 Then XP1=8010 : DrawWidth=2
Line (XP1,YCF)-(XS6,YCF),vbGreen   'hor. line
Line (XS6,YS6)-(XS6,YCF),vbGreen   'vert.line
DrawWidth=1 : Line (XDC,YDC)-(XS6,YS6),vbBlue
End Sub
```

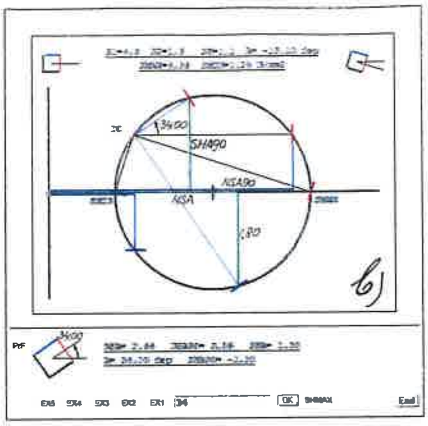
Next a short red plane line with coordinates X1, Y1, X2 and Y2 like done before on page .
For NSA90 and SHA90 same procedure.
Clicking SHMAX calcultes the maximum possible shear stress, it's angle A, the square right bottom. See the striped lines.

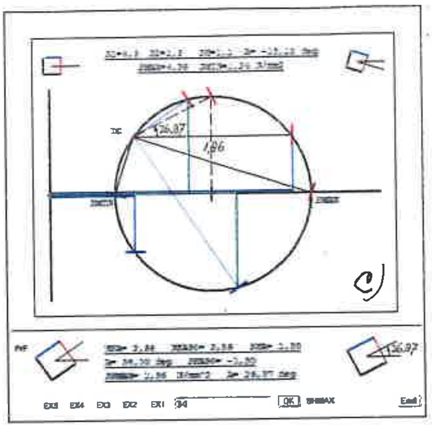
```
Private Sub LEX1_Click()
SC1=SC1+1
If SC1=1 Then
S1=4.6 : S2=1.6 : SH=1.1
ElseIf SC1=2 Then
SH=-1.1
ElseIf SC1=3 Then
S1=-4.6 : S2=-1.6 : SH=1.1
ElseIf SC1=4 Then
SH=-1.1

ElseIf SC1=5 Then
S1=1.6 : S2=4.6 : SH=1.1
ElseIf SC1=6 Then
SH=-1.1
ElseIf SC1=7 Then
S1=-1.6 : S2=-4.6 : SH=1.1
ElseIf SC1=8 Then
SH=-1.1
End If
```

Private Sub LEX1 Click()

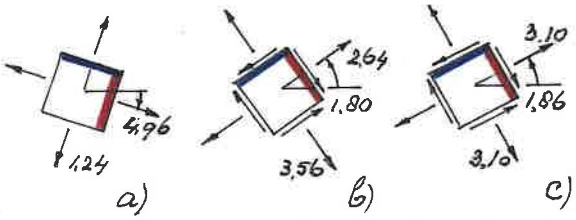
Eight possible combinations of normal stresses S1 and S2 and shear stress SH. The first click on EX1 gives a) the print of the left, see example on page 4 . With SCR=1 below the vertical zero line.
Print b) appears with 34 Enter a positive value of 34 degrees, as assumed to the left. Text and square at the left appeared below the circle. See the green lines, NSA, NSA90 and SHA have b een calculated. SCR=1 disappeared because the circle is drawn again with Cls at the start, page 5 .
With a click on SHMAX maximum shear stress SHMAX (page 4), its angle A and normal stress are calculated, see the striped drawn lines in print b). Radias R=Sqr(((S1-S2)/2)^2+SH^2) is for all the eight cases the same, circles all drawn with RA=2100 tw. But...see EX2 next page.








```
RA = 2100
STRESSCIRCLE1
CurrentX = 780: CurrentY = 6300
FontBold = True
Print "SC1=" & SC1
If SC1 = 8 Then SC1 = 0
FontBold = False
End Sub
```



SC1=4   S1=-4.6   S2=-1.6   SH=-1.1   N/mm2
For angle A1 -54 Enter, and a click on SHMAX does appear the print on the left.

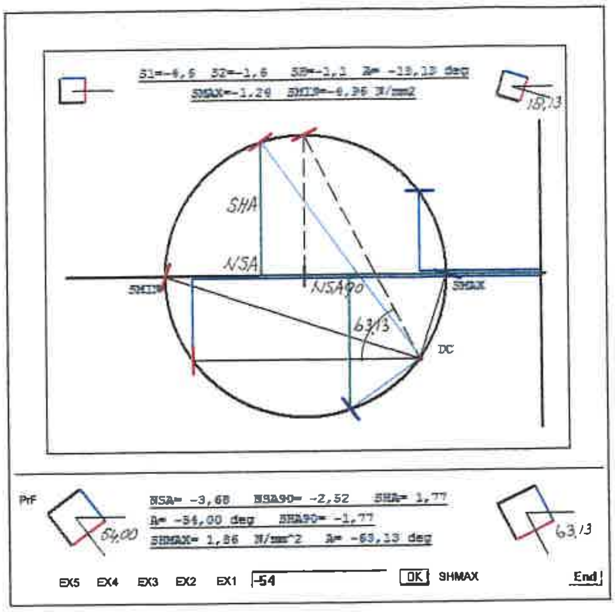
NSA= (S1+S2)/2 + ((S1-S2)/2)*cos2A - SH*sin2A
A=-54 deg   2A=-108   Cos2A=-0,309   Sin2A=-0,951

-3,10 +(-1,5)*(-0,309)-(-1,1)*(-0,951)= -3,69

With A+90 is 2A=-108+180=72 degrees,
Cos2A= 0,309   Sin2A= 0,951
NSA90=-3,10 +(-1,5)*0,309 -(-1,1)*0,951 = -2,51

SHA=((S1-S2)/2*Sin2A+SH*Cos2A
=(-1,5)*(-0,951)+(-1,1)*(-0,309)=
=   2,43   +   0,34   = 1,77 N/mm2

SHMAX= R   R=Sqr(((S1-S2)/2)^2+SH^2)
R= Sqr((-1,5)^2+(-1,1)^2= Sqr(3,46)= 1,86 N/mm2



Note!
A1 Enter, always with -90 <= A1 <= 90
If not then the square left bottom has a wrong position.... can be repaired... code... yes...

```
Private Sub LEX2_Click()
SC2=SC2+1
A1=-50      'deg

If SC2=1 Then _
S1=4.6 : S2=-1.6 : SH=1.1
If SC2=2 Then SH=-1.1
If SC2=3 Then _
S1=-4.6 : S2=-1.6 : SH=1.1
If SC2=4 Then SH=-1.1

If SC2=5 Then
S1=1.6 : S2=-4.6 : SH=1.1
ElseIf SC2=6 Then
SH=-1.1
ElseIf SC2=7 Then
S1=-1.6 : S2=-4.6 : SH=1.1
ElseIf SC2=8 Then
SH=-1.1
End If

RA=2100
R=Sqr(((S1-S2)/2)^2+SH^2)

RR=3.289 : RA=(R/RR)*RA

STRESSCIRCLE1              page 5
CurrentX=780 : CurrentY=6300
FontBold=True
Print "SC2=" & SC2 & _
"    R= " & Format(R,"0.000")
If SC2=8 Then SC2=0
FontBold=True
End Sub
```

Private Sub LEX2 Click()

Like LEX1, another eight combinations of S1, S2 and SH, exchanging values positive or negative. The radius R of the stress circle is

If - Then for SC2=1 To SC2=4 written in another way, just done to show this possibility.

Given angle A1=-50 deg, -50 Enter.

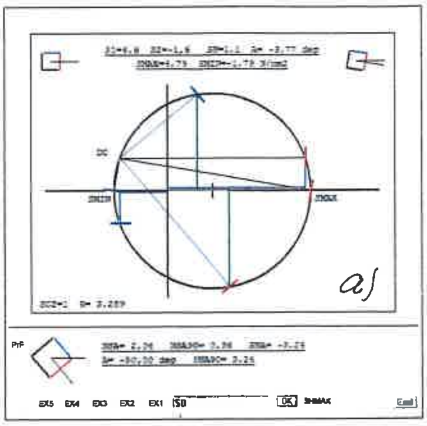
R= Sqr(((S1-S2)/2)^2+SH^2)   N/mm2

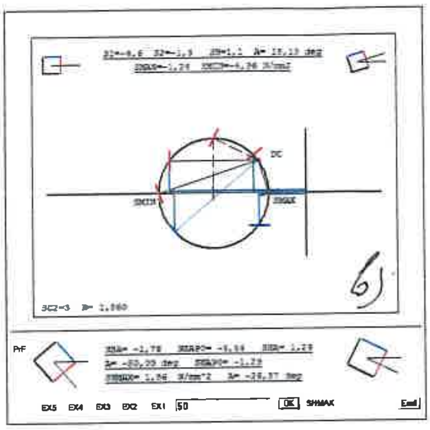
SC2=1
R= Sqr(((4.6-(-1.6))/2)^2+1.1^2)
 = Sqr(3.1^2+1.1^2)= Sqr(9.61+1.21)= 3,289
SC2=2
R= Sqr(((4.6-(-1.6))/2)^2+(-1.1)^2)
 = Sqr(3.1^2+1.1^2)= Sqr(9.61+1.21)= 3,289
SC2=3
R= Sqr(((-4.6-(-1.6))/2)^2+1.1^2)
 = Sqr((-1.5)^2+1.1^2)= Sqr(2.25+1.21)= 1.860
SC2=4 with R= 1.860 as well.
SC2=5/6 R= 3.289 and SC2=7/8 R= 1.860.
(EX1 all R=1,860, drawn with RA=2100 tw)

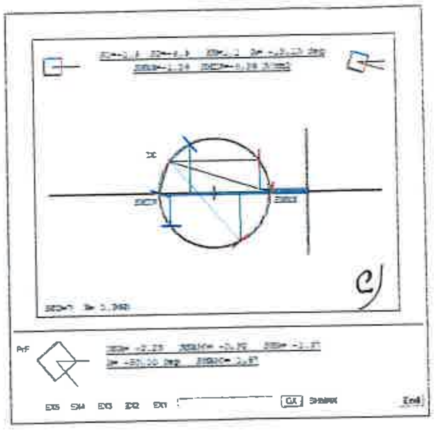
Until now all circles were drawn with a radius of RA=2100 twips. To draw the 'real' circle some lines of code are added.

RR= 3.289 the largest radius,
RA=(R/RR)*RA is RA=(R/RR)*2100 twips.

Prints below a) SC2=1, b) SC2=3, c) SC2=8.







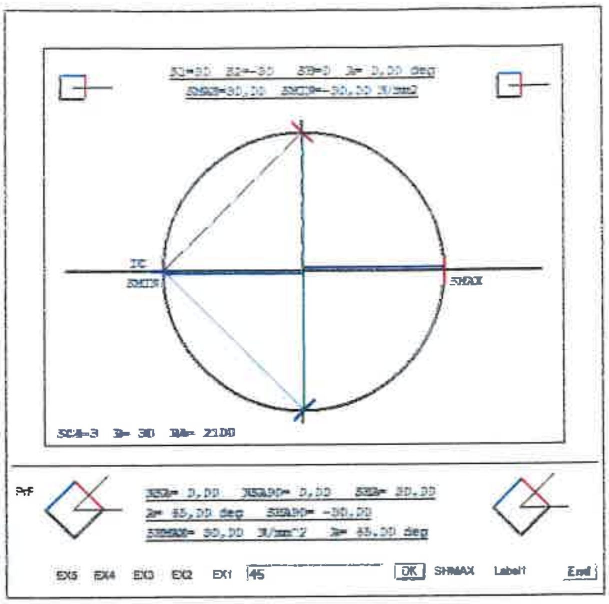
```
Private Sub LEX3_Click()
SC3=SC3+1
If SC3=1 Then
S1=-90 : S2=-60 : SH=-17 : A1=16
ElseIf SC3=2 Then
S1=90 : S2=60 : SH=27
ElseIf SC3=3 Then
S1=-60 : S2=70 : SH=33 : A1=-20
ElseIf SC3=4 Then
S1=60 : S2=-90 : SH=-21
End If

RA=2100                'circle radius
R=Sqr(((S1-S2)/2)^2+SH^2)

STRESSCIRCLE1              page 5
CurrentX=780 : CurrentY=6300
FontBold=True
```

These eight cases are given, the maximum rsdius RR calculated, RR=3.289. This maximum must be known to be able to print each of the eight circles in proportion, with RA=(R/RR)*RA.

Private Sub LEX3 Click()

Four possible cases with given S1, S2 and SH. Also an angle A1. All circles are drawn with RA=2100 tw. R is calculated and printed,
SC3=1 R= 22.7   SC3=2 R= 30.9   SC3=3 R= 72.9
SC3=4 R= 215.4

```
Print "SC3=" & SC3 & _
"    R= " & Format(R,"0.0")
If SC3=8 Then SC3=0
FontBold=True
End Sub
```



```
Private Sub LEX4_Click()
SC4=SC4+1
If SC4=1 Then
S1=70 : S2=70 : SH=30
ElseIf SC4=2 Then
S1=80 : S2=-80 : SH=0
ElseIf SC4=3 Then
S1=30 : S2=-30 : SH=0
ElseIf SC4=4 Then
S1=15 : S2=15 : SH=50

ElseIf SC4=5 Then
S1=-40 : S2=-40 : SH=-60
ElseIf SC4=6 Then
S1=0 : S2=0 : SH=45
ElseIf SC4=7 Then
S1=0 : S2=0 : SH=-65
ElseIf SC4=8 Then
S1=-55 : S2=-55 : SH=-25
End If
```

Private Sub LEX4 Click()

With first eight clicks the circles are drawn with radius RA=2100 twips. On the left the circle after third click, 45 Enter, (line SC4=3 R=30 RA=2100 does not disappear in this special case, see rem page 5) SHMAX click.

If CS=0 Then, see code left bottom.

At the start CS=0, at each SC4 of the eight clicks radius
RI(SC4)=Sqr(((S1-S2)/2)^2+SH^2) is calculated and compared with RR4,
If RR4<RI(SC4) Then RR4=RI(SC4) to determine the largest radius, being finally RR4.

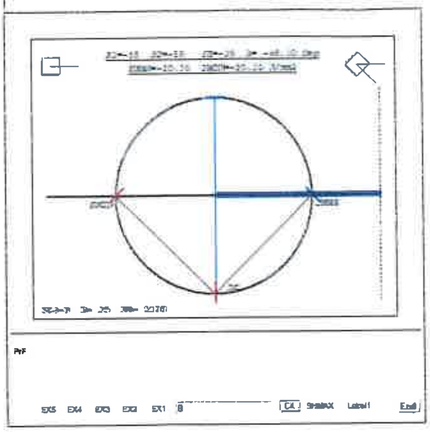
SC4=	1	2	3	4	5	6	7	8
R=	30	80	30	50	60	45	65	25

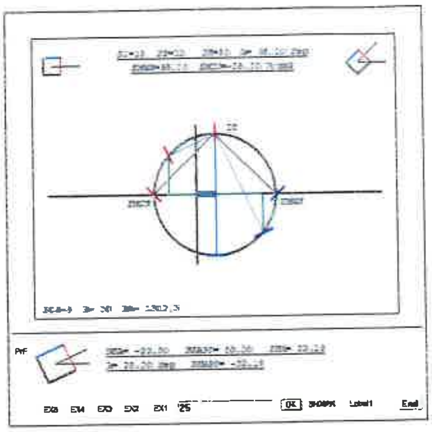
Largest R is RR4=80.
If SC4 has become SC4=8 then CS=1, and just before End Sub becomes SC4=0 for the second round.

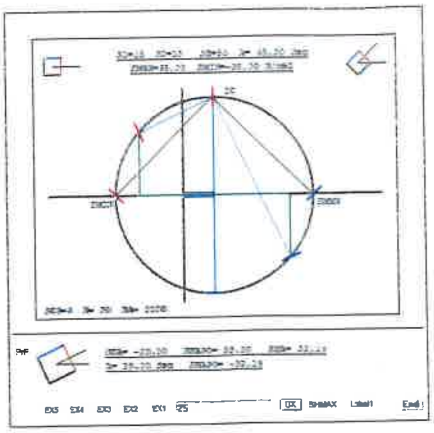
ElseIf CS=1 Then

Another eight clicks on EX4 draws the circles with a radius <= RR4 is <= RA=2100.
Print b) after the fourth click in the second round, CS4=4.
RA= (RI(SC4)/RR4)*RA= (50/80)*2100= 1313 <2100.
A1=25 was put in with A=25 Enter
When going on clicking SC4 becomes SC4=8,
If SC4=8 Then CS=0 and before End Sub follows
If SC4=8 Then SC4=0 for the third round to draw all circles with same radius RA=2100, see c) like b) but with RA=2100. In the mean time the maximum value RR4 is once again determined, not necessary....

A click on SHMAX gives in few cases visible results, a striped line and text. When DC not on the vertical zero line.







```
RA=2100
If CS=0 Then
RI(SC4)=Sqr(((S1-S2)/2)^2+SH^2)
If RR4<RI(SC4) Then RR4=RI(SC4)
If SC4=8 Then CS=1

ElseIf CS=1 Then
RA=(RI(SC4)/RR4)*RA
If SC4=8 Then SC4=0
End If
```

STRESSCIRCLE1 page 5

```
CurrentX=780 : CurrentY=6300
FontBold=True
Print "SC4=" & SC4 & "  R= " & _
Format(RI(SC4),"0" & & _
"  RA " & Format(RA,"0")
FontBold=False
If SC4=8 Then SC4=0
End Sub
```

```
Private Sub LEX5_Click()
RA=2100
SC5=SC5+1
If SC5=1 Then                1st case
T=T+1 : N=SC5

If T=1 Then
S1=-70 : S2=120 : SH=80
RR5=Sqr(((S1-S2)/2)^2+SH^2)
RI(1)=RR5

Else
If T=2 Then SHATRANDOM

RA=(Abs(R1(T))/RR5)*RA
SH=SH1(T)
End If
SC5=SC5-1
If T=4 Then SC5=1
ElseIf SC5=2 Then            2nd case
T=T+1 : N=SC5

If T=1 Then
S1=4.9 : S2=28.3 : SH=-8.8
RR5=Sqr(((S1-S2)/2)^2+SH^2)
RI(1)=RR5

Else
If T=2 Then SHATRANDOM

RA=(Abs(R1(T))/RR5)*RA
SH=SH1(T)
End If
SC5=SC5-1
If T=4 Then SC5=2
End If

STRESSCIRCLE1               page 5

CurrentX=780 : CurrentY=6300
FMT="0"
If N=2 Then FMT="0.00"
FontBold=True
Print "SC5=" & N & "  T=" & T _
& "  R= " & Format(RI(T),FMT) _
& "  RA= " & Format(RA,"0")
FontBold=False

If SC5=2 Then SC5=0
If T=4 Then T=0
End Sub

Private Sub SHATRANDOM()
If Int(2*Rnd)=0 Then _
S=S1 : S1=-S2 : S2=-S

For I=2 To 4
B=1 : If Int(2*Rnd)=1 Then B=-1
SH1(I)=B*Int(SH*Rnd)+1)

R=Sqr(((S1-S2)/2)^2+SH1(I)^2)
RI(I)=R
If RR5<R Then RR5=R
Next I
End Sub

Private Sub LRANDOMIZE_Click()
Randomize
End Sub
```

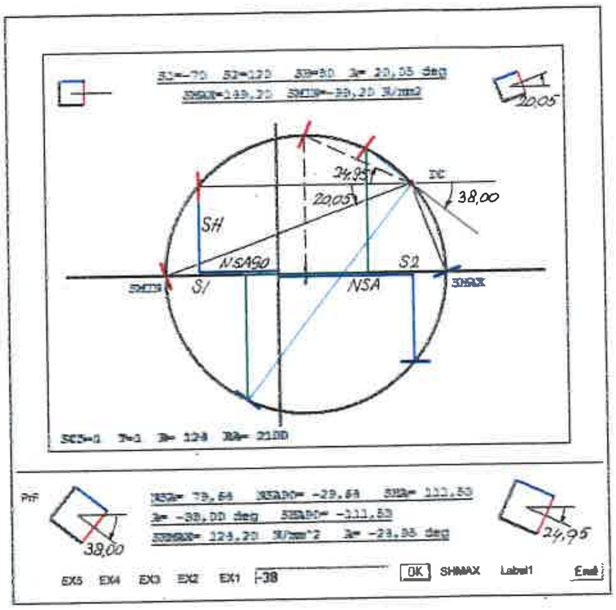
Private Sub LEX5 Click()   Click and see...

Another click example. Two cases CS5=1 and CS5=2 with start values for S1, S2 and SH.
If SC5=1 Then, first click on EX5.
N=SC5 to print "SC5=" & N, see Print.
T counts the times a stress circle is drawn for SC5=1, T=4 times.

If T=1 Then
Radius RR5 is calculated, stored with RI(1)=RR5 for later use.
After End If becomes SC5=SC5-1=0 for a second click on EX5 on behalf of T=2, etc.

Second click on EX5, SC5=SC5+1=1. T=T+1=2
Else
If T=2 Then SHATRANDOM left bottom to determine three more SH1() three more RI() after manupulating shear stress SH with at random function Rnd, and the largest radius RR5 of all four.
Each time after Else for a T the radius RA for the concerning stress circle is calculated with RA=Abs(RI(T))/RR5)*RA, (RA=2100 at the start) becomes SH=SH1(T) to draw circle T with STRESSCIRCLE1.

A possible circle to appear could look like shown below. With -38 Enter and a click on SHMAX. S1=-70, S2=120, SH=80, A=-38.



Private Sub SHATRANDOM()

Int(2*Rnd), at random numbers 0 and 1. If 0 then S=S1 : S1=-S2 : S2=-S,.... just like that, no 'smart' reason.
For I=2 To 4 B becomes B=1 or B=-1, and with Int(SH*Rnd)+5 numbers 0+5 to SH-1+5 follows SH1(I)=B*Int(SH*Rnd)+5.
Radius R calculated, R1(I)=R to compare with RR5, finally RR5 is found. All RA depend on it with maximum RA=2100.

At each start of the program the at random determined valus are the same in same order.
Just click Randomize to change the values to appear.

Plane stresses and Mohr's stress circle without sign conventions.

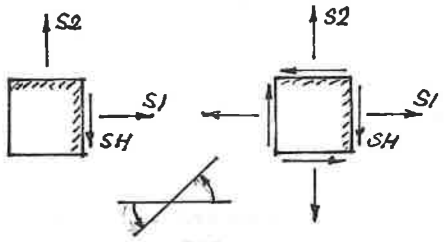


Fig.1.

Fig.1.
Assumptions of the names and directions of normal stresses S1 and S2 and shear stress SH determined after experimenting.
Next the other stresses are added for equilibrium. A rotation angle A is assumed to the left, belonging to Mohr's formulas.
With this figure the values could be e.g.
S1= 16 N/mm^2, SH= 5 N/mm^2, S2= 8 N/mm^2.

Drawing the circle of Mohr.

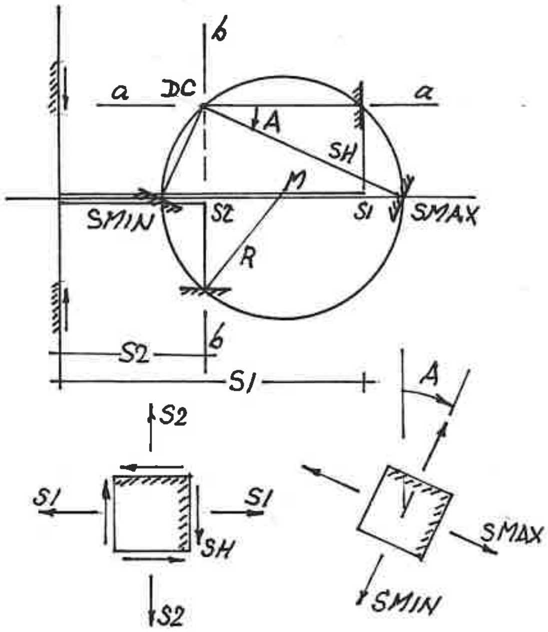


Fig.2

Fig.2.
S1 to the right. SH always belonging to S1, plotted above the zero line because of the assumed orientation of SH in fig.1.
Next S2 to the right and SH, opposite directed to SH of S1, below the zero line.
(S1+S2)/2 gives the centre M of the circle. The circle can be drawn.
The lines a-a and b-b intersect the circle in DC the direction centre.
The pricipal stresses with shear stress zero are SMIN and SMAX. The lines through DC and SMIN and SMAX show the rotation over angle A.

R=Sqr(((S1-S2)/2)^2+SH^2)

SMAX=(S1+S2)/2+R   SMIN=(S1+S2)/2-R

Angle A=(-Atn(SH/((S1-S2)/2))/2

(S1+S2)/2=(16+8)/2= 12   (S1-S2)/2=(16-8)/2= 4

Radius R=Sqr(((S1-S2)/2)^2+SH^2)
R= Sqr(4^2+5^2)=Sqr(41)= 6,4

SMAX= 12,0+6,4= 18,4 N/mm^2
SMIN= 12,0-6,4= 5,6 N/mm^2

A= (-Atn(5/4)/2= (-Atn(1.25))/2 = -25,67 deg,
a negativ answer, so not to the left as assumed but to the right.

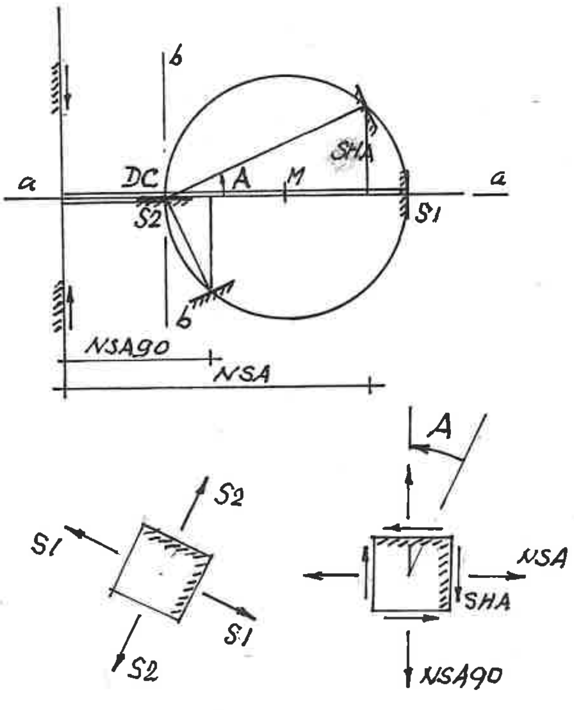


Fig.3.

Fig.3.
Now only the principal stresses S1 is SMAX and S2 is SMIN to start with, SH=0. With M the centre the circle can be drawn.
S1= 18,4 N/mm2, S2=5,6 N/mm2, SH= 0 N/mm2.

The stresses under angle A to the left are determined, with A= 25,76 deg. See the two lines drawn through DC to find NSA, SHA and NSA90.

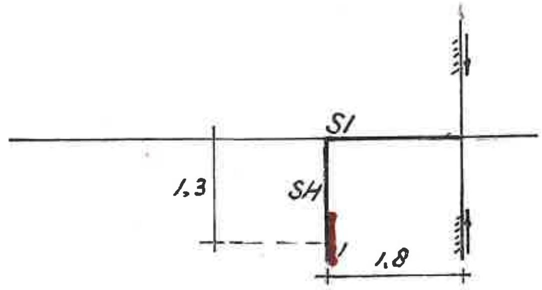
NSA= (S1+S2)/2 + ((S1-S2/2)*Cos2A - SH*Sin2A

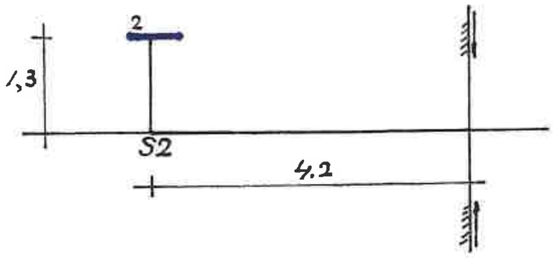
SHA= ((S1-S2)/2)*Sin2A + SH*Cos2A

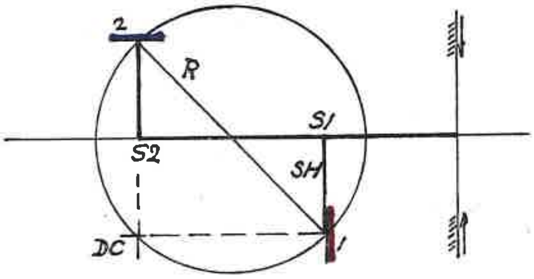
NSA90 with angle A+90, so 2A is 2(A+90).

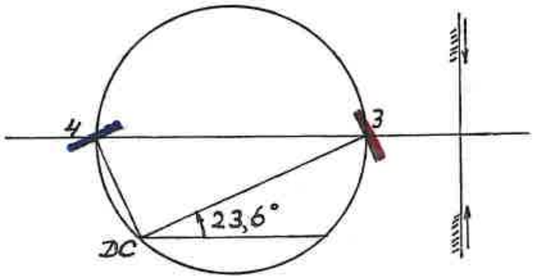
The results with the formulas are
NSA= 16 N/mm^2, SHA= 5 N/mm^2, NSA90= 8 N/mm2.

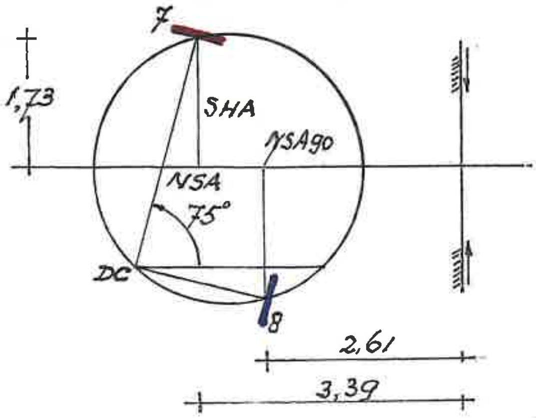
The first situation has appeared.





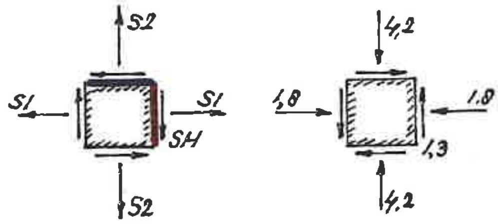






Example.

S1= -1,8 N/mm2   S2=-4,2 N/mm2   SH= -1,3 N/mm2

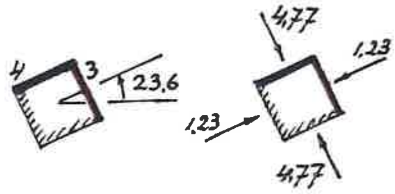


On the left the assumptions, on the right the stresses drawn with their real directions.

R=Sqr(((S1-S2)/2)^2+SH^2)

R=Sqr(((-1,8-(-4,2))/2)^2+(-1,3)^2)

=Sqr(1,2^2+(-1,3)^2)=Sqr(1,44+1,69)= 1,77



(S1+S2)/2= (-1,8+(-4,2))/2= -3

SMAX=(S1+S2)/2+R   SMAX= -3+1,77= -1,23   N/mm2

SMIN=(S1+S2)/2-R   SMIN= -3-1,77= -4,77   N/mm2

Angle A=(-Atn(SH)/((S1-S2)/2))/2

A=(-Atn(-1,3)/1,2)/2)=

=(-Atn(-1,09)/2= -(-47,2)/2= 23,6 deg

Stresses under 75 degrees to the left.

A= 75 deg   2A= 150 deg   (S1-S2)/2= 1,2

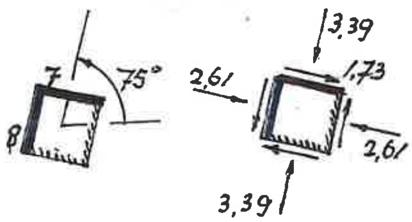
NSA=   (S1+S2)/2 +((S1-S2)/2)*Cos2A -SH*Sin2A,

NSA=   -3 +1,2*Cos150 -(-1,3)*Sin150

-3 +1,2*(-0.866) +1,3*0.5= -3,39 N/mm2

NSA90= (S1+S2)/2 -((S1-S2)/2)*Cos2A +SH*Sin2A,

NSA90= -3 +1,04 -0,65= -2,61 N/mm2



SHA=   ((S1-S2)/2)*Sin2A + SH*Cos2A,   A=75

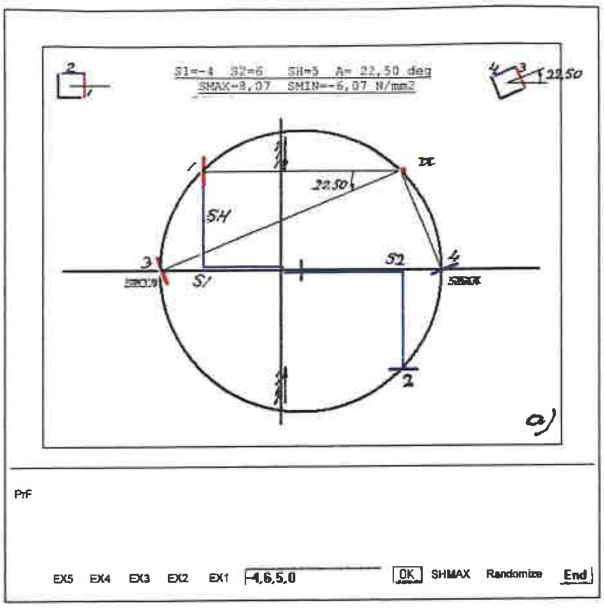
SHA=   1,2*Sin150 +(-1,3)*Cos150=

1,2)*0,5 +(-1,3)*(-0,866)= 1,73 N/mm2

R=Sqr(((S1-S2)/2)^2+SH^2)

SMAX=(S1+S2)/2+R   SMIN=(S1+S2)/2-R

Angle A=(Atn(SH)/((S1-S2)/2))/2



a) S1= -4 N/mm2   S2= 6 N/mm2   SH= 5 N/mm2

A=0   -4,6,5,**0** Enter,   S1,S2,SH,A Enter.

(S1+S2)/2=(-4+6)/2= 1   (S1-S2)/2=(-4-6)/2= -5

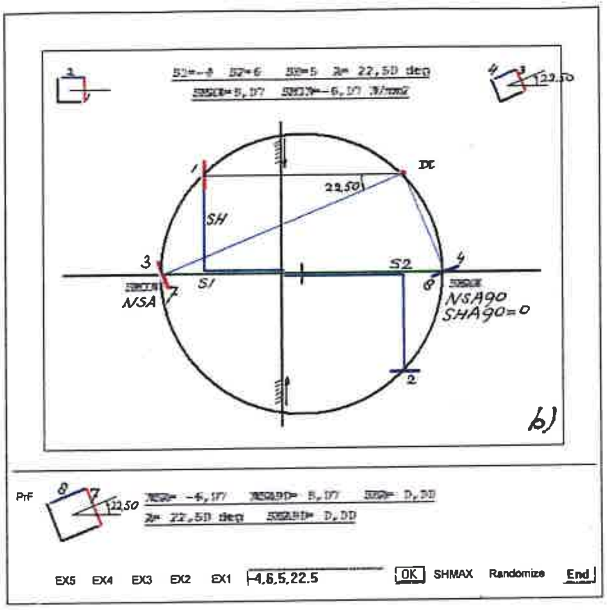
Radius R=Sqr((-5)^2+5^2)= Sqr(50)= 7,07 N/mm2

SMAX= 1+7,07= 8,07 N/mm2

SMIN= 1-7,07= -6,07 N/mm2

A=(-Atn(5)/-5)/2= (-Atn(-1))/2= 45/2= 22,50 deg

A= 22,50 deg, that's to the left as assumed.



b) S1= -4 N/mm2   S2= 6 N/mm2   SH= 5 N/mm2

A=22.50 deg   -4,6,5,**22.50** Enter.   2A=45.00

NSA=   (S1+S2)/2 +((S1-S2)/2)*Cos2A -SH*Sin2A,

NSA=   1 +(-5)*Cos45.00 -5*Sin45.00
       1 +(-5)*0.707 -5*0.707= 1-7.07= -6.07

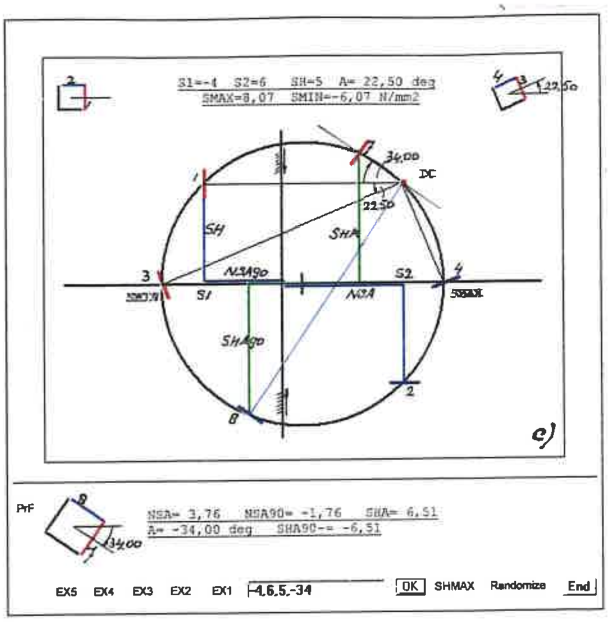
NSA90= (S1+S2)/2 -((S1-S2)/2)*Cos2A +SH*Sin2A,

NSA90= 1 +(-5)*(-0.707)-5*(-0.707)=1+7.07=8.07

SHA= ((S1-S2)/2)*Sin2A + SH*Cos2A,   A=22.50

SHA=   -5*Sin45 +5*Cos45=
       -5*0,707 +5*0,707= 0

See the horizontal green line(s).



c) S1= -4 N/mm2   S2= 6 N/mm2   SH= 5 N/mm2

Suppose A=-34 degrees, that's to the right.

A=-34 deg   -4,6,5,**-34** Enter.   2A=-68

NSA=   (S1+S2)/2 +((S1-S2)/2)*Cos2A -SH*Sin2A,

NSA= 1 +(-5)*Cos(-68) -5*Sin(-68)
     1+(-5)*0,375-5*(-0,927)=1-1.88+4,64= 3,76

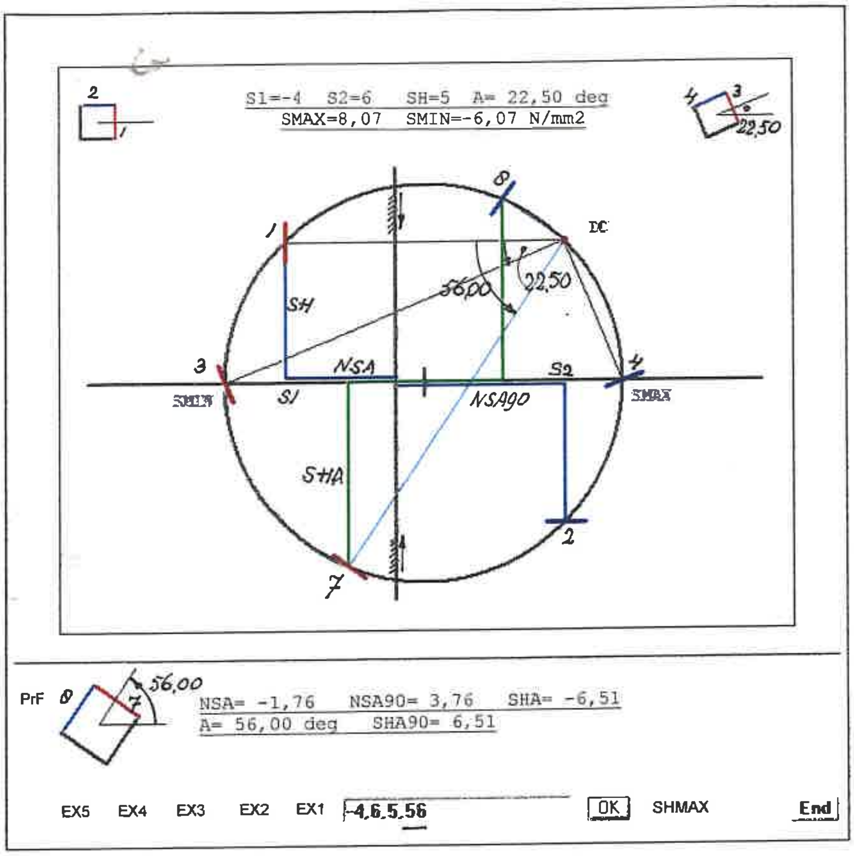
NSA90= (S1+S2)/2 -((S1-S2)/2)*Cos2A +SH*Sin2A,

NSA90= 1 +1,88 -4,64= -1,76 N/mm^2

SHA= ((S1-S2)/2)*Sin2A + SH*Cos2A,   A=-34 deg

SHA=   -5*Sin(-68) +5*Cos(-68)=
       -5*(-0,927) +5*0,375= 4,64 +1,88= 6,52

SHA90= -6,52 N/mm^2



S1=-4 N/mm^2 S2=6 N/mm^2

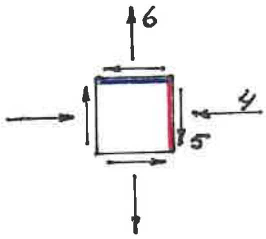
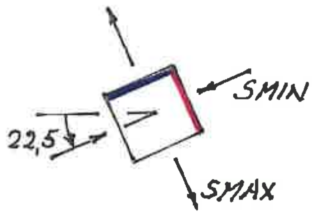
SH= 5 N/mm2

SH=5 belongs to S1, see the little red line 1, and SH=-5 belongs to S2, the little blue line 2

Stresses NSA and NSA90 are drawn and calculated for angle 56 deg to the left.

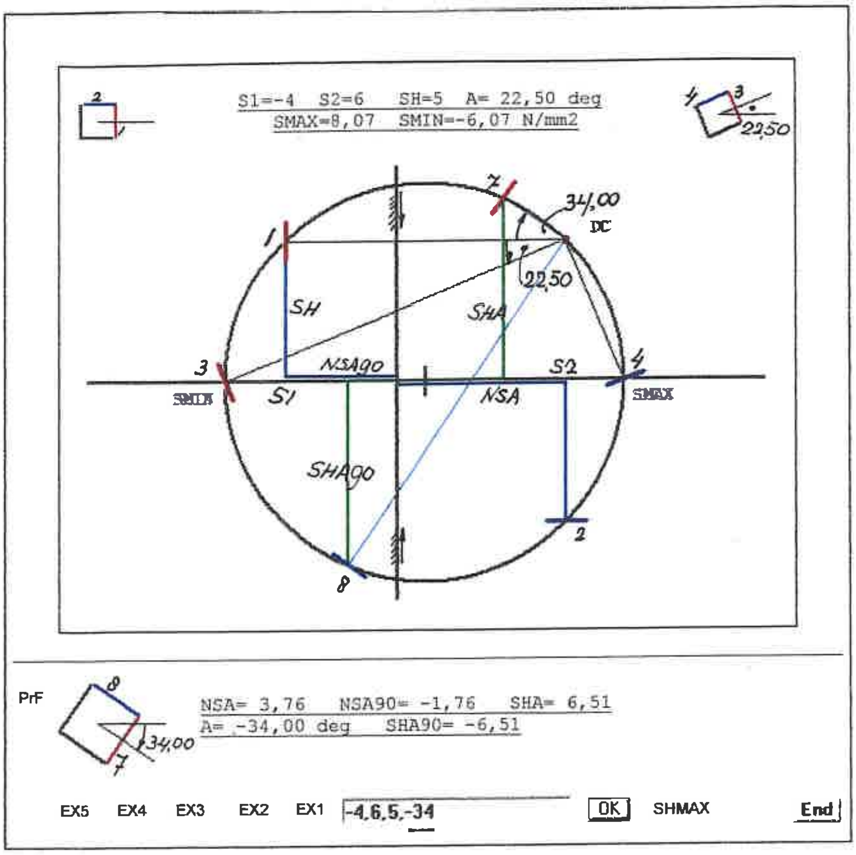
The green lines belong to NSA, NSA90 and SHA.

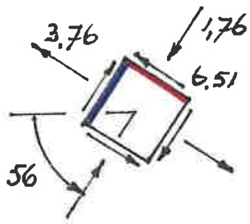
SHA=-6,51 belongs to NSA, see the little red line 7 and SHA90=6,51 to NSA90, the blue little line 8.

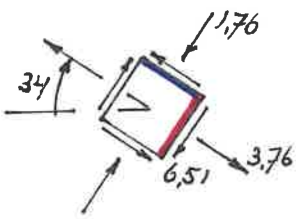



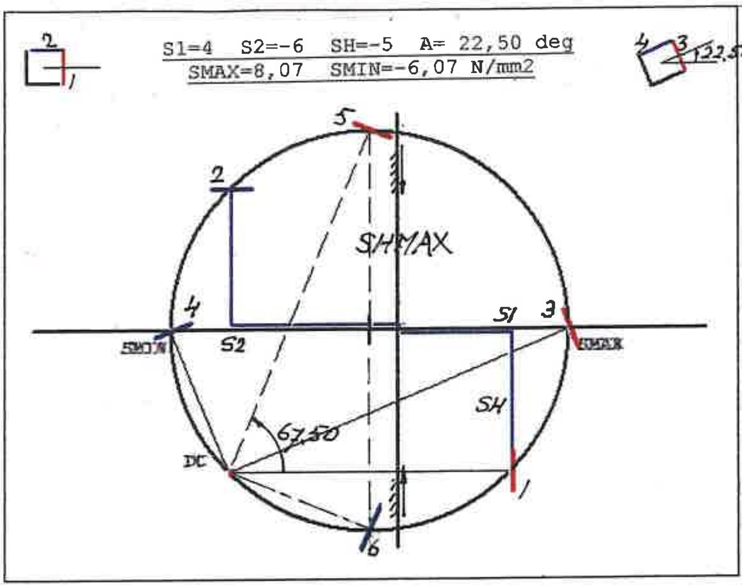
First case above 56 deg.
NSA=-1,76 NSA90= 3,76
SHA=-6,51 SHA90= 6,51

Second case below -34 deg.
NSA= 3,76 NSA90=-1,76
SHA= 6,51 SHA90=-6,51



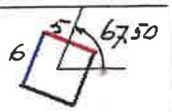






PrF

SHMAX= 7,07 N/mm^2 A= 67,50 deg



S1= 4 N/mm^2 S2=-6 N/mm62

SH=-5 N/mm^2

4,-6,-5,0 Enter,

SMAX=8,07 N/mm^2,

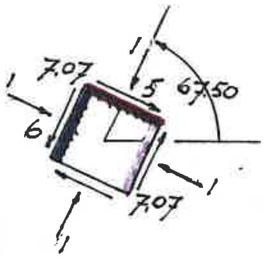
SMIN=-6,07 N/mm^2 and

angle A= 22,50 deg.

A click on SHMAX on the right of OK, gives SHMAX= 7,07 N/mm^2 the maximum shear stress, with angle A= 67,50 deg.
2A= 135 deg

The formula for NSA gives

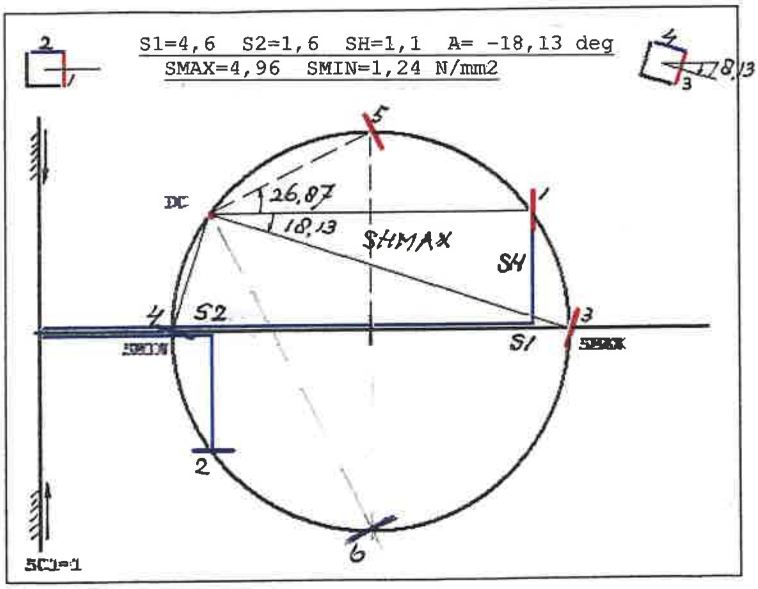
NSA= -1,00 N/mm^2.



NSA= (S1+S2)/2 +((S1-S2)/2)*Cos2A -SH*Sin2A

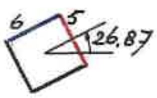
NSA= (4+(-6))/2 +((4-(-6)/2)*Cos135 -(-5)*Sin135

NSA= -1 +5(-0,707) -(-5)(0,707) = -1 N/mm^2



PrF

SHMAX= 1,86 N/mm^2 A= 26,87 deg



S1= 4,6 N/mm^2 S2= 1,6 N/mm62

SH= 1,1 N/mm^2

4.6,1.6,1.1,0 Enter,

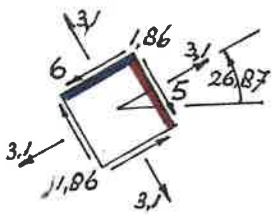
SMAX=4,96 N/mm^2,

SMIN=1,24 N/mm^2 and

angle A= -18,13 deg.

A click on SHMAX on the right of OK, gives SHMAX= 1,86 N/mm^2 the maximum shear stress, with angle A= 26,87 deg.
2A= 53,74 deg
The formula for NSA gives

NSA= 3,10 N/mm^2.



NSA= (S1+S2)/2 +((S1-S2)/2)*Cos2A -SH*Sin2A

NSA= (4,6+1,6)/2 +((4,6-1,6)/2)*Cos53,74 -1,1*Sin53,74

NSA= 3,1 +1,5(-0,591) -1,1(0,806)

NSA= 3,1 +0,887 -0,887= 3,1 N/mm^2

§ 78. Directions of the Principal Stresses

In the preceding section we determined only the magnitude of the principal stresses for an arbitrarily selected element without concerning ourselves with their direction. The results obtained were good

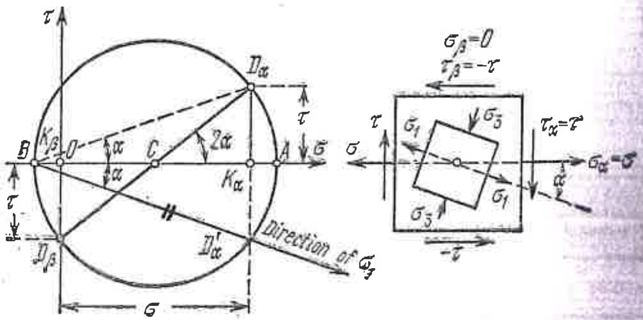
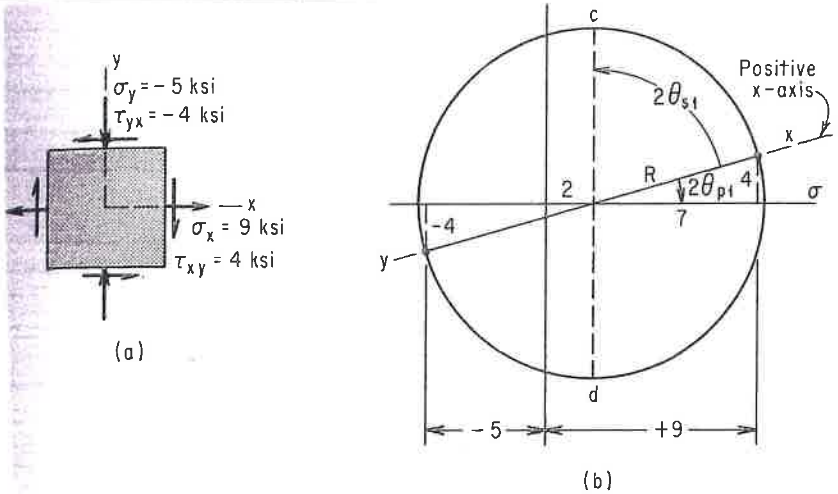
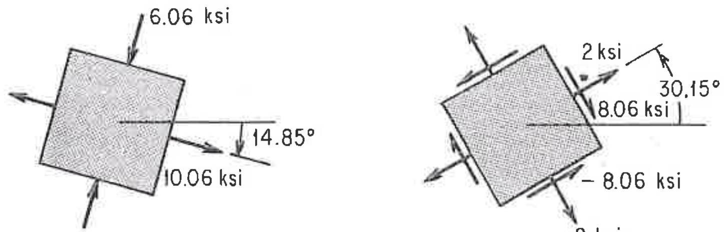
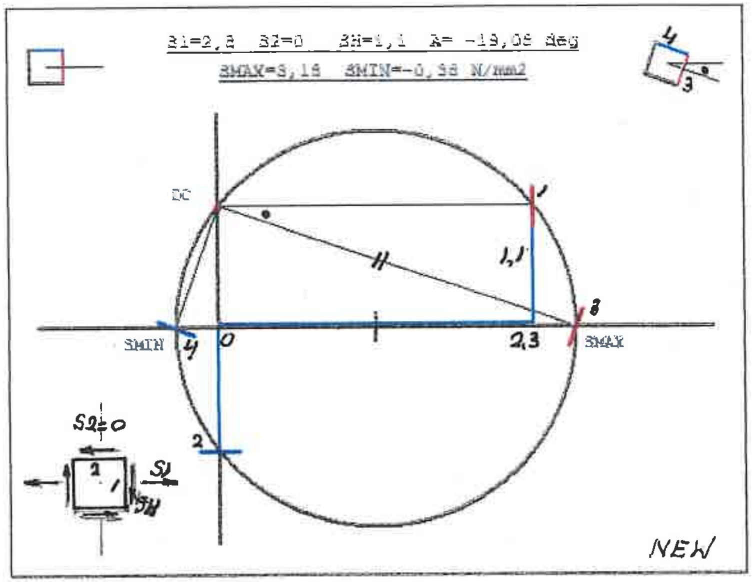


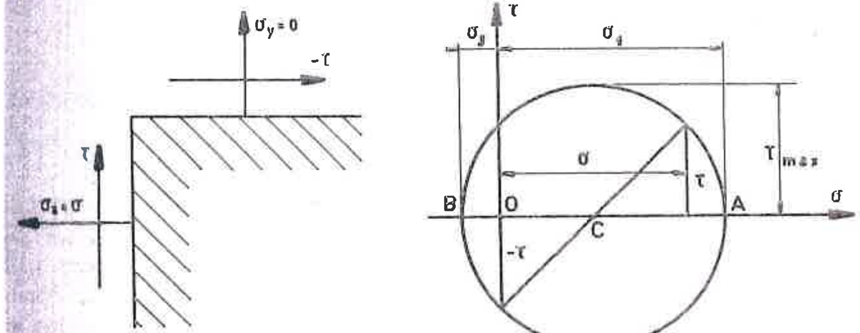
Fig. 197



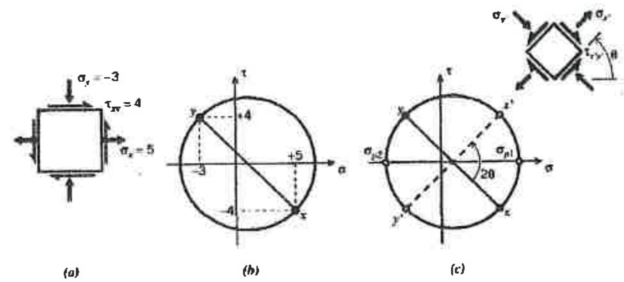




Remarquons que ces contraintes principales sont toujours de signes contraires.



Draw the stress square, noting the values on the x and y faces; Fig. 5(a) shows a hypothetical case for illustration. *For the purpose of Mohr's circle only,* regard a shear stress acting in a clockwise-rotation sense as being positive, and counter-clockwise as negative. The shear stresses on the x and y faces must then have opposite signs. The normal stresses are positive in tension and negative in compression, as usual.



their σ-τ values are the stresses on the rotated x'-y' axes as shown in Fig. 5(c).

There is nothing mysterious or magical about the Mohr's circle; it is simply a device to help visualize how stresses and other second-rank tensors change when the axes are rotated.

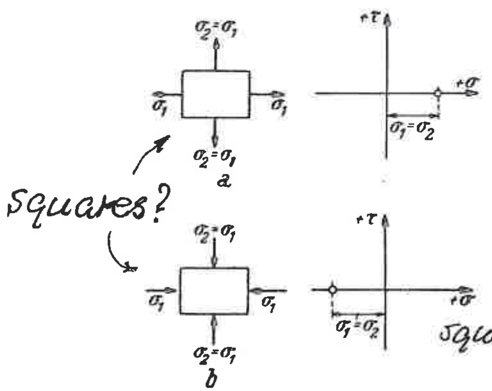


Abb. 9. Die Spannungszustände, a) ebener allseits gleicher Zug, b) ebener allseits

Beispiele. 1. *Allseits gleicher Zug bzw. allseit: gleicher Druck in der Ebene.* Nehmen wir an daß beide Hauptspannungen einander gleich und positiv sind, dann ergibt sich für die Normal- bzw. für die Schubspannung auf einem beliebigen Flächenelement aus den Gl. (5, 21)

$$\sigma_n = \sigma_1 = \sigma_2, \quad \tau_n = 0.$$

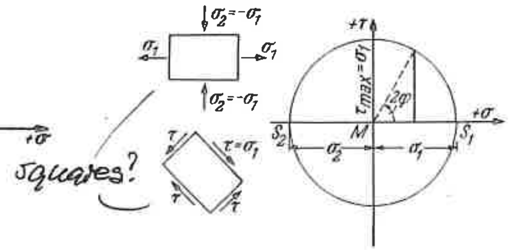


Abb. 10. Der ebene Spannungszustand „reiner Schub"

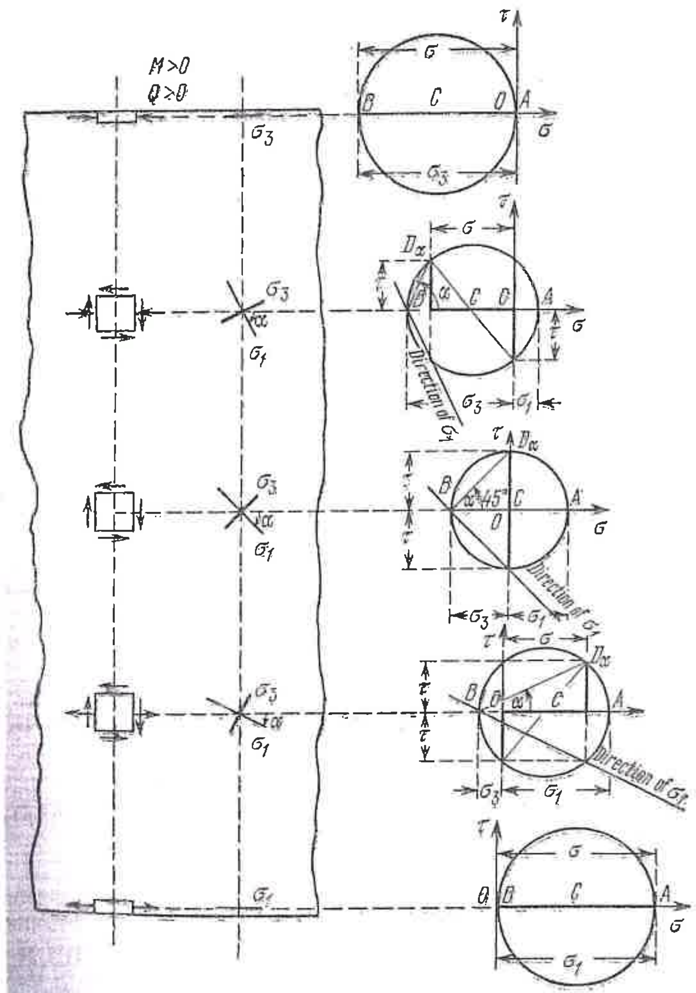


Fig. 198

direction of the principal stresses may be determine elp of the stress circle (Fig. 197). Suppose σ_α and τ_α, a ne perpendicular to the axis of the beam, are positive:

$$\sigma_\alpha = +\sigma = \frac{Mz}{J}$$

```
These parts here below found in a book. Talking even about
'two set of sign comventions', and many times the words
'positive' and 'negative' appear.... Really necessary???
```

Sign conventions

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The previous derivation for the equation of the Mohr Circle using Figure 4 follows the engineering mechanics sign convention. **The engineering mechanics sign convention will be used for this article.**

Physical-space sign convention

From the convention of the Cauchy stress tensor (Figure 3 and Figure 4), the first subscript in the stress components denotes the face on which the stress component acts, and the second subscript indicates the direction of the stress component. Thus τ_{xy} is the shear stress acting on the face with normal vector in the positive direction of the x-axis, and in the positive direction of the y-axis.

In the physical-space sign convention, positive normal stresses are outward to the plane of action (tension), and negative normal stresses are inward to the plane of action (compression) (Figure 5).
In the physical-space sign convention, positive shear stresses act on positive faces of the material element in the positive direction of an axis. Also, positive shear stresses act on negative faces of the material element in the negative direction of an axis. A positive face has its normal vector in the positive direction of an axis, and a negative face has its normal vector in the negative direction of an axis. For example, the shear stresses τ_{xy} and τ_{yx} are positive because they act on positive faces, and they act as well in the positive direction of the y-axis and the x-axis, respectively (Figure 3). Similarly, the respective opposite shear stresses τ_{xy} and τ_{yx} acting in the negative faces have a positive sign because they act in the negative direction of the x-axis and y-axis, respectively.

Mohr-circle-space sign convention

In the Mohr-circle-space sign convention, normal stresses have the same sign as normal stresses in the physical-space sign convention: positive normal stresses act outward to the plane of action, and negative normal stresses act inward to the plane of action.

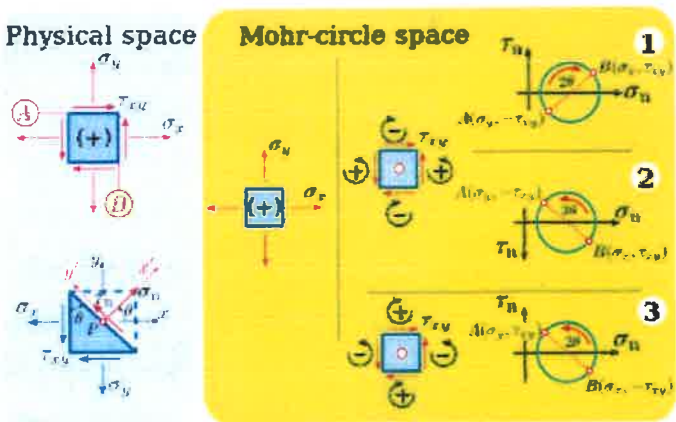
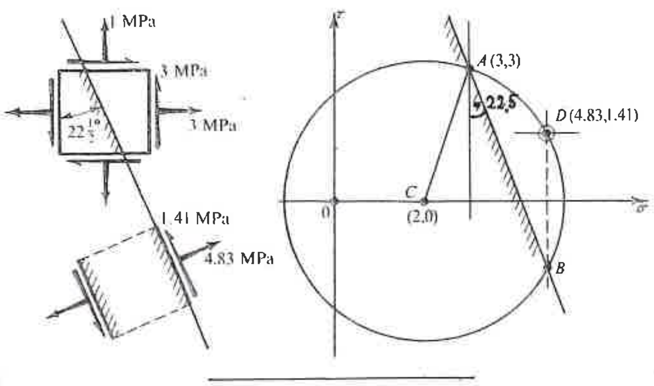
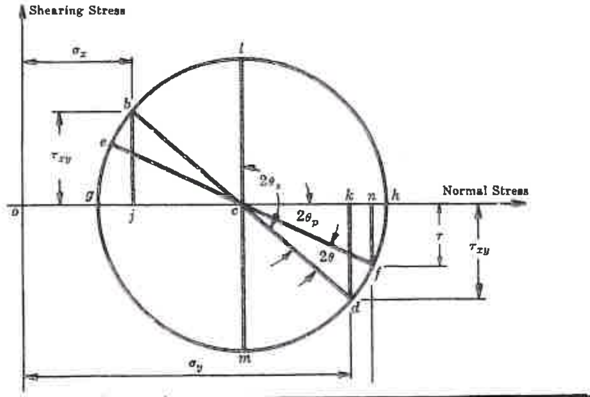


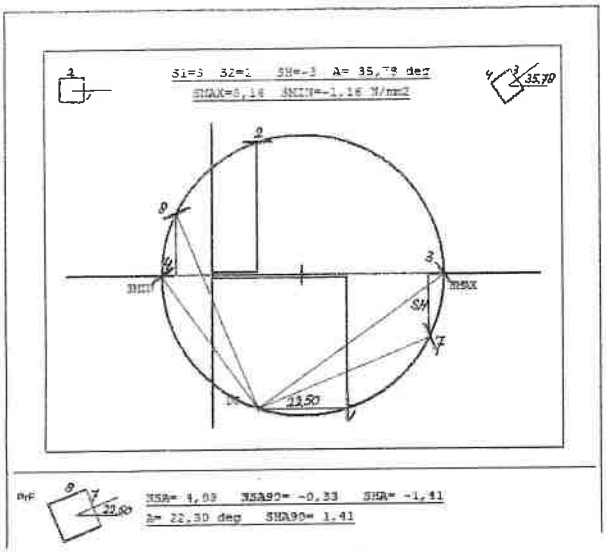
Figure 5. Engineering mechanics sign convention for drawing the Mohr circle. This article follows sign-convention # 3, as shown.

Shear stresses, however, have a different convention in the Mohr-circle space compared to the convention in the physical space. In the Mohr-circle-space sign convention, positive shear stresses rotate the material element in the counterclockwise direction, and negative shear stresses rotate the material in the clockwise direction. This way, the shear stress component τ_{xy} is positive in the Mohr-circle space, and the shear stress component τ_{yx} is negative in the Mohr-circle space.





A plane element is subject to the stresses shown in Fig. 17-53. Using Mohr's circ the principal stresses and their directions, (b) the maximum shearing stresses and the planes on which they occur.



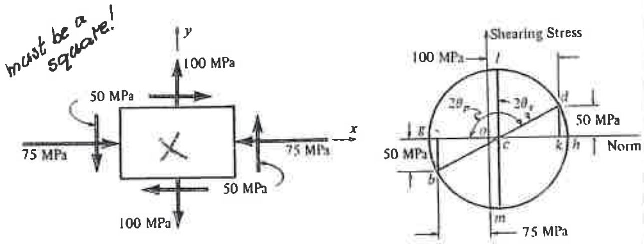
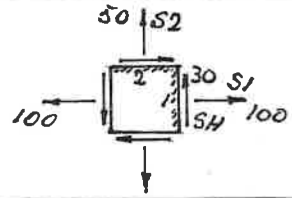


Fig. 17-53

Again we refer to Problem 17.14 for the procedure for constructing Mohr's circle. In ac sign convention outlined there, the shearing stresses on the vertical faces of the element are the horizontal faces positive. Thus the stress condition of $\sigma_x = -75$ MPa, $\tau_{xy} = -50$ MPa exist:

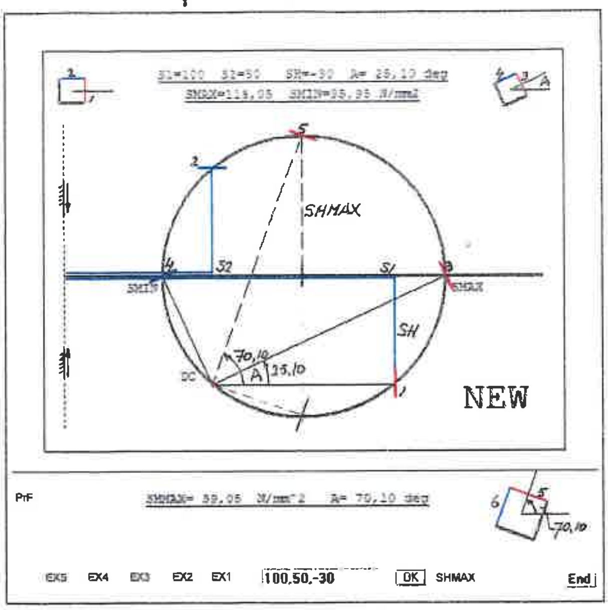


Shear stress SH 'belongs' to normal stress S1 as assumed. Here that SH is directed upward not as assumed downward, therefore SH below the horizontal zero line.

R=Sqr(((S1-S2)/2)^2+SH^2)

SMAX=(S1+S2)/2+R   SMIN=(S1+S2)/2-R

Angle A=(-Atn(SH/((S1-S2)/2))/2



S1= 100 N/mm^2   S2= 50 N/mm^2   SH= -30 N/mm2

(S1+S2)/2= (100+50)/2= 150/2= 75

(S1-S2)/2= (100-50)/2= 50/2= 25

R= Sqr((25)^2+(-30)^2)=Sqr(625+900)
R= Sqr(1525)= 39,1

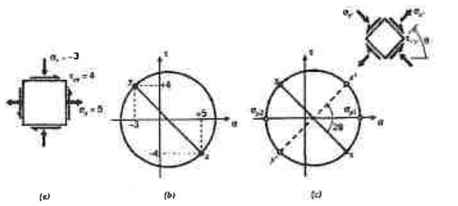
SMAX= 75,0+39,1= 114,1 N/mm^2
SMIN= 75,0-39,1= 35,9 N/mm^2

A= (-Atn(-30/25)/2=
   (-Atn(-1,20))/2 = 25,1 deg, a positiv answer, so to the left as assumed.

The dotted vertical line replaces the real line lying 1,7 cm to the left.

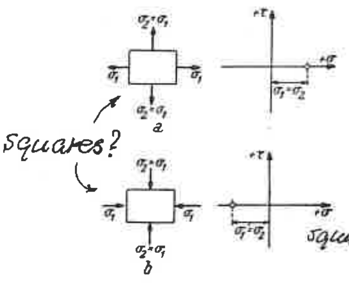
A click on SHMAX gives the striped line and the stress square turned 70,1 degrees to the left.

Draw the stress square, noting the values on the x and y faces; Fig. 5(a) shows a hypothetical case for illustration. *For the purpose of Mohr's circle only,* regard a shear stress acting in a clockwise-rotation sense as being positive, and counter-clockwise as negative. The shear stresses on the x and y faces must then have opposite signs. The normal stresses are positive in tension and negative in compression, as usual.



their σ-τ values are the stresses on the rotated x'-y' axes as shown in Fig. 5(c).

There is nothing mysterious or magical about the Mohr's circle; it is simply a device to help visualize how stresses and other second-rank tensors change when the axes are rotated.



Beispiele. 1. *Allseits gleicher Zug bzw. allseit: gleicher Druck in der Ebene.* Nehmen wir an daß beide Hauptspannungen einander gleich und positiv sind, dann ergibt sich für die Normal- bzw. für die Schubspannung auf einem beliebigen Flächenelement aus den Gl. (5, 21)

$$\sigma_n = \sigma_1 = \sigma_2, \quad \tau_n = 0.$$

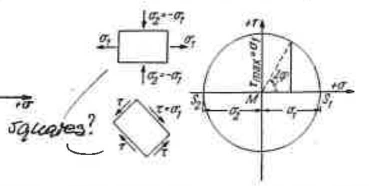


Abb. 9. Die Spannungszustände, a) ebener allseits gleicher Zug, b) ebener allseits

Abb. 10. Der ebene Spannungszustand „reiner Schub"

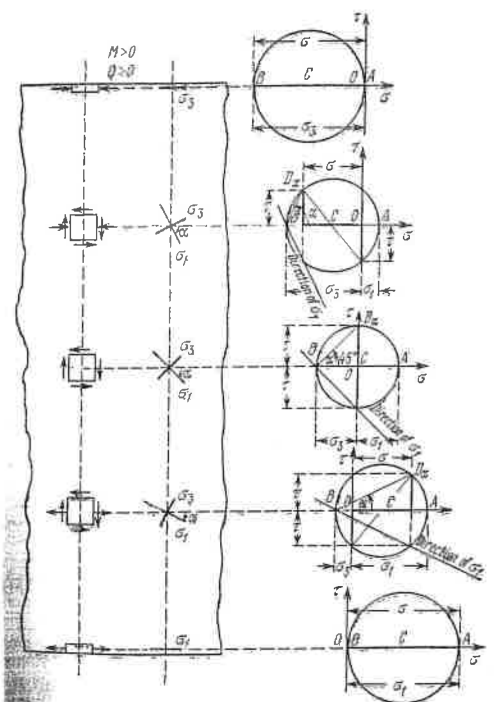


Fig. 198

direction of the principal stresses may be determin... of the stress circle (Fig. 197). Suppose σ_α and τ_α, a... perpendicular to the axis of the beam, are positive:

$$\sigma_\alpha = +\sigma = \frac{M_z}{J}$$

§ 78. Directions of the Principal Stresses

In the preceding section we determined only the magnitude of the principal stresses for an arbitrarily selected element without concerning ourselves with their direction. The results obtained were good

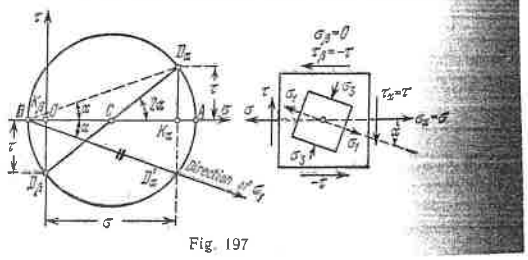
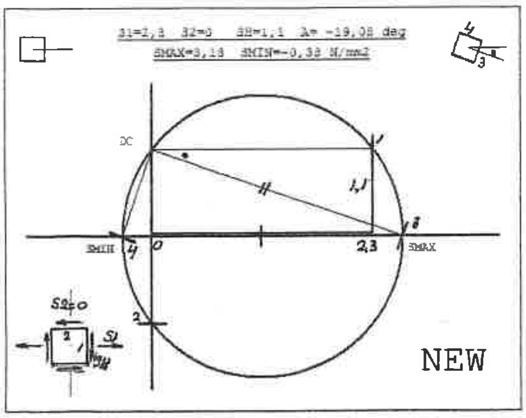
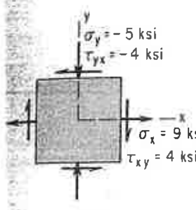
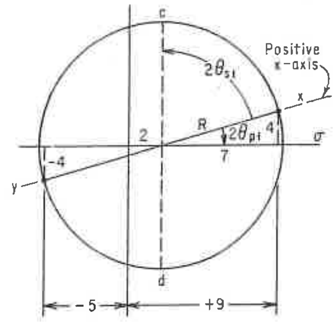


Fig. 197

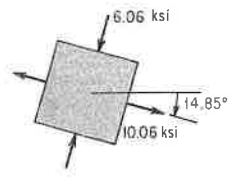




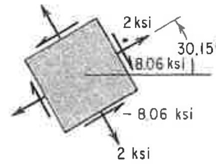
(a)



(b)

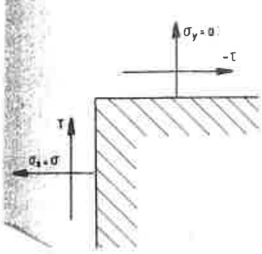


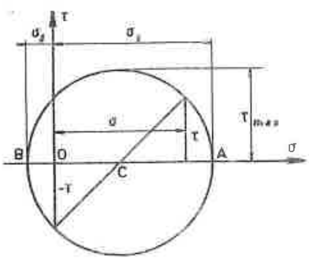
Principal normal stresses

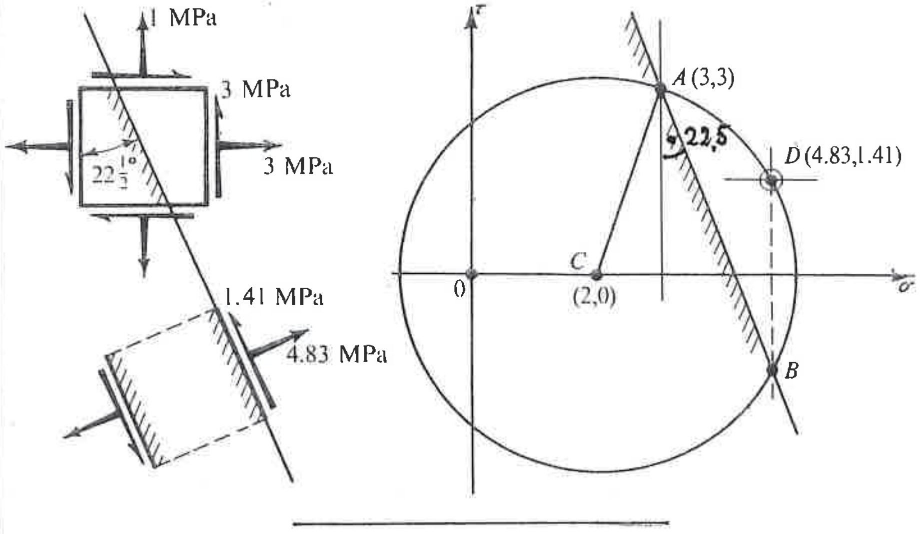


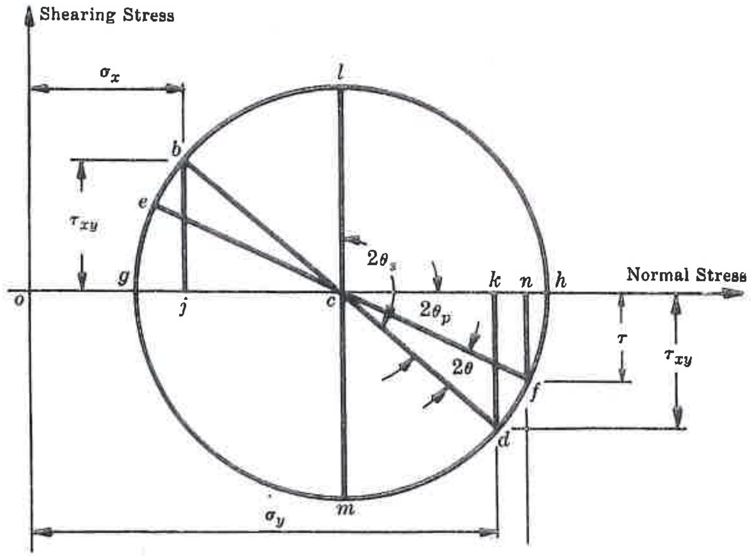
Principal shearing stresses

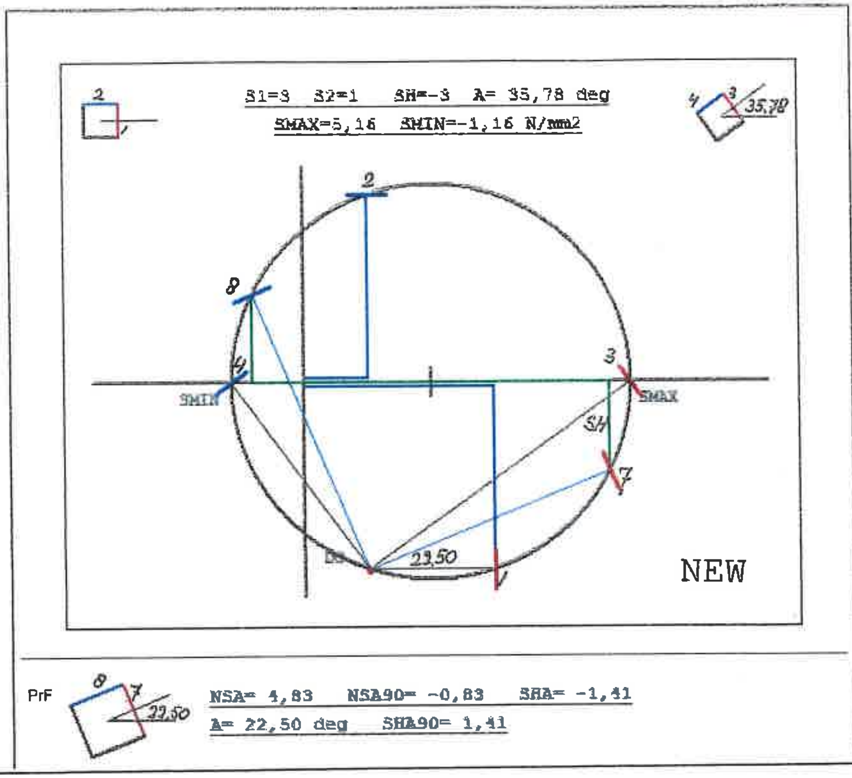
Remarquons que ces contraintes principales sont toujours de signes contraires.











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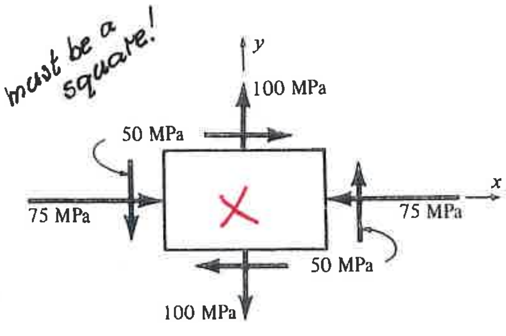

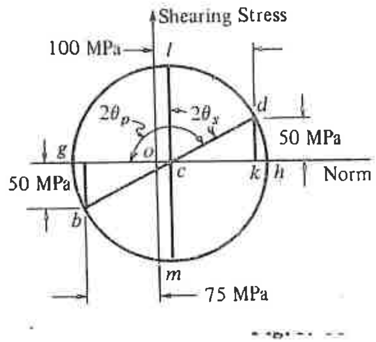

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These parts here below found in a book. Talking even about 'two set of sign comventions', and many times the words 'positive' and 'negative' appear.... Really necessary???

Sign conventions

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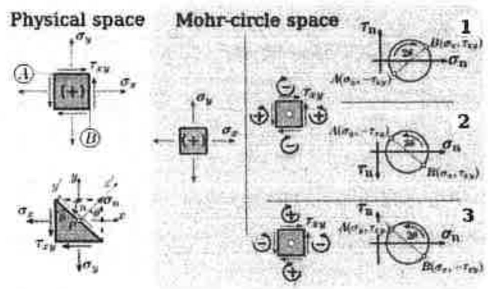
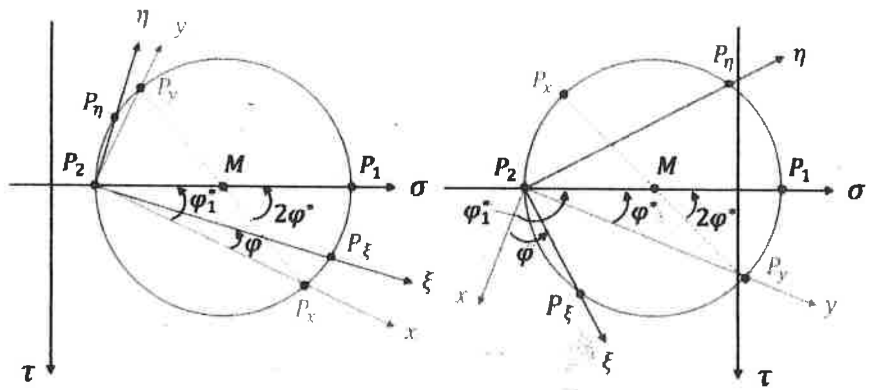


Figure 5. Engineering mechanics sign convention for drawing the Mohr circle. This article follows sign-convention # 3, as shown.

a) $\sigma_x > \sigma_y ; \tau_{xy} > 0$ b) $\sigma_x > \sigma_y ; \tau_{xy} < 0$



1 Verbindungen von P_2 mit P_x und P_y legen $x - y$ -Achsen fest! Richtungssinn von x beliebig, unter Beachtung eines Rechtssystems folgt der Richtungssinn von y.

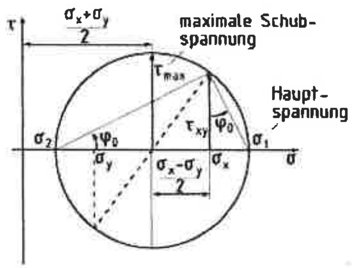
2 Von x -Achse ausgehend für gegebenen Winkel φ die ξ -Achse (\xi = Xi) zeichnen

3 Unter Beachtung des Richtungssinnes folgt die η -Achse (η = Eta)
→ Merke: Aus x wird Xi und aus y wird Eta!

4 Schnittpunkte der $\xi - \eta$ -Achse mit Kreis legen Punkte P_ξ und P_η fest

5 Abgreifen der Spannungen $P_\xi = (\sigma_\xi,\ \tau_{\xi\eta})$ und $P_\eta = (\sigma_\eta,\ -\tau_{\xi\eta})$ ↑

Mohrscher Spannungskreis



Mohrscher Spannungskreis

Der *Mohrsche Spannungskreis* ermöglicht die geometrische Transformation von Spannungen zu den Koordinatenachsen (σ_x, σ_y und τ_{xy}) in die herrschenden *Spannungen* einer Scheibe (σ_ξ, σ_η und $\tau_{\xi\eta}$) und unter einem beliebigen Winkel φ_0. Ein Kreis mit dem Radius *r* wird in einem $\tau - \sigma$ -Diagramm gezeichnet. Für die Konstruktion des *Mohrschen Spannungskreises* müssen σ_x, σ_y und τ_{xy} bekannt sein.

1.5 Van spanningscirkel naar cirkel van Mohr

Going from stress circle to circle of Mohr with first and second figure shown on the left.
Two pages of the book Sterkteleer 2.

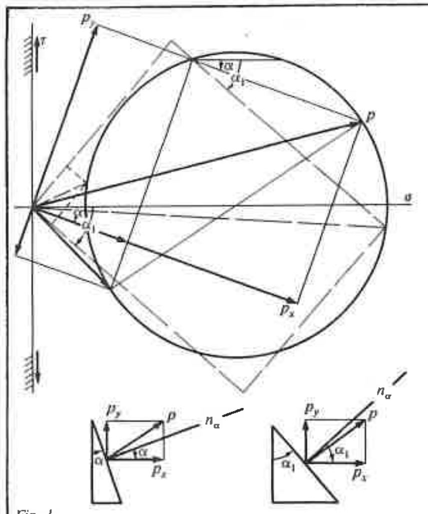


Fig. 1

Fig. 1
De spanningscirkel met de spanningsvectoren p, p_x en p_y voor twee α-vlakjes, $\alpha_l > \alpha$. Bij toename van α draait p_x *rechtsom* en het α-vlakje *linksom*. (In de figuur zijn eveneens p_x en p_y van het $\alpha+90$-vlakje weergegeven.)

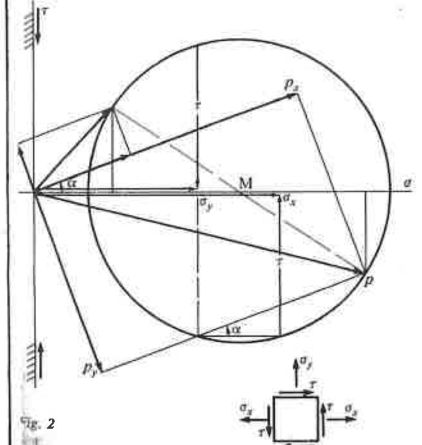


Fig. 2

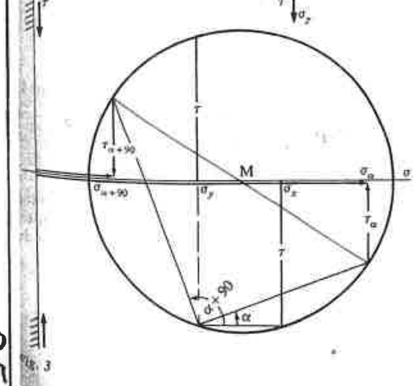


Fig. 3

Fig. 2 en 3
Door spiegeling om de σ-as zijn beide α-richtingen linksom geworden, zie figuur. De schuifspanning met oriëntatie ⇅ is nu naar boven uitgezet, die met ⇅ naar beneden. De oriëntatie van de spanningen op de kubusvlakjes blijven zoals ze waren! De nieuwe cirkel voldoet nu aan de wiskundige constructie, de z.g. cirkel van Mohr.

De formules voor de spanningen moeten nu worden gewijzigd om aan Mohr te voldoen. De spanningsformule σ_α is (zie hiervoor):

$$\sigma_\alpha = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha + \tau \sin 2\alpha$$

Van het verticale vlakje ⇅τ was τ naar boven uitgezet, en nu naar beneden, dus negatief geworden bij Mohr; σ_α is even groot gebleven, dan volgt de formule van Mohr voor σ_α.

$$\sigma_\alpha = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha - \tau \sin 2\alpha$$

Dezelfde redenering geldt voor $\sigma_{\alpha+90}$.
De schuifspanningsformule τ_α was

$$\tau_\alpha = \frac{-\sigma_x + \sigma_y}{2}\sin 2\alpha + \tau \cos 2\alpha$$

τ_α is bij Mohr naar beneden uitgezet, dus negatief geworden, zodat τ_α gewijzigd wordt in

$$\tau_\alpha = -\left(\frac{-\sigma_x + \sigma_y}{2}\sin 2\alpha + \tau \cos 2\alpha\right)$$

maar óók is de τ tegengesteld uitgezet, zodat de formule van Mohr voor τ_α wordt:

$$\tau_\alpha = -\left(\frac{-\sigma_x + \sigma_y}{2}\sin 2\alpha - \tau \cos 2\alpha\right) \quad \text{of tenslotte}$$

$$\tau_\alpha = \frac{\sigma_x - \sigma_y}{2}\sin 2\alpha + \tau \cos 2\alpha$$

Een zelfde redenering geldt voor $\tau_{\alpha+90}$.

De Mohr-formules voor de spanningen:

$$\sigma_\alpha = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha - \tau \sin 2\alpha$$

$$\sigma_{\alpha+90} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha + \tau \sin 2\alpha$$

$$\tau_\alpha = \frac{\sigma_x - \sigma_y}{2}\sin 2\alpha + \tau \cos 2\alpha$$

$$\tau_{\alpha+90} = -\left(\frac{\sigma_x - \sigma_y}{2}\sin 2\alpha + \tau \cos 2\alpha\right)$$

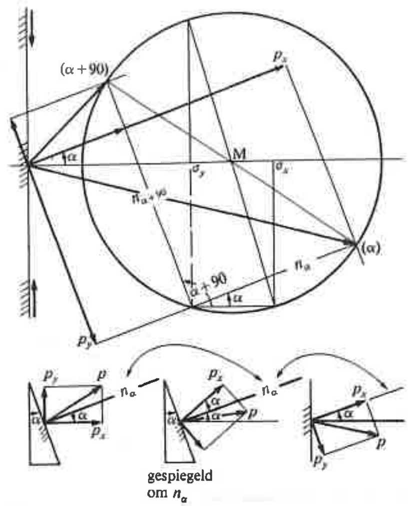


Fig. 4

Fig. 4
De spanningen σ_α en τ_α zijn afgebeeld in het punt (α) van de cirkel, $\sigma_{\alpha+90}$ en $\tau_{\alpha+90}$ in punt ($\alpha+90$). Door de spiegeling om de σ-as is de p_x-richting van het α-vlakje dezelfde geworden als de normaal n_α van dat vlakje. Dit α-vlakje is bij (α) in de cirkel getekend. Dezelfde redenering geldt voor het $\alpha+90$-vlakje.
Beide normalen n_α en $n_{\alpha+90}$ snijden elkaar in het *'richtingencentrum R.C. van de normalen'*. Men kan dus onder elke hoek α (en $\alpha+90$) een lijn door R.C. trekken die de cirkel snijdt en zo de spanningen σ_α en τ_α (en $\sigma_{\alpha+90}$ en $\tau_{\alpha+90}$) geeft.

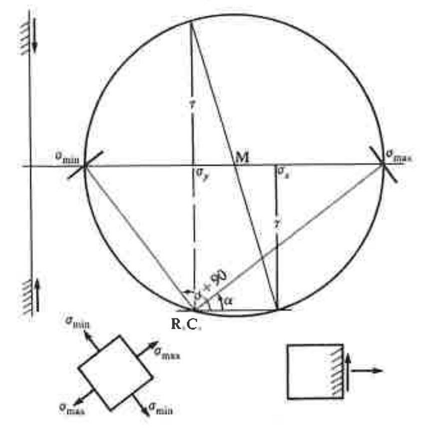


Fig. 5

Fig. 5
Trekt men twee lijnen door R.C. naar de snijpunten van de cirkel met de σ-as, dan vindt men de *hoofdspanningen* σ_{max} *en* σ_{min} (σ_{max} altijd rechts op de cirkel, σ_{min} links) en de bijbehorende vlakjes en hoeken. De grootte van spanningen en hoek α kunnen door opmeten worden gevonden.

De berekening verloopt als volgt:
de uiterste waarden van σ_α vindt men door $\frac{d\sigma_\alpha}{d\alpha} = 0$ te stellen.
De Mohrformule voor σ_α was

$$\sigma_\alpha = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha - \tau \sin 2\alpha \quad \text{zodat}$$

$$\frac{d\sigma_\alpha}{d\alpha} = 0 + \frac{\sigma_x - \sigma_y}{2}(-2\sin 2\alpha) - \tau(2\cos 2\alpha)$$

$$= -(\sigma_x - \sigma_y)\sin 2\alpha - 2\tau\cos 2\alpha = 0 \quad \text{dan is}$$

$$\frac{\sin 2\alpha}{\cos 2\alpha} = -\frac{2\tau}{\sigma_x - \sigma_y} \quad \text{of} \quad \boxed{\tan 2\alpha = \frac{-2\tau}{\sigma_x - \sigma_y}}$$

(de τ is die van het verticale vlakje, hier in de Mohrcirkel naar beneden uitgezet, dus negatieve waarde invullen). De gevonden waarde voor α ingevuld geeft σ_α en $\sigma_{\alpha+90}$ waarvan één σ_{max} en de ander σ_{min} zal zijn (zie ook blz. 29).
(Als men $\tau_\alpha = 0$ stelt vindt men
$\frac{\sigma_x - \sigma_y}{2}\sin 2\alpha + \tau\cos 2\alpha = 0$, en
ten slotte $\tan 2\alpha = \frac{-2\tau}{\sigma_x - \sigma_y}$)