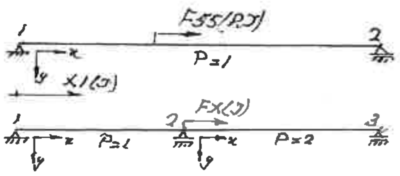
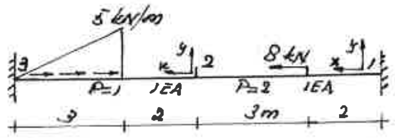
**Part 4**

Axially loaded continuous beams/members.

Each member has two member ends, each member end is connected with a joint.
The relation between member end forces and joint displacements depend on the strain stiffness EA of the members. They deliver the equations with the joint displacements as unknowns to be solved.





When the joint displacements are known the member end forces can be calculated.
Eor each member 2 equations are written,
each member delivers a stiffness matrix 2 x 2
which will be placed in the construction matrix.

Follows now the code of some basic subroutines written and explained. To be copied if wanted.


Private Sub MEMBER() 15
Calculation of primary forces.

Private Sub N5XX() 17
Calculation of the normal force each m.

Private Sub N5G() 19
Calculation of the normal force each G m.

Private Sub MEMBERMATS5AXMEMBER() 21
Stiffness matrix S5, size 2 x 2.

Private Sub CONSTRMATCCAXMEMBER() 20
Matric CC composed of matrices S5.
N9 joints, matrix CC size N9*1.

Private Sub **AXMAINCALC()** 22-25
Solving the equations with GAUSS. Part 12.
N9 joints with displacement UH(I),
N9*1 equations to be solved

Composing constructie stiffness matricX CC
with member stiffness matrices S5 with

**Program AXCC111()** and 37-41

**Program AXCC222()**. 42

## 1.1. The relation between member end forces and joint displacements.

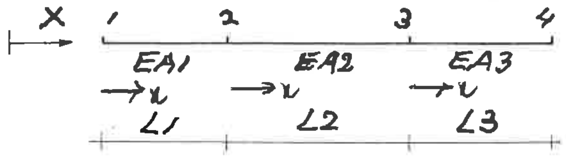



Fig.1.

Fig.1.
The drawn construction consists of (only) two members. The member ends are connected with the joints 1, 2 and 3, numbered from left to right.
E is the modulus of elasticity.
A1 and A2 are two cross-sectional areas.
EA is the strain stiffness, is E times A.
EA1 for member 1 and EA2 for member 2.
The member lengths are L1 and L2.
The member axes x are assumed to be directed from lowest to highest member end number, so to the right. The construction axis X is assumed to the right as well, but not necessary. For more see page      about x- and X axis.

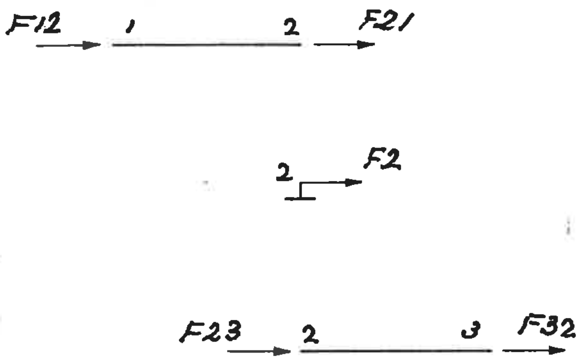



Fig.2.

Fig.2.
On the member ends of the from the joints loosened members act memberend forces, F12 and F21, F23 and F32. The assumption for their directions is to the right, according to the member axes. (Member axes and construction axis are not related, do not depend on each other.)

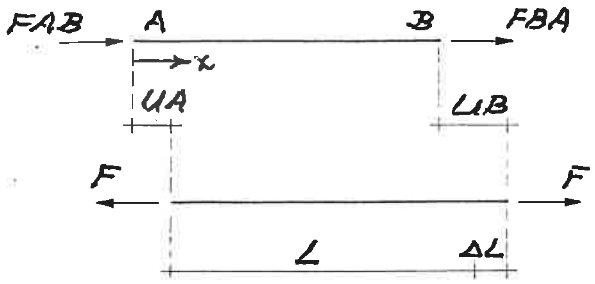



Fig.3.

Fig.3.
The joint displacements UA and UB, being also member end displacements, are assumed to be directed to the right, as the member axis x.
Now there are two possibilities to derive the same relation between memberend forces and joint displacements.

The first possibility.

Is UB larger then UA, then the member will become $\Delta L$=UB-UA longer. The member is a tension member. On the member ends act tension forces equal in magnitude, the forces F as the figure shows.
With Hooke's law is $\Delta L$=FL/EA.
(F times L divided by E times A.)
From which follows  F=(EA/L)$\Delta L$.
With member stiffness factor R=EA/L becomes F=R$\Delta L$. Then with $\Delta L$=UB-UA follows
F=R(UB-UA)  or  F=R(-UA+UB).

Memberend forces FAB and F at member end A are the 'same' forces. F=-FAB or FAB=-F.
With FAB=-F follows FAB=-R(-UA+UB)  or

FAB=R(UA-UB).  1)

Memberend forces FBA and F at member end B are the 'same' forces. F=FBA or FBA=F.

With FBA=F follows FBA=R(-UA+UB).  2)

Now two equations are found which give the relation between
memberend forces FAB and FBA, and
the joint displacements UA an UB, by using member stiffness factor R=EA/L.

$$\begin{bmatrix} FAB \\ FBA \end{bmatrix} = \begin{bmatrix} R & -R \\ -R & R \end{bmatrix} \cdot \begin{bmatrix} UA \\ UB \end{bmatrix}$$

$\underline{f}$ S5 $\underline{u}$

$$\begin{bmatrix} S5(1,1) & S5(1,2) \\ S5(2,1) & S5(2,2) \end{bmatrix}$$

Fig.4.

Fig.4.
The two equations can be represented in matrix form in which is

$\underline{f}$ the force vector (or force column),

S5 the member stiffness matrix, and

$\underline{u}$ the displacement vector (or -column).

An element of $\underline{f}$ is equal to a row of matrix S5 multiplied by column $\underline{u}$.

FAB= S5(1,1)*UA + S5(1,2)*UB

R*UA -R*UB

FBA= S5(2,1)*UA + S5(2,2)*UB

-R*UA +R*UB

These memberend forces FAB and FBA arise in consequence of the joint displacements UA and UB.

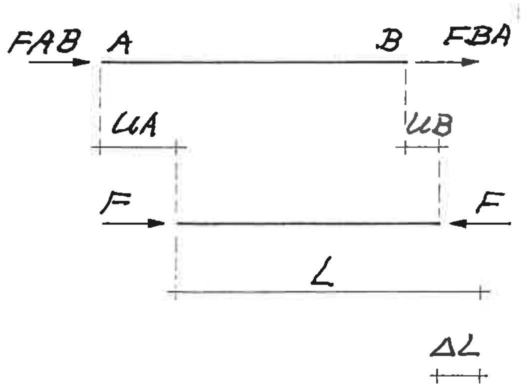



Fig.5.

<u>The second possibility.</u>

Fig.5.
Not displacement UB is larger then UA, but now UA is larger then UB. The member will become ΔL=UA-UB shorter. It is a compression member. At the member ends act equal compression forces F because the member is in equilibrium.
With Hooke's law is ΔL=FL/EA, or F=(EA/L)Δ

With member stiffness factor R=EA/L

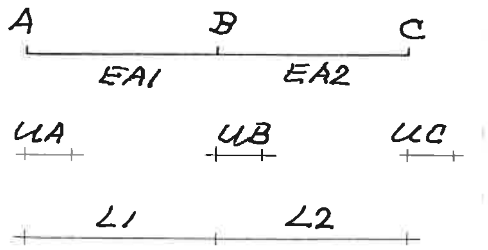
becomes F=RΔL zodat <u>F=R(UA-UB).</u>

At member end A both memberend forces represent one single force.
Then FAB=F so that <u>FAB=R(UA-UB).</u> 1)

To memberend B applies the same.
Then FBA=-F so that FBA=-R(UA-UB) or

<u>FBA=R(-UA+UB).</u> 2)

One finds the same two equations as in the case of the tension member of fig.3. (The relation between $\underline{f}$ , S5 and $\underline{u}$ for the same member cannot be different ofcourse.)
The relation between memberend forces and joint displacements is determined by strain stiffness EA and member length L, so by member stiffness factor R=EA/L.
If the construction consists of one single member then construction stiffness matrix CC (following page) is the same member stiffness matrix S5.




R1=EA1/L1 R2=EA2/L2

$$\begin{bmatrix} FAB \\ FBA \end{bmatrix} \quad \begin{bmatrix} R1 & -R1 \\ -R1 & R1 \end{bmatrix} \quad \begin{bmatrix} UA \\ UB \end{bmatrix}$$

$$\begin{bmatrix} FBC \\ FCB \end{bmatrix} \quad \begin{bmatrix} R2 & -R2 \\ -R2 & R2 \end{bmatrix} \quad \begin{bmatrix} UB \\ UC \end{bmatrix}$$

Fig.6.

Fig.6.
If the construction consists of two members then one gets two sets of two equations on the left represented in matrix form. Both sets of two equations can be united to one single set of three equations with the three unknown displacements UA, UB and UC.

## 1.2. From member matrices S5 to construction matrix CC.

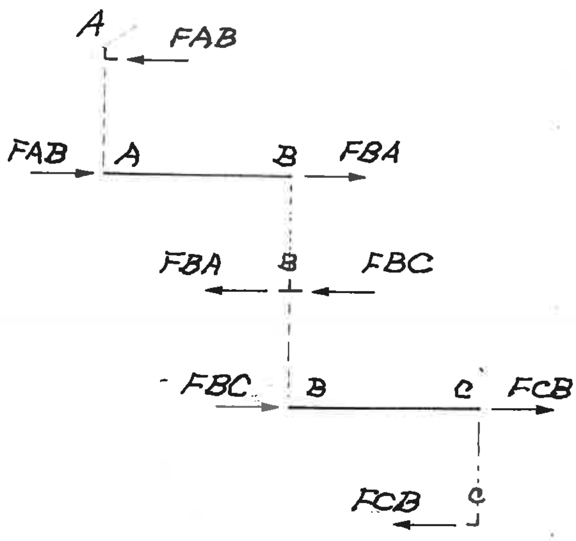



Fig.7.

Fig.7.
Joints and members are seperated/loosened from each other. The on the member ends working memberend forces are, according assumption, directed to the right. On the joints act opposite directed memberend forces equal in magnitude, directed to the left.

On joint A works, see fig.6,

FAB = R1*UA -R1*UB + 0*UC 1)

On joint B works

FBA+FBC= -R1*UA +R1*UB +R2*UB -R2*UC
= -R1*UA + (R1+R2)*UB -R2*UC 2)

On joint C works

FCB = 0*UA -R2*UB +R2*UC 3)

$$\underbrace{\begin{bmatrix} FAB \\ FBA+FBC \\ FCB \end{bmatrix}}_{\underline{f}} = \underbrace{\begin{bmatrix} R1 & -R1 & 0 \\ -R1 & R1+R2 & -R2 \\ 0 & -R2 & R2 \end{bmatrix}}_{CC} \cdot \underbrace{\begin{bmatrix} UA \\ UB \\ UC \end{bmatrix}}_{\underline{u}}$$

Thus arise three equations which are on the left represented in matrix form.
With forcevector f,
construction stiffness matrix CC, and
displacement vector u.
Both sets of two equations are extended to a set of three equations as given here below, which will be added.

FAB= R1*UA-R1*UB+ 0*UC 1')
FBA=-R1*UA+R1*UB+ 0*UC 2')
0 = 0*UA+ 0*UB+ 0*UC 3')

0 = 0*UA+ 0*UB+ 0*UC 1'')
FBC= 0*UA+R2*UB-R2*UC 2'')
FCB= 0*UA-R2*UB+R2*UC 3'')

Adding equation 1') and 1'') gives equation 1) as shown here above, FAB= ... And so on.

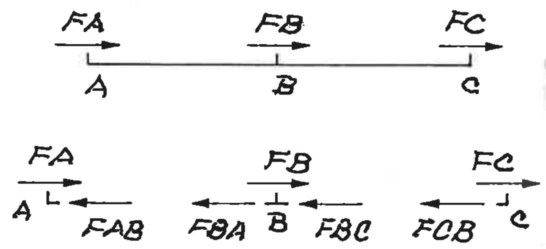



Fig.8.

Fig.8.
On the joints work the memberend forces, and the joint load forces FA, FB, FC of which the assumption for the direction is to the right. Each seperated joint must be in equilibrium.

Σ hor. joint A =0

FA-FAB=0 ⇒ FAB=FA

Σ hor. joint B =0

FB-FBA-FBC=0 ⇒ FBA+FBC=FB

Σ hor. joint C =0

FC-FCB=0 ⇒ FCB=FC

$$\underbrace{\begin{bmatrix} R1 & -R1 & 0 \\ -R1 & R1+R2 & -R2 \\ 0 & -R2 & R2 \end{bmatrix}}_{CC} \cdot \underbrace{\begin{bmatrix} UA \\ UB \\ UC \end{bmatrix}}_{\underline{u}} = \underbrace{\begin{bmatrix} FA \\ FB \\ FC \end{bmatrix}}_{\underline{f}}$$

In this way one gets a set of equations from which the displacements can be solved.

Example.

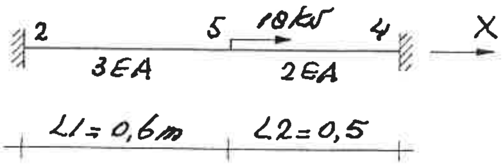



Fig.1.

Fig.1.
The three joint numbers are arbitrarily chosen, they are memberend numbers as well.
The strain stifnesses are expressed in EA.
The member stiffness factors are

R1=EA1/L1=3EA/0,6=5EA and

R2=EA2/L2=2EA/0,5=4EA.

$$\begin{bmatrix} F25 \\ F52 \end{bmatrix} = EA \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix} \cdot \begin{bmatrix} U2 \\ U5 \end{bmatrix}$$

$$\begin{bmatrix} F54 \\ F45 \end{bmatrix} = EA \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \cdot \begin{bmatrix} U5 \\ U4 \end{bmatrix}$$

The joint load forces are
F2=0 kN,   F5=18 kN  and  F4=0 kN.

As on the preceding page the set of three equations can be found. To come to a solution one, or two, displacements must be known. Here of both cantilevers. The displacements of joint 2 and 4 are given, are prescribed. U2=0 and U4=0. To compute the unknown displacement U5 only one equation is needed. Three equations will be kept, but some elements of CC and $\underline{f}$ will be changed,

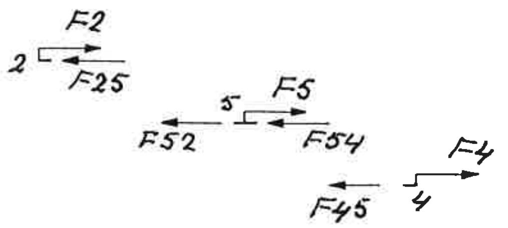



Fig.2.

Fig.2.
Joint displacement U2 is prescribed, so not an unknown. Then first row and first column of construction matrix CC are filled with zeros, but the element on the main diagonal is made CC(1,1)=1.
The elements of force vector $\underline{f}$ do not change because displacement U2 is zero.
The same for U4. Third row and third column are filled with zeros and CC(3,3) becomes 1.
The first and third element of force vector $\underline{f}$ are zero because the joint load forces F2 and F4 are zero. If they were not zero then they would have been made zero because U2=0 and U4=0.

$$EA \underbrace{\begin{bmatrix} 5 & -5 & 0 \\ -5 & 5+4 & -4 \\ 0 & -4 & 4 \end{bmatrix}}_{CC} \underbrace{\begin{bmatrix} U2 \\ U5 \\ U4 \end{bmatrix}}_{\underline{u}} \underbrace{\begin{bmatrix} 0 \\ 18 \\ 0 \end{bmatrix}}_{\underline{f}}$$

$$EA \begin{bmatrix} 1 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U2 \\ U5 \\ U4 \end{bmatrix} \begin{bmatrix} 0 \\ 18 \\ 0 \end{bmatrix}$$

In this way the number of equations remains the same. And in this way the sets of equations in programs

CC · $\underline{u}$ = $\underline{f}$  will be prepared to solve the set

AA · $\underline{x}$ = $\underline{b}$  with the elimination method of

GAUSS.

See program GAUSSNEQUATIONS.

Written out the equations become

EA(1*U2 +0*U5 +0*U4)= 0   ⇒   U2=0

EA(0*U2 +9*U5 +0*U4)=18   or

EA(9*U5)=18   ⇒   U5=2/EA.

EA(0*U2 +0*U5 +1*U4)= 0   ⇒   U4=0

And so the equations are solved.
(Numbering the joints from left to right with 2, 5 and 4 might be a little bit 'strange', but it is possible. Ofcourse, when programming a regular way of numbering is necessary to avoid problems.)

Now as the displacements are known all the memberend forces can be computed.

$$\begin{bmatrix} F25 \\ F52 \end{bmatrix} = EA \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix} \begin{bmatrix} U2 \\ U5 \end{bmatrix}$$


Fig.3a.


Fig.3b.

Fig.3a en 3b.
With the two equations for the first member follows when U2=0 and U5=2/EA

F25=EA(5*U2-5*U5)
=EA(5*0 -5*2/EA)=EA(-10/EA)=-10 kN.

The answer for F25 is negative. Thus the member end force is not directed to the right as assumed but directed to the left. The force does not press on the member end 2 but pulls at the member end.

F52=EA(-5*U2+5*U5)
=EA(-5*0 +5*2/EA)=EA(10/EA)=10 kN

A positive answer for F25. So this member end force is directed to the right as assumed. The force pulls at member end 5.
One sees now that the member is a tension member.

$$\begin{bmatrix} F54 \\ F45 \end{bmatrix} = EA \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \begin{bmatrix} U5 \\ U4 \end{bmatrix}$$


fig.4a.


fig.4b.

Fig. 4a en 4b.
In the same way for the second member with U5=2/EA en U4=0.

F54=EA(4*U5-4*U4)
=EA(4*2/EA-4*0)=EA(8/EA)=8kN

A positive answer for F54. Thus the member end force is directed as assumed to the right. The force presses on memberend 5.

F45=EA(-4*U5+4*U4)
=EA(-4*2/EA+4*0)=EA(-8/EA)=-8kN

A negative answer. Thus member end force F45 is not directed to the right as assumed, but to the left. The force presses on member end 4.
The member is a compression member.

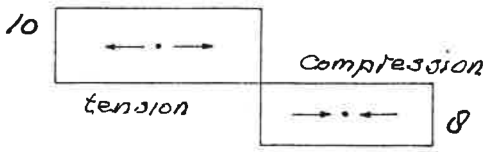



Fig.5.

Fig 5.
Normal force diagram.

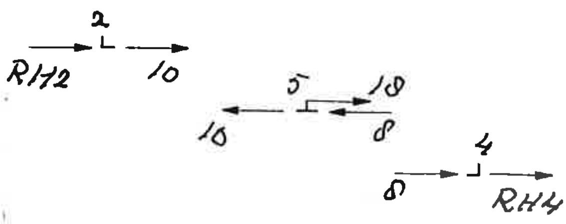



Fig.6.

Fig.6.
Now the memberend forces acting on the joints are drawn with their real directions, equal in magnitude but opposite directed to those of fig. 3b and 4b.
The reactions are assumed to be directed to the right and are found with horizontal equilibrium of the joints.

Σ hor. joint 2 =0
RH2+10=0 ⇒ RH2=-10 kN

Σ hor. joint 4 =0
RH4+8=0 ⇒ RH4= -8 kN

For both ractions a negativ answer. So they are not directed to the right as assumed but to the left.
Joint 5 is in equilibrium, 18-10-8=0.

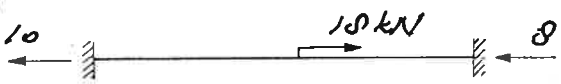



Fig.7.

Fig.7.
The construction is in equilibrium, Σ hor. =0.

## 1.3. Joint load forces and hold forces.

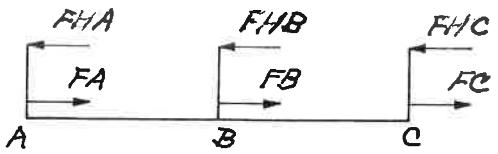



Fig.1.

Fig.1.
The construction consists of two members and three joints. While unloaded the joints A, B and C are hold at place with the hold forces FHA, FHB and FHC. Assumed directions to the left.
Next the joint load forces FA, FB and FC are applied. Assumed directions to the right.

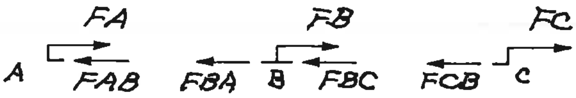



Fig.2.

Fig.2.
When the joints are released the hold forces are not there anymore and the joint load forces become active. The construction deforms, the members deform and the joints displace. At the member ends arise member end forces directed to the right according assumption to the right.
On the joints act forces as large as the memberend forces but opposite directed, thus to the left.

$$\begin{bmatrix} FAB \\ FBA+FBC \\ FCB \end{bmatrix} \text{ becomes } \underbrace{\begin{bmatrix} FA \\ FB \\ FC \end{bmatrix}}_{\underline{f}}$$

Fig.3.

Fig.3.
As shown on page 4 force vector $\underline{f}$ will be filled with joint load forces using horizontal equilibrium.
After that the unknown displacements UA, UB and UC are solved out of the equations.

Joint load forces, and

member load forces, and

hold forces.

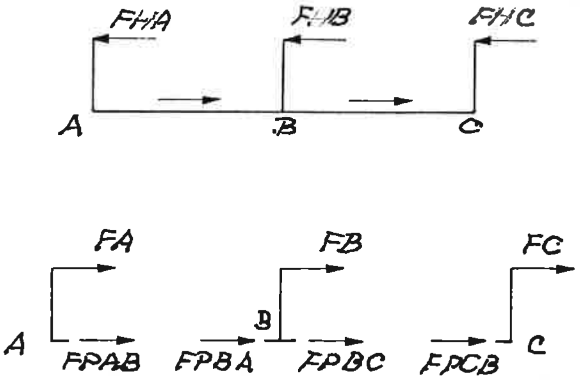


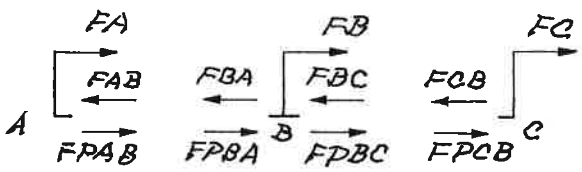

Fig.4.

Fig.4.
When also member loads are applied between the joints with an asssumed direction to the right then the hold forces must become larger to keep the joints at their places.

Because of these member load forces on the still hold joints will work to the right directed forces FPAB, FPBA, FPBC and FPCB.

These forces are called primary forces.

These forces are computed as the reactions of the on both ends fixed members.




$$\begin{bmatrix} FAB \\ FBA+FBC \\ FCB \end{bmatrix} \text{ becomes } \underbrace{\begin{bmatrix} FA+FPAB \\ FB+FPBA+FPBC \\ FC+FPCB \end{bmatrix}}_{\underline{f}}$$

Fig.5.

Fig.5.
As above the elements of force vector $\underline{f}$ follow from the equilibrium of the joints.

Σ hor. joint A =0
FA+FPAB-FAB=0 ⇒ FAB=FA+FPAB

Σ hor. joint B =0
FB+FPBA+FPBC-FBA-FBC=0 ⇒ FBA+FBC=FB+FPBA+FPBC

Σ hor. joint C =0
FC+FPCB-FCB=0 ⇒ FCB=FC+FPCB

Thus, force vector $\underline{f}$ is now filled with joint-load forces plus primary forces.

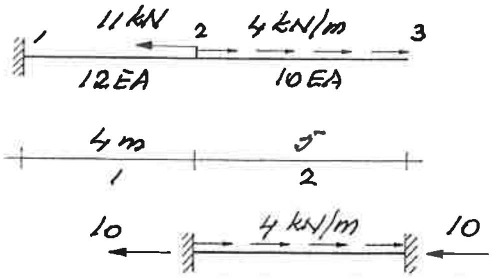
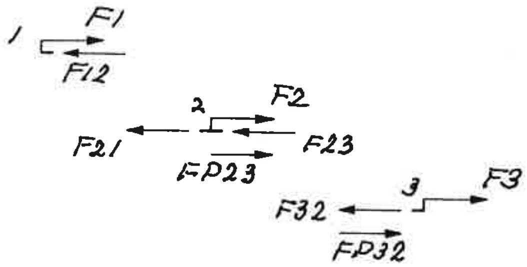



Fig.1.

$$\begin{bmatrix} F12 \\ F21 \end{bmatrix} = EA \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} U1 \\ U2 \end{bmatrix}$$

$$\begin{bmatrix} F23 \\ F32 \end{bmatrix} = EA \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} U2 \\ U3 \end{bmatrix}$$




$$\underset{\underline{f}}{\begin{bmatrix} F12 \\ F21+F23 \\ F32 \end{bmatrix}} = EA \underset{CC}{\begin{bmatrix} 3 & -3 & 0 \\ -3 & 3+2 & -2 \\ 0 & -2 & 2 \end{bmatrix}} \underset{\underline{u}}{\begin{bmatrix} U1 \\ U2 \\ U3 \end{bmatrix}}$$

Fig.2.

$$EA \underset{CC}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & -2 \\ 0 & -2 & 2 \end{bmatrix}} \underset{\underline{u}}{\begin{bmatrix} U1 \\ U2 \\ U3 \end{bmatrix}} = \underset{\underline{f}}{\begin{bmatrix} 0 \\ -1 \\ 10 \end{bmatrix}}$$

Fig.3.

<u>Example.</u>

Fig.1.
The construction consists of two members and three joints which are numbered from left to right.
The member stiffness factors are
R1=EA1/L1=12EA/4=3EA and R2=EA2/L2=10EA/5=2EA.

The joint load forces are
F1=0 kN, F2=-11 kN and F3=0 kN.
Along member 2 acts a uniformly distributed load of 4 kN/m directed to the right.
The reactions of this member hold at both ends are (5*4)/2=10 kN, they are directed to the left, on the joints in opposite direction, thus to the right.
In the following calculation rate number EA is omitted.

Fig.2 en 3.
The elements of force vector <u>f</u> follow with Σ hor. =0 of the joints.

The primary forces are
FP23=10 kN and FP32=10 kN.

Σ hor. joint 1 =0
F1-F12=0 ⇒ <u>F12</u>=F1=0 kN

Σ hor. joint 2 =0
F2+FP23-F21-F23=0
⇒ <u>F21+F23</u>=F2+FP23=-11+10=-1 kN

Σ hor. joint 3 =0
F3+FP32-F32 ⇒ <u>F32</u>=F3+FP32=0+10=10 kN

Fig.3.
The displacement of joint 1 is known, is prescribed, is U1=0. Therefore the first row and first column of construction matrix CC are filled with zeros except the diagonal element, this becomes CC(1,1)=1. (See page 4 .)

The first element of force vector <u>f</u> is zero, F12=F1=0.
Multiplication of the first row of CC with <u>u</u> delivers
1*U1 +0*U2 +0*U3=0 (this zero is the first element of <u>f</u>) and gives U1=0. That's correct, but if this first element of <u>f</u> is not zero, thus if F1<>0, then would become U1<>0; in that case one must correct U1 and make it zero.
Because the whole set of three equations will be used in the programmatic solution......later.
At this moment without programming. Then there are two equations left to solve.

```
 5*U2-2*U3=- 1        2)
-2*U2+2*U3= 10   +    3)
 3*U2      =  9   from which  U2=3,
```

and given in eq. 2) follows
5*3-2*U3=-1 or -2*U3=-16 so that U2=8.
(And with EA then U2=3/EA and U3=8/EA.)

The answers for U2 en U3 are positive, joint 2 en 3 displace as assumed to the right.

## 1.4. Calculation of the member end forces.

$$\begin{bmatrix} F12 \\ F21 \end{bmatrix} = EA \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} U1 \\ U2 \end{bmatrix}$$


Fig.4a.

Fig.4a.
Also now calculating omitting stiffness EA, it 'disappears'.
With U1=0 and U2=3 follow with 'row times column',
F12= 3*0-3*3=-9 kN
F21=-3*0+3*3= 9 kN

These are member end forces as result of displacements alone. As there are no member loads these forces are the final member end forces.


Fig.4b.

Fig.4b.
The member end forces as they really act on the member ends.
A negative answer for F12, so not directed to the right as assumed, but to the left.
A positive answer for F21, so as assumed directed to the right.

$$\begin{bmatrix} F23 \\ F32 \end{bmatrix} = EA \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} U2 \\ U3 \end{bmatrix}$$


Fig.5a.

Fig.5a.
Next member 2 with U2=3 and U3=8.
F23= 2*3-2*8= 6-16=-10 kN
F32=-2*3+2*8=-6+16= 10 kN

These are member end forces as reult of displacements alone.


Fig.5b.

Fig.5b.
The memberend forces as they really act on the member ends.

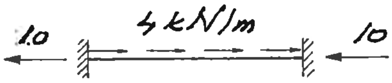



Fig.5c.

Fig.5c.
As result of member loads alone, arise on the before holded/fixed member ends forces of 10 kN directed to the left.


Fig.5d.

Fig.5d.
The member end forces as result of displacements alone, fig.5b, and
memberend forces as result of member loads alone, fig.5c,
when added they deliver the final member end forces of member 2.
At member end 2 a force of 20 kN which pulls on the member end, and at member end 3 a force equal to zero.

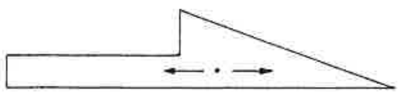


Fig.6.

Fig.6.
The normal force diagram. The members are subjected to tension.

$$EA \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} U1 \\ U2 \\ U3 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 3 \\ 8 \end{bmatrix}$$

CC    u    f

Fig.7.
The elements of the total force vector f are calculated using the original, not altered, construction matrix CC.

K1=F12     = 3*0 -3*3 +0*0= 0- 9+ 0= -9 kN

K2=F21+F23= -3*0 +5*3 -2*8= 0+15-16= -1 kN

K3=F32     = 0*0 -2*3 +2*8= 0- 6+16= 10 kN

These are the so-called joint forces as result of the displacements alone, assumed direction to the left.

Fig.7.

More about this later, see page .

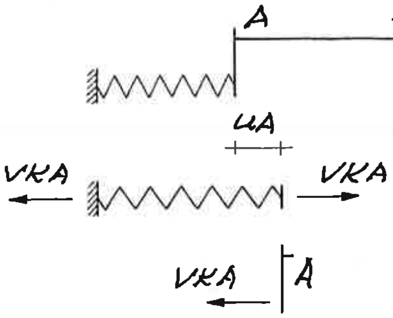




Fig. 1a.

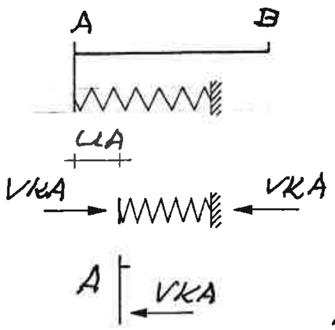



Fig. 1b.

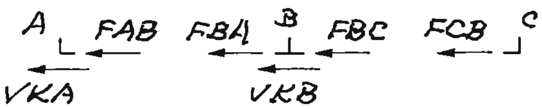



Fig. 2.

$$\begin{bmatrix} FAB+\underline{VKA} \\ FBA+FBC+\underline{VKB} \\ FCB \end{bmatrix} =$$

$\underline{f}$

$$\begin{bmatrix} R1+\underline{SA} & -R1 & \\ -R1 & R1+R2+\underline{SB} & -R2 \\ & -R2 & R2 \end{bmatrix} \cdot \begin{bmatrix} UA \\ UB \\ UC \end{bmatrix}$$

CC $\quad$ $\underline{u}$

## 1.5. The elastic/springy support.

Fig.1a.
Joint A is supposed to be elasticly supported. A spring is drawn at A, a bit large, and the axis does not coincide with the member axis, just to be more clear.

If joint A displaces UA to the right the spring will be stretched.
The member will exercise on the spring end on the right a spring force VKA to the right.

On joint A itself a spring force VKA is exercised to the left.

With SA as spring constant follows VKA ← =SA*UA.

Fig.1b.
In this case the spring is pushed in if joint A displaces UA as assumed to the right.
Then the member will exercise on the spring end on the left a spring force VKA to the right.

On joint A itself a spring force VKA is exercised to the left.

Fig.2.
Then on joint A act member end force FAB=R1*UA-R1*UB, see page 3 , and spring force VKA=SA*UA.

The first element of the force vector becomes FAB+VKA=R1*UA+SA*UA-R1*UB=(R1+$\underline{SA}$)*UA-R1*UB.

Is also joint B elastic supported then if joint B displaces UB to the right the on the joint acting spring force VKB is directed to the left.
Then VKB ← =SB*UB.

On joint B act now member end force
FBA=-R1*UA+R1*UB, and member end force
FBC= R2*UB-R2*UC, and spring force
VKB= SB*UB.
The second element of $\underline{f}$ becomes

FBA+FBC+VKB=-R1*UA+R1*UB+R2*UB-R2*UC+SB*UB or

=-R1*UA+(R1+R2+$\underline{SB}$)*UB-R2*UC.

If there's no spring at C then the third element of force vector $\underline{f}$
FCB=-R2*UB+R2*UC.

The relation between member end forces and spring forces, and the joint displacements, is given on the left in matrix form.

The spring constants SA and SB are stiffness factors like R1=EA1/L1 and R2=EA2/L2.

EA1/L1 dimension, [kN/m^2]*[m^2]/m is [kN/m].

So one can see now that when a joint is elasticly supported then the concerning spring constant is added to the belonging/concerning diagonal element of construction matrix CC.

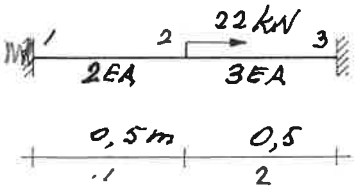



Fig.1.

$$\begin{bmatrix} F12 \\ F21 \end{bmatrix} = EA \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \cdot \begin{bmatrix} U1 \\ U2 \end{bmatrix}$$

$$\begin{bmatrix} F23 \\ F32 \end{bmatrix} = EA \begin{bmatrix} 6 & -6 \\ -6 & 6 \end{bmatrix} \cdot \begin{bmatrix} U2 \\ U3 \end{bmatrix}$$

S5

$$\underset{\underline{f}}{\begin{bmatrix} F12+VK1 \\ F21+F23 \\ F32 \end{bmatrix}} = EA \underset{C}{\begin{bmatrix} 4 & -4 & 0 \\ -4 & 4+6 & -6 \\ 0 & -6 & 6 \end{bmatrix}} \cdot \underset{\underline{u}}{\begin{bmatrix} U1 \\ U2 \\ U3 \end{bmatrix}}$$

Fig.2.

$$EA \underset{C}{\begin{bmatrix} 6 & -4 & 0 \\ -4 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \cdot \underset{\underline{u}}{\begin{bmatrix} U1 \\ U2 \\ U3 \end{bmatrix}} = \underset{\underline{f}}{\begin{bmatrix} 0 \\ 22 \\ 0 \end{bmatrix}}$$

Fig.3.

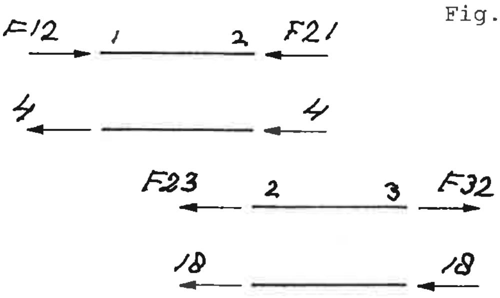



Fig.4.

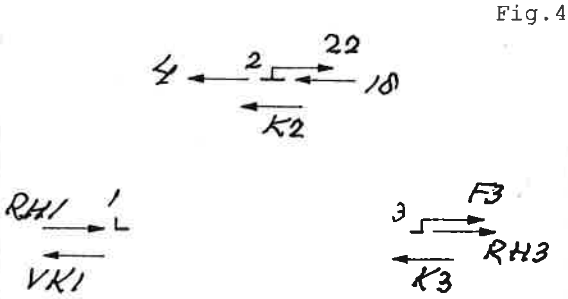



Fig.5.

Example.

Fig.1.
The strain stiffness factors are

R1=EA1/L1=2EA/0,5= 4EA   and

R2=EA2/L2=3EA/0,5= 6EA.

Joint 1 is elasticly supported. The spring constant S1, a stiffness factor as well, is here expressed in strain stiffness EA, S1=2EA.
The displacement of joint 3 is prescribed, U3=0.
In the calculation EA is omitted. The displacements one will find finally in 1/EA.

Fig.2.
The first element of force vector f is member end force F12 plus spring force VK1.
The first diagonal element C(1,1) is stiffness factor R1=4 plus spring constant S1=2.

Fig.3.
Joint load forces F1=0 and F2=0, and F2=22 kN are as assumed directed to the right.
The second element of f then becomes 22 kN.
The two equations to find U1 and U2 now are

```
 6*U1- 4*U2= 0         1)
-4*U1+10*U2=22         3)  times 1.5 gives

-6*U1+15*U2=33         3')   1) + 3') gives

      11*U2=33   thus  U2=3/EA,  in 1) follows
 6*U1-4*3=0    so that U1=2/EA.
```

Fig.4 en 1.
With f = S5 u the member end forces for each member can be determined.

```
Member 1.
F12= 4*U1-4*U2= 4*2-4*3= 8-12= -4  kN
F21=-4*U1+4*U2=-4*2+4*3=-8+12=  4  kN
Member 2.
F23= 6*U2-6*U3= 6*3-6*0= 18-0= 18  kN
F32=-6*U2+6*U3=-6*3+6*0=-18+0=-18  kN
```

The forces acting on the member ends and on joint 2 are drawn with their real directions.
Member 1 is a tension member and member 2 a compression member.
Joint 2 is in equilibrium.
Calculation of the joint forces. They are the elements of f of fig.2.

```
K1=F12+VK1= 6*2- 4*3+0*0= 12-12=  0  kN
K2=F21+F23=-4*2+10*3-6*0= -8+30= 22  kN
K3=F32    = 0*2- 6*3+6*0        =-18  kN
```

Fig.5.
The spring force is reaction force as well. Is assumed that the direction of reaction force RH1 to the right, and of the spring force to the left, see preceding page, then is
RH1=-VK1=-S1*UA=-2EA*2/EA=-4  kN. Minus 4, so not as assumed to the right but to the left.
RH3 follows with Σ hor. joint 3 =0.
RH3+F3-K3=0    or    RH3+0-(-18)=0  so that
RH3=-18  kN, thus directed to the left.

## 1.6. About construction axis X and member axis x.

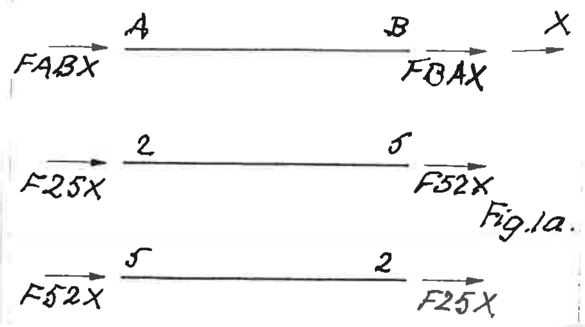



Fig.1b.

$$\begin{bmatrix} F25X \\ F52X \end{bmatrix} = \begin{bmatrix} R & -R \\ -R & R \end{bmatrix} \begin{bmatrix} U2 \\ U5 \end{bmatrix} \qquad \begin{bmatrix} UA \\ UB \end{bmatrix}$$

$$\begin{bmatrix} F52X \\ F25X \end{bmatrix} = \begin{bmatrix} R & -R \\ -R & R \end{bmatrix} \begin{bmatrix} U5 \\ U2 \end{bmatrix} \qquad \begin{bmatrix} UA \\ UB \end{bmatrix}$$

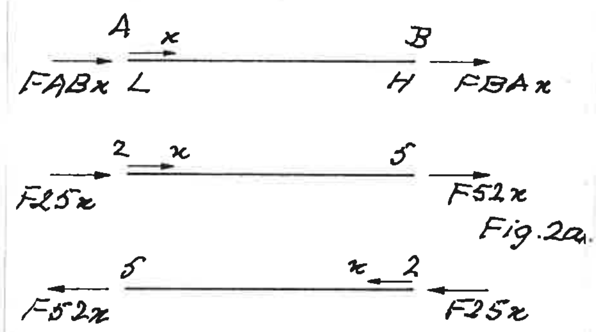



Fig.2b.

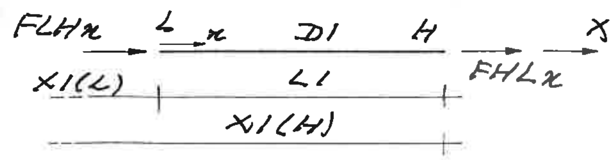



Fig.3a.

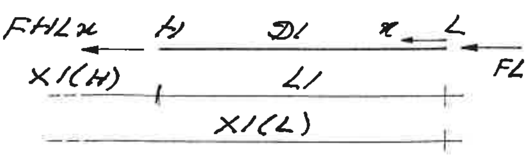



Fig.3b.

Fig. 1a and 1b.
FABX and FBAX are member end forces with respect to construction axis X, assumed to be directed to the right. The capital letter X indicates these forces.

Fig. 2a and 2b.
It is assumed that the origin of member axis x is A and that this axis is directed from A to B. The member end forces FABx and FBAx (indicated by the small letter x) are directed according to the direction of the x-axis.
Is the lowest member end number L equal to A, and the highest member end number H equal to B, then the member end forces FABx and FBAx are directed according to the x-axis from L to H.
There are no member loads between the joints, or member ends.
Member end forces w.r.t. construction axis X and member axis x are equal.

Fig.1a and 2b.

F25X= F25x   F52X= F52x

Fig.1b and 2a.

F25X=-F25x   F52X=-F52x

One may say that the second member 2a resp. 2b is equal to the first member 1a resp. 1b, which is turned over 180 degrees about A.

Fig. 3a, 2a and 1a.
If the coördinates X1(L) and X1(H) of the joints, or member ends, are given then one can write
D1=X1(H)-X1(L)  and the member length becomes
L1=SQR(D1^2) and is  C=D1/L1.
D1 is positive, thus C=D1/L1=+1, also positive.
The member end forces w.r.t. member axis x are

<u>FLHx= FLHX·C</u> = F25X·C  is  F25X  and

<u>FHLx= FHLX·C</u> = F52X·C  is  F52X.

Fig.3b, 2b and 1b.
Now the member end numbers are exchanged.
The member axis x is now directed to the left, and construction axis X is directed to the right as is assumed.
D1=X1(H)-X1(L)  Also now, first the coordinate with the highest joint number H, and then the coordinate with the lowest joint number L.
L1=SQR(D1^2) and  C=D1/L1.
D1 is negative, then C=D1/L1=-1, also negative.
The member end forces can be found with the formulas here above.

<u>FLHx= FLHX·C</u> = F25X·C  is  -F25X  and

<u>FHLx= FHLX·C</u> = F52X·C  is  -F52X.

Negative answers in this case. Member end forces FLHx and FHLx are not directed to the left according to the assumed direction of the x axis, but to the right.

## 1.7. Primary forces as result of member load forces along the member.

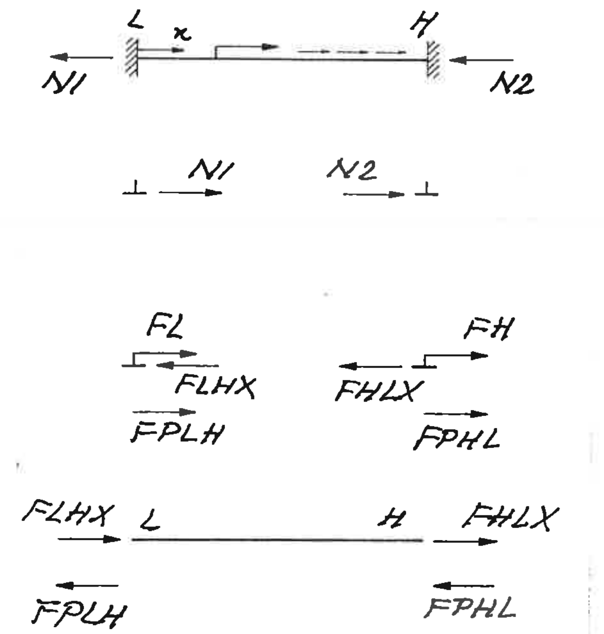



Fig.4a.

Fig.4a.
It is assumed that the concentrated loads and distributed loads are directed as the member axis x from the lowest member end number L to the highest member end number H.

While unloaded the member is hold at both member ends. Then the loads are applied. The reactions which arise are directed to the left, N1 at L and N2 at H. On the joints these forces are directed to the right.

The member loads deliver on the joints acting primary forces FPLH and FPHL with an assumed direction as the construction axis X to the right. So on the member ends they are opposite directed, to the left.

Force vector f is now filled with joint load forces and primary forces:
the L-th element with FL+FPLH= FL+N1·C  and
the H-th element with FH+FPHL= FH+N2·C.
See fig.3a preceding page.
C=D1/L1=+1
Forces N1 and FPLH acting on joint L, and the forces N2 and FPHL acting on joint H have the same direction.
When the member end forces FLHX and FHLX as result of the joint displacements are calculated they together with the primary forces will give the final member end forces.

FLHX  becomes  FLHX-FPLH= FLHX-N1·C
FHLX  becomes  FHLX-FPHL= FHLX-N2·C

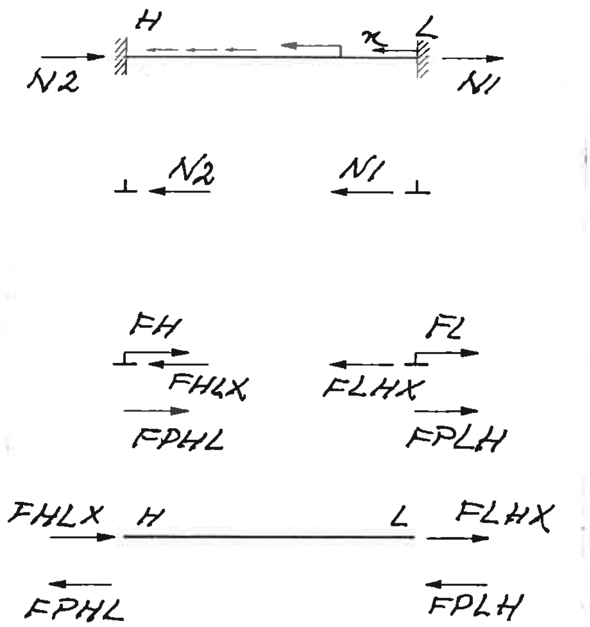



Fig. 4b.

Fig.4b.
Now the member end numbers are exchanged. The member axis x is directed from lowest to highest memberend number, thus to the left.
The assumption for the direction of the member loads is that of the member axis x. The reaction forces N1 and N2 acting on the member ends are now directed to the right, so on the joints to the left.

The elements of the force vector are now also:
the L-th element with FL+FPLH= FL+N1·C  and
the H-th element with FH+FPHL= FH+N2·C.
See fig. 3b.
C=D1/L1=-1
The on joint L acting forces N1 and FPLH are opposite directed, and
the on joint H acting forces N2 and FPHL are opposite directed.

For the final member end forces follows:

FLHX  becomes  FLHX-FPLH= FLHX-N1·C  and
FHLX  becomes  FHLX-FPHL= FHLX-N2·C.

The on the member ends acting forces N1 and FLHX have the same direction and must be summed/added. That's right because C=-1.
The same applies for N2 and FHLX.

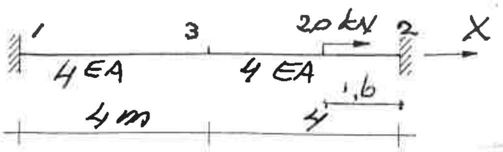



Fig.1.

$$\begin{bmatrix} FLHX \\ FHLX \end{bmatrix} = EA \begin{bmatrix} R & -R \\ -R & R \end{bmatrix} \begin{bmatrix} UL \\ UH \end{bmatrix}$$

$\underline{f}$ S5 $\underline{u}$

$$\begin{bmatrix} F13X \\ F31X \end{bmatrix} = EA \begin{bmatrix} 1 & -1 \\ -1 & \underline{1} \end{bmatrix} \begin{bmatrix} U1 \\ U3 \end{bmatrix}$$

$$\begin{bmatrix} F23X \\ F32X \end{bmatrix} = EA \begin{bmatrix} 1 & -1 \\ -1 & \underline{1} \end{bmatrix} \begin{bmatrix} U2 \\ U3 \end{bmatrix}$$

$$\begin{bmatrix} F13X \\ F23X \\ F31X+F32X \end{bmatrix} = EA \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & \underline{2} \end{bmatrix} \begin{bmatrix} U1 \\ U2 \\ U3 \end{bmatrix}$$

$\underline{f}$ CC $\underline{u}$

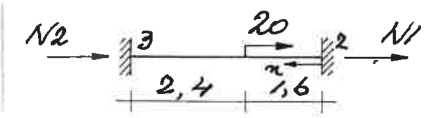



Fig.2a.

$$\begin{bmatrix} F13X \\ F23X \\ F31X+F32X \end{bmatrix} = \begin{bmatrix} F1 \\ F2+FP23 \\ F3+FP32 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 12 \end{bmatrix}$$

$$EA \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} U1 \\ U2 \\ U3 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 12 \end{bmatrix}$$

CC $\underline{u}$ $\underline{f}$

Fig.3.

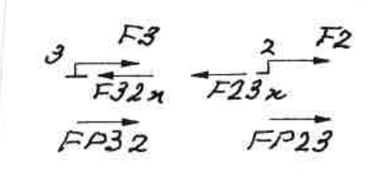



Fig.2b.

Example.

Fig.1.
The construction consists of two members with equal length and the same strain stifness 4EA. The three joints are numbered 1, 2 and 3 (step 1) but arbitrarely to explain the preceding page.

The member stiffness factor is
R=4EA/L1=4EA/4=1.

On the left for both members the relation
$\underline{f}$ = S5 $\underline{u}$ is given.
Both sets of equations will be composed to a set of three equations with memberend forces and displacements in order 1, 2, 3.

The joint load forces are all zero.
F1=0 kN  F2=0 kN  F3=0 kN

Fig.2a.
The second member is loaded with a to the right directed member load force of 20 kN.
Member axis x is directed from L to H, from joint 2 to joint 3. For the member loads it was assumed that they are directed as the member axis x. From that followed the assumption that the forces N1 en N2 are opposite directed to the x-axis. N1 at the lowest, N2 at the highest memberend number.

N1=-(20*2,4)/4= -12 kN and
N2=-(20*1,6)/4= -8 kN. (see formula page 45)

Fig.2b.
On joint 1 acts F1=0, and

on joint 2 acts F2=0 and the primary force FP23=N1·C, and

on joint 3 acts F3=0 and the primary force FP32=N2·C.

The origin of construction axis X is joint 1, so the joint coordinates are known.

X1(1)= 0 m  X1(2)= 8 m  X1(3)= 4 m

For the second member is
D1=X1(H)-X1(L)=X1(3)-X1(2)=4-8= -4 m, and is
C=D1/L1=-4/4=-1.

Then the primary forces become
FP32=N2·C= (-8)(-1)= 8 kN and
FP23=N1·C=(-12)(-1)= 12 kN.
The load of 20 kN causes on the joints 2 and 3 primary forces directed to the right indeed.

Fig.3.
The force vecor is filled with joint load forces and primary forces.
Construction matrix CC will be altered because of the prescribed displacements U1=0 and U2=0.
Finally there is one single equation left.

EA(0*U1+0*U2+2*U3)=12 from which follows
U3=6/EA.

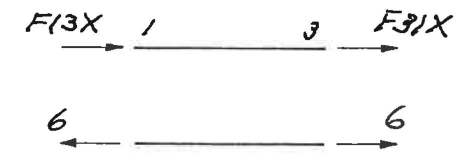



Fig.4.

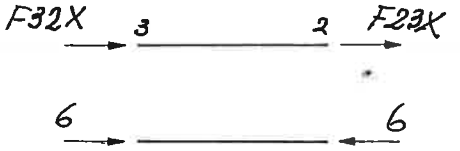



Fig.5a.

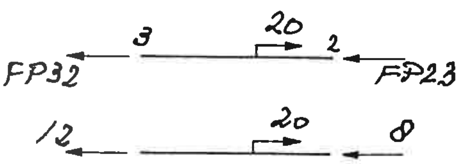



Fig.5b.


Fig.5c.

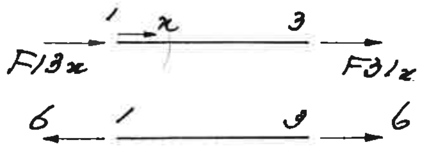



Fig.6a.

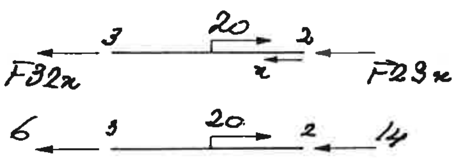



Fig.6b.

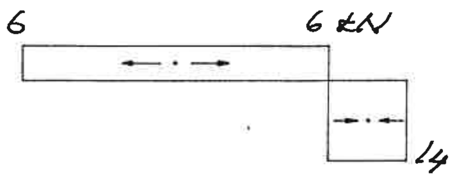



Fig.7.

1.8. Member end forces with respect to construction axis X.

Fig.4.
By means of $\underline{f}$ = S5 $\underline{u}$ for the first member one finds
F13X=EA(1*U1-1*U3)
=EA(0-1(6/EA))=-6 kN,
a negative answer, so not as assumed directed to the right but to the left, and
F31X=EA(-1*U1+1*U3)
=EA(0+1(6/EA))= 6 kN,
a positive answer, so as assumed directed to the right.
There are no member loads so that these forces are the final member end forces.
Fig.5a.
For the second member follow
F23X=EA(1*U2-1*U3)
=EA(0-1(6/EA))=-6 kN, not directed to the right but to he left, and
F32X=EA(-1*U2+1*U3)
=EA(0+1(6/EA))= 6 kN, directed to the right as assumed.
These are member end forces caused by the displacements alone!
Fig.5b.
The on the joints acting primary forces are assumed to be directed to the right like is assumed for joint load forces and the X-axis.
On the member ends act forces equal in magnitude but opposite directed, so to the left.
On the preceding page was found
FP23= 8 kN and FP32=12 kN which are the result of the member load.
The final member end forces are found by adding fig.5a and 5b.

Fig.5c.
F23X becomes F23X-FP23=-6-(-8) =-14 kN, not directed to the right but to the left.
The force pushes on member end 2.
F32X becomes F32X-FP32= 6-(-12)= -6 kN, not directed to the right but to the left.
The forse pulls on member end 3.

<u>Member end forces w.r.t. member axis x.</u>

Fig.6a. (see also fig.4.) <u>The first member.</u>
D1=X1(H)-X1(L)=X1(3)-X1(2)=4-0= 4
C=D1/L1=4/4=+1
<u>FLHx= FLHX·C</u> F13x= F13X·C=-6(+1)=-6 kN, so not as assumed from L to H, as the x-axis, but opposite directed, to the left.
<u>FHLx= FHLX·C</u> F31x= F31X·C= 6(+1)= 6 kN, so as assumed directed as the x-axis to the right.
Fig.6b. (see also fig.5c.) <u>The second member.</u>
D1=X1(H)-X1(L)=X1(3)-X1(2)=4-8=-4
C=D1/L1=-4/4=-1
<u>FLHx= FLHX·C</u> F23x= F23X·C=-14(-1)=14 kN, so as the x-axis directed to the left.
<u>FHLx =FHLX·C</u> F32x= F32X·C= -6(-1)= 6 kN, so as the member axis x directed to the left.

Fig.7.
The normal force diagram. Left of the concentrated load a tension force of 6 kN, right of it a compression force of 14 kN.

## 1.9. Private Sub MEMBER()

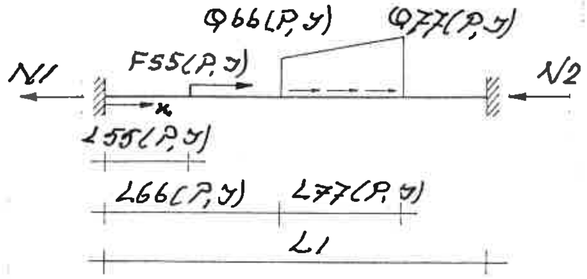



Fig.1.

Subroutine for the calculation of the reactions N1 and N2, of the at both ends holded/fixed member as result of member loads along the member. The primary forces are opposite directed to N1 and N2, see page 12.
For a member P there are
NFA(P) concentrated loads, and
NQA(P) distributed loads.

Fig.1 en 2a.
For each load case the reactions N3 and N4 are calculated and added to preceding values of N1 and N2; in the beginning they are set N1=0 and N2=0.


Fig. 2a.

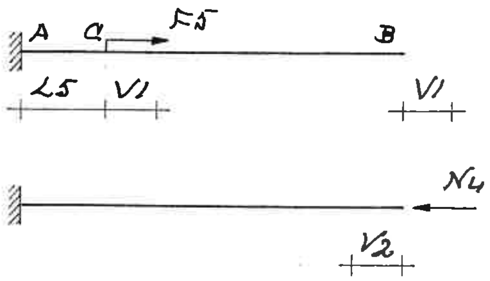



Fig.2b.

The concentrated loads.

Fig.2a en 2b.
For I=1 To NFA(P)
The load forces are F55(P,I) and the distances are L55(P,I). For convenience is written
F5=F55(P,I) : L5=L55(P,I)
On the member fixed on the left act a load F5 and the unknown reaction N4 at member end B.
Acts alone force F5 then part AC becomes longer and C will displace over V1 to the right.

With Hooke is ΔL=FL/EA, then follows
V1=F5*L5/EA
(EA is modulus of elasticity E times member cross-section A.)
Part CB is not loaded, so also B displaces over V1 to the right.

Acts alone the to the left directed force N4, then B will displace over V2 to the left.
V2=N4*L1/EA

The displacement of B must be zero, thus
V1-V2=0 or F5*L5/EA-N4*L1/EA=0 so that
N4=F5*L5/L1.

W Σ hor. =0 follows N3-F5+N4=0 so that
N3=F5-N4.

The calculated forces N3 and N4 are added to the preceding values of N1 and N2. Then the new values become
N1=N1+N3 : N2=N2+N4.
And then the following concentrated load with Next I.

```
Private Sub MEMBER()
'Calculation of the reactions due to
'member loads along the member.
N1=0 : N2=0
'The concentrated loads.
For I=1 To NFA(P)
F5=F55(P,I) : L5=L55(P,I)
N4=F5*L5/L1 : N3=F5-N4
N1=N1+N3 : N2=N2+N4
Next I
```

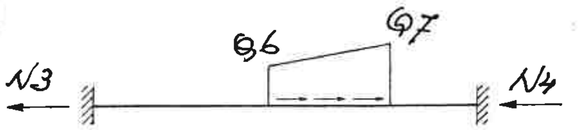



Fig. 3a.

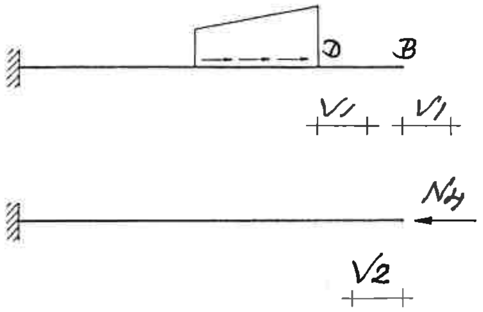



Fig.3b.

The distributed loads. (see also next page)

Fig.3a.
This time N3 and N4 are calculated for each distributed load.
For I=1 To NQA(P)
Q6=Q66(P,I) : Q7=Q77(P,I)
L6=L66(P,I) : L7=L77(P,I)

Fig.3b.
If the member fixed at the left end is loaded only with the distributed load, then D and B will displace over V1 to the right.

```
'The distributed loads.
For I=1 To NQA(P)
Q6=Q66(P,I) : L6=L66(P,I)
Q7=Q77(P,I) : L7=L77(P,I)
F=.5*(Q6+Q7)*L7 : V3=F*L6/EA

V5=Q7*L7^2/(2*EA)
V6=(Q7-Q6)*L7^2/(6*EA)
V1=V3+V5-V6

N4=V1*EA/L1 : N3=F-N4
N1=N1+N3 : N2=N2+N4
Next I

End Sub
```

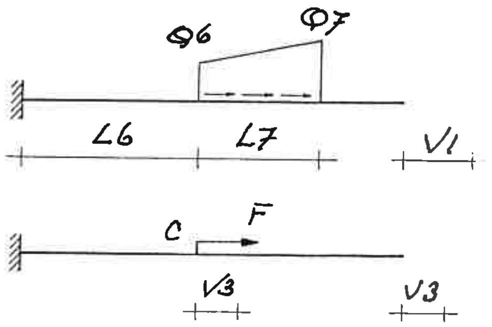



Fig.4.

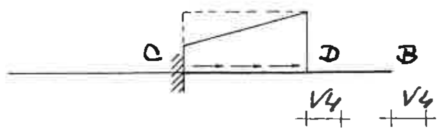



Fig.5a.

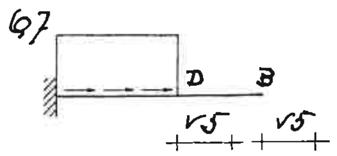



Fig.5b.

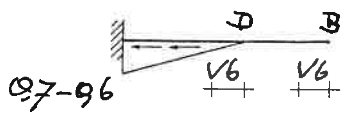



Fig.5c.

Fig.4.
The forces along the member of the trapezium like divided distributed load delivers force F at C.

The area of the trapezium is
F=0.5*(Q6+Q7)*L7

Acts only force F then C and B will displace over V3 to the right.

With Hooke is ΔL=FL/EA so that V3=F*L6/EA.

Fig.5a,5b en 5c.
Next the member is thought to be clamped at C and is the displacement of D calculated due the forces along part CD.
The displacement of D, and of B, is V1=V3+V4.

To find V4 the trapezium like load is divided in a rectangular load directed to the right, and a triangular load directed to the left.

The rectangle.

Fig.5b.
The to the right directed forces of te rectangular load do displace D an B over V5 to the right. With the formula of page 45 follows V5=Q7*L7^2/(2*EA).

The triangle.

Fig.5c.
The triangular load with forces directed to the left give D and B a displacement over V6 to the left. With the formula follows
V6=(Q7-Q6)*L7^2/(2*EA).

The effect V4 over CD due to the trapezium is equal the effect V5 of the rectangle lessened with the effect V6 of the triangle.
V4=V5-V6 With V1=V3+V4 follows V1=V3+V5-V6.

Finally the effect Of force N4 which gives a displacement of B over V2 to the left, fig.3b.
V2=N4*L1/EA

The displacement of B must be zero.

V1-V2=0 of V1-N4*L1/EA=0 from which
N4=V1*EA/L1.

Σ hor. =0 gives N3-F+N4=0 so that
N3=F-N4.

The this way calculated N3 and N4 are added to the previous values of N1 and N2,

N1=N1+N3 : N2=N2+N4

and then the next distributed load with

Next I.

And the end of the subroutine with

End Sub.

## 1.10. Private Sub N5XX()

Calculation of the normal force at X meter from memberend L.

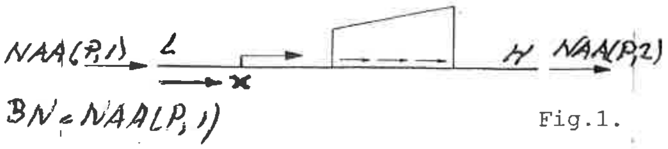



Fig.1.

Fig.1.
The main calculation, see page 22 , delivers for a member P
member end force NAA(P,1) at member end L, and member end force NAA(P,2) at member end H.

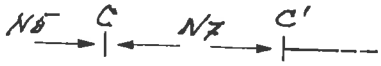



Fig.2a.

Fig.2a.
On cross-section C acts from left onto right a normal force N5 with a direction as assumed for the member end forces NAA(P,1) and NAA(P,2).
On section C' acts from left onto right a normal force N7 with a direction as well as assumed for the member end forces NAA(P,1) and NAA(P,2).
On section C acts from right onto left a normal force as large as N7 but opposite directed.
When applying a real section and seperating the two parts then on the sections of the two parts always act forces as large as but opposite directed.

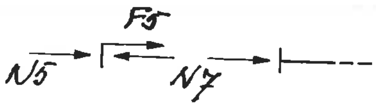



Fig.2b.

```
Private Sub N5XX()
'Calculation of the normal force
'at X meter from member end L.

N5=BN : N7=BN

'The concentrated loads.
For I=1 To NFA(P)
F5=F55(P,I) : L5=L55(P,I)

If X>L5 Then
N5=N5+F5 : N7=N7+F5
ElseIf X=L5 Then
N7=N7+F5
End If

Next I
```

Fig.2b.
Acts on section C a load force F5, as assumed directed from L to H, then follows with horizontal equilibrium of section C
N5+F5-F7=0  which gives  N7=N5+F5.

Said in another way, the on section C' acting as assumed to the right directed normal force N7 is equal the resultant of N5 and F5, so that N7=N5+F5.
Acts one more member load force F5 then N7 becomes larger, and F5 is added to the previous value of N7 with N7=N7+F5.

The start values of N5 and N7, when X=0, so at member end L, are BN=NAA(P,1) which must be given before calling subroutine N5G page . With this subroutine normal forces are calculated every G meter using the just considered subroutine N5XX.

Normal force NAA(P,1) pushes on member end L because the force is directed as assumed from L to H. That happens to be the same direction as that of member axis x at L. Also the normal forces N5 and N7 are compression forces as assumed. (It could have been different if the assumptions would have been different.)

The concentrated loads.
For I=1 To NFA(P)
F5=F55(P,I) : L5=L55(P,I)

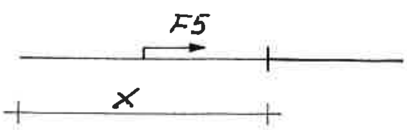



Fig.3a.

Fig.3a.
If X<L5 then N5 and N7 stay equal.
If X>L5 then become N5=N5+F5 and N7=N7+F5.

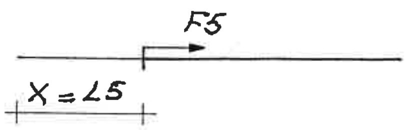



Fig.3b.

Fig.3b.
ElseIf X=L5 Then    In that case N5 left of the section does not change, but N7 right of the section does, so that N7=N7+F5.
End If and then

Next I for the following concentrated load.

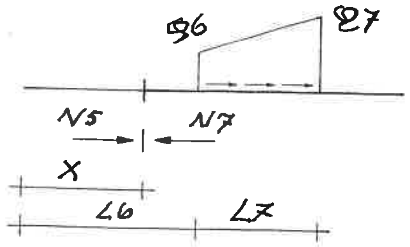



Fig.4a.

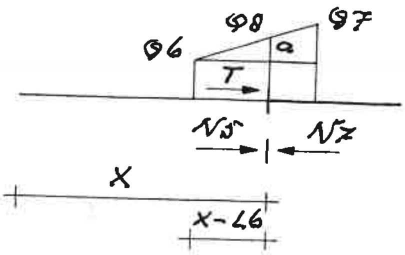



Fig.4b.

```
'The distributed loads.
For I=1 To NQA(P)
Q6=Q66(P,I) : L6=L66(P,I)
Q7=Q77(P,I) : L7=L77(P,I)
If X>L6 Then
If X>L6 And X<=L6+L7 Then
Q8=Q6+(Q7-Q6)*(X-L6)/L7
T=0.5*(Q6+Q8)*(X-L6)
N5=N5+T : N7=N7+T
ElseIf X>L6+L7 Then
T=0.5*(Q6+Q7)*L7
N5=N5+T : N7=N7+T
End If
End If
Next I

[N5=D*N5 : N7=D*N7]

End Sub
```

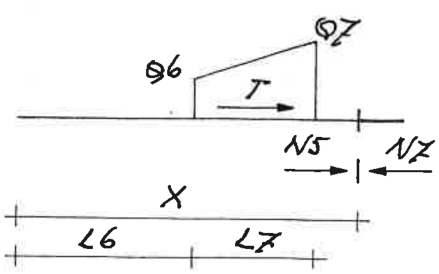



Fig.4c.

The distributed loads.

For I=1 To NQA(P)
Q6=Q66(P,I) : L6=L66(P,I)
Q7=Q77(P,I) : L7=L77(P,I)

Fig.4a.
Is X<=L6 then N5 and N7 do not change because the distributed loads is applied 'just on the right side' of the section.
Fig.4b en 4c.
When X is larger than L6, then N5 and N7 change and one of the two possible calculations is carried out. Therefore the first If-End ~~If~~ with If X>L6 Then.

The first possibility.
Fig.4b.
If X>L6 And X<=L6+L7 Then
A part of the distributed load, the trapezium left of the section does change N5 and N7. For that Q8 is calculated.
With congruence of triangles follows

a/(Q7-Q6)=(X-L6)/L7 from which follows

a=(Q7-Q6)*(X-L6)/L7 so that

Q8=Q6+(Q7-Q6)*(X-L6)/L7.

The to the right directed resultant T of the distributed load left of the section is equal the area of the concerning trapezium.

T=0.5*(Q6+Q8)*(X-L6) N5 and N7 change with

N5=N5+T : N7=N7+T.

The second possibility.
Fig.4c.
ElseIf X>L6+L7 Then
Now the total distributed load is on the left side of the section. Resultant T then becomes

T=0.5*(Q6+Q7)*L7 and also now follows

N5=N5+T : N7=N7+T. And then

Next I for the following distributed load.

(With the assumed directions for N5 and N7 follows that they are compression forces. If after calculation of N5 the answer is positiv, then the assumption was correct so the force is a compression force. The same applies for N7. Saying before a calculation that a compression force is 'negativ', is premature.
But if one wants as result of the calculation shown here a 'negative' answer when it concerns a compression force, then it is possible, but now and not earlier. When one writes before the calling of this subroutine D=-1, and after Next I N5=D*N5 : N7=D*N7, only then a negative answer means that the force is a compression force.
Or... adjust the assumptions, N5 and N7 pull at the section, then with opposite direction, not! L-H as the member axis........, and then adjust the code for calculations........One may do so.)

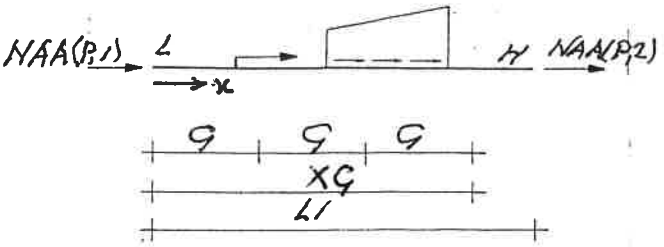



Fig. 1.

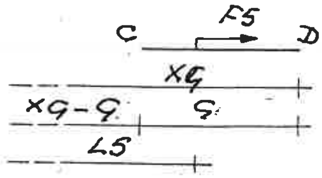



fig. 2.

```
Private Sub N5G()
'Calculation of the normal forces
'every G meter.
L=LL(P) : H=HH(P)
D1=X1(H)-X1(L)   L1=Sqr(D1^2)
NA=0     (or L1=L11(P))
For XG=0 To L1+G Step G
If XG=0 Then
X=XG : N5XX              page
NA=NA+1 : LA(P,NA)=X
NAL(P,NA)=N5 : NAR(P,NA)=N7
ElseIf XG>0 And XG<=L1 Then
C1=1
For I1=1 To NFA(P)
L5=L55(P,I1)
If L5>XG-G And L5<=XG Then
X=L5 : N5XX
NA=NA+1 : LA(P,NA)=X
NAL(P,NA)=N5 : NAR(P,NA)=N7
If L5=XG Then C1=0
End If
Next I1
If C1=1 Then
X=XG : N5XX
NA=NA+1 : LA(P,NA)=X
NAL(P,NA)=N5 : NAR(P,NA)=N7
End If
ElseIf XG>L1 Then
For I1=1 To NFA(P)
L5=L55(P,I1)
If L5>XG-G And L5<L1 Then
X=L5 : N5XX
NA=NA+1 : LA(P,NA)=X
NAL(P,NA)=N5 : NAR(P,NA)=N7
End If
Next I1
If XG-G<L1 Then
X=L1 : N5XX
NA=NA+1 :LA(P,NA)=X
NAL(P,NA)=N5 : NAR(P,NA)=N7
End If
End If
Next XG : NAC(P)=NA
End Sub
```

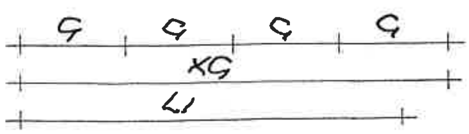



Fig.3.

1.11. Private Sub N5G()

Calculation of the normal forces in successive sections each G meter, and at the places of the concentrated loads.

Fig.1.
The number of sections is counted with NA which will be stored at the end with NAC(P)=NA. To begin with NA=0.
Member length is L1=L11(P), or calculated.

For XG=0 To L1+G Step G
With subroutine N5XX are calculated at distance X from member end L,
normal force N5 left of the section, and
normal force N7 right of the section.
First the calculation for XG=0.

If XG=0 Then
First becomes X=XG and then follows subroutine N5XX.

NA increases with 1 with NA=NA+1=0+1=1 and distance X is stored with LA(P,NA)=X.
N5 and N7 are kept with
NAL(P,NA)=N5 : NAR(P,NA)=N7.
(Or start values at member end L with X=0 and N5XX, NAL(P,NA)=BN and NAR(P,NA)=BN.)

Fig.2 and fig.1.
ElseIf XG>0 And XG<=L1 Then
With C1=1 it is assumed that there's no load force F5 with distance L5=XG.
Then for all loads I1=t To NFA(P) is checked if there is a load force after section C up to and including section D with
If L5>XG-G And L5<=XG.

If a load force is found then X=L5 and the normal forces N5 and N7 are calculated with subroutine N5XX.
For I1= and not For I= because I is used in subroutine N5XX.

If case L5=XG Then C1=0 which means that for that section a calculation is carried out, thus a second calculation omitting. If C1 stays C1=1 then for that section follows after Next I1 the calculation of the concerning N5 and N7 for X=XG.

Fig.3.
ElseIf XG>L1 Then
The last part of the member is checked, again
For I1=1 To NFA(P) and then
If L5>XG-G And L5<L1.

At distance L1, the member end, will not act a concentrated load but on the connected joint can act a joint load force.
Again all member load forces are checked and when L5 satisfies If-And-Then then X=L5 and follows again N5XX.
After Next I1 another check If XG-G<L1 Then.
If so then X=L1 and again N5XX.
After Next XG the total number of sections NAC(P)=NA and finally
End Sub.

## 1.12. Private Sub CONSTRMATCCAXMEMBER()


Fig.1.

```
        J                          J1

I  | 1,1  1,2 |          I1  | 1,1  1,2 |
   | 2,1  2,2 |              | 2,1  2,2 |

        S5                     member 1

                             | 2,2  2,3 |
                             | 3,2  3,3 |

                               member 2

                             | 3,3  3,4 |
                             | 4,3  4,4 |

                               member 3

                             | 4,4  4,5 |
                             | 5,4  5,5 |

                               member 4
```

```
Private Sub CONSTRMATCCAXMEMBER

N=N9
For I=1 To N : For J=1 To N
CC(I,J)=0 : Next J : Next I
FOR P=1 To P9 : L=LL(P) : H=HH(P)
EA=EAA(P)
MEMBERMATS5AXMEMBER
TT(1)=L : TT(2)=H
For I=1 To 2 : I1=TT(I)
For J=1 To 2 : J1=TT(J)
CC(I1,J1)=CC(I1,J1)+S5(I,J)
Next J
Next I
Next P

End Sub.
```

```
     1  2  3  4  5         1  2  3  4  5
1  | x  x           |    | .  .           |
2  | x  x           |    | .  x  x        |
3  |                |    |    x  x        |
4  |                |    |                |
5  |                |    |                |

         CC
   after member 1          after member 2

                         | .  .           |
                         | .  .  .        |
                         |    .  .  .     |
                         |       .  x  x  |
                         |          x  x  |

                           after member 4
```

Fig.1.
The construction consists of P9=4 members and N9=5 joints. There is 1 displacement for each joint, the number of equations then is N=N9.
The dimensions of construction stiffness matrix CC are N x N. First all elements of CC are set zero.
For I=1 To N : For J=1 To N
CC(I,J)=0 : Next J : Next I
For each member P=1 up to and including P9 is the lowest member end number L=LL(P) and the highest member end number H=HH(P), and the strain stiffnes is EA=EAA(P).

With the subroutine
MEMBERMATS5AXMEMBER (see next page)
for a member member stiffness matrix S5 is filled, which will be put in matrix CC after that, first the first row with I=1 and next the second row with I=2.

With I1=TT(1) the row number of matrix CC, and with J1=TT(2) the column number of CC is determined.
TT(1)=L : TT(2)=H
With CC(I1,J1)=CC(I1,J1)+S5(I,J) the elements of matrix CC are formed.
The new value CC(I1,J1) is equal the 'old' preceding value of CC(I1,J1) added with the value S5(I,J).
On the left the matrices S5 of the 4 members are given of which the elements are indicated with row and column numbers of matrix C.

For member P=1 with L=LL(1)=1 and H=HH(1)=2 follow
TT(1)=L=1 and TT(2)=H=2.
The first row of S5 to C.

```
I=1   I1=TT(I)=TT(1)=1
J=1   J1=TT(J)=TT(1)=1   CC(1,1)= 0  +S5(1,1)
J=2   J1=TT(J)=TT(2)=2   CC(1,2)= 0  +S5(1,2)
```

The second row of S5 to C.

```
I=2   I1=TT(I)=TT(2)=2
J=1   J1=TT(J)=TT(1)=1   CC(2,1)= 0  +S5(2,1)
J=2   J1=TT(J)=TT(2)=2   CC(2,2)= 0  +S5(2,2)
```

For member P=2 with L=LL(2)=2 and H=HH(2)=3 follow
TT(1)=L=2 and TT(2)=H=3.
The first row of S5 to C.

```
I=1   I1=TT(1)=2
J=1   J1=TT(1)=2        CC(2,2)=CC(2,2)+S5(1,1)
J=2   J1=TT(2)=3        CC(2,3)=  0   +S5(1,2)
```

S5(2,2) of member 1 coincides with S5(1,1) of member 2.

The second row of S5 to C.

```
I=2   I1=TT(2)=3
J=1   J1=TT(1)=2        CC(3,2)=  0   +S5(2,1)
J=2   J1=TT(2)=3        CC(3,3)=  0   +S5(2,2)
```

And so on.
Three times two elements of S5's coincide on the main diagonal of matrix CC.

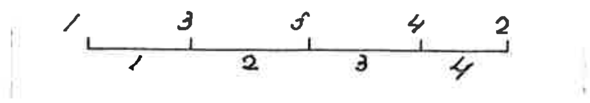


Fig.2.

J

$$I\begin{bmatrix}1,1 & 1,2\\ 2,1 & 2,2\end{bmatrix}$$

S5

J1

$$I1\begin{bmatrix}1,1 & 1,3\\ 3,1 & \underline{3,3}\end{bmatrix}\qquad\begin{bmatrix}\underline{3,3} & 3,5\\ 5,3 & \underline{5,5}\end{bmatrix}$$

member 1 — member 2

$$\begin{bmatrix}\underline{4,4} & 4,5\\ 5,4 & \underline{5,5}\end{bmatrix}\qquad\begin{bmatrix}2,2 & 2,4\\ 4,2 & \underline{4,4}\end{bmatrix}$$

member 3 — member 4

CC after menber 1

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | x | | x | | |
| 2 | | | | | |
| 3 | x | | x | | |
| 4 | | | | | |
| 5 | | | | | |

after member 2

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | . | | . | | |
| 2 | | | | | |
| 3 | . | | <u>x</u> | | x |
| 4 | | | | | |
| 5 | | | x | | x |

after member 3

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | . | | . | | |
| 2 | | | | | |
| 3 | . | | . | | . |
| 4 | | | | x | x |
| 5 | | | . | x | <u>x</u> |

after member 4

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | . | | . | | |
| 2 | | x | | x | |
| 3 | . | | . | | . |
| 4 | | x | | <u>x</u> | . |
| 5 | | | . | . | . |

```
Private Sub MEMBERMATS5AXMEMBER()
D1=X1(H)-X1(L)
L1=Sqr(D1^2)

R=EA/L1
S5(1,1)=R : S5(1,2)=-R
S5(2,1)=-R : S5(2,2)=R

End Sub
```


$$\begin{bmatrix}FLHX\\ FHLX\end{bmatrix}=\begin{bmatrix}R & -R\\ -R & R\end{bmatrix}\cdot\begin{bmatrix}UHL\\ UHH\end{bmatrix}$$

S5

Fig.3.

Fig.2.
Suppose that the numbering of the joints is a bit irregular as shown on the left, then the construction matrix CC will look different. Here below row and column numbers are give for each member.

| P | L | TT(1) | H | TT(2) |
|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 |
| 2 | 3 | 3 | 5 | 5 |
| 3 | 4 | 4 | 5 | 5 |
| 4 | 2 | 2 | 4 | 4 |

In the four member matrices S5 the elements are indicated with row numbers I1 and column numbers J1 of matrix CC.

Member 1 with row and column numbers 1 and 3.
Matrix S5 is placed in C.
CC(1,1)= 0 +S5(1,1) CC(1,3)= 0 +S5(1,2)
CC(3,1)= 0 +S5(2,1) <u>CC(3,3)=</u> 0 +S5(2,2)

Member 2 with row and column numbers 3 and 5.
Matrix S5 of member two is placed in CC.
CC(3,3)=<u>CC(3,3)</u>+S5(1,1) CC(3,5)= 0 +S5(1,2)
CC(5,3)= 0 +S5(2,1) <u>CC(5,5)=</u> 0 +S5(2,2)

Member 3 with row and column numbers 4 and 5.
<u>CC(4,4)=</u> 0 +S5(1,1) CC(4,5)= 0 +S5(1,2)
CC(5,4)= 0 +S5(2,1) CC(5,5)=<u>CC(5,5)</u>+S5(2,2)

Member 4 with row and column numbers 2 and 4.
CC(2,2)= 0 +S5(1,1) CC(2,4)= 0 +S5(1,2)
CC(4,2)= 0 +S5(2,1) CC(4,4)=<u>CC(4,4)</u>+S5(2,2)

The values of the third, fifth and fourth diagonal element change two times.

The elements of the main diagonals of the member matrices S5 arrive on the main diagonal of matrix CC. When the last matrix S5 has been placed in CC all elements on the main diagonal have become unequal to zero.
(With continuous beams, and frames, it is possible that there are still zeros on the main diagonal after construction matrx CC has been composed.)
Elements left and right of the main diagonals of the matrices S5 arrive at places outside the main diagonal of CC and never coincide.

Next the subroutine with which the member stiffness matrices S5 are filled.

## 1.13. Private Sub MEMBERMATS5AXMEMBER()

Fig.3.
D1=X1(H)-X1(L) is the member length which can be negative, the case for member 4. Thderefore member length L1 is calculated with L1=Sqr(D1^2).
The elements of member stiffness matrix S5 are the stiffness factors R=EA/L with a + or - sign as given on the left, see page 2 .

S5(1,1)=R : S5(1,2)=-R : S5(2,1)=-R : S5(22)=R

End Sub

## 1.14. Private Sub AXMAINCALC()

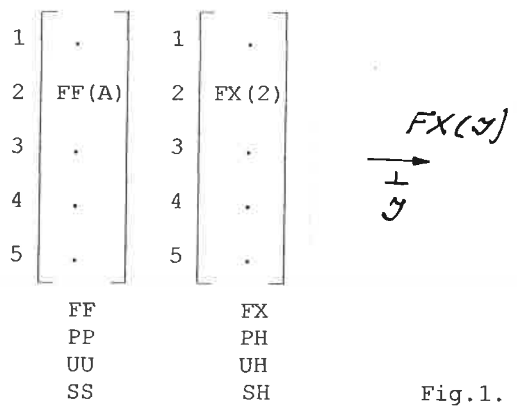



Fig.1.

```
Private Sub AXMAINCALC()

'1.Composition of construction ma-
'trix CC with member matrices S5.

CONSTRMATCCAXMEMBER

'2. Elements of force vector FF.
'2a. Joint load forces  FX(I).
N=1*N9
For I=1 To N9
A=1*I
FF(A)=FX(I)
PP(A)=PH(I)
UU(A)=UH(I)
SS(A)=SH(I)
Next I
```

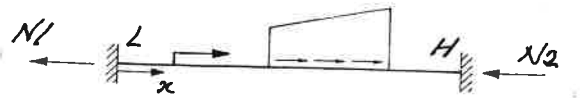



Fig.2a.

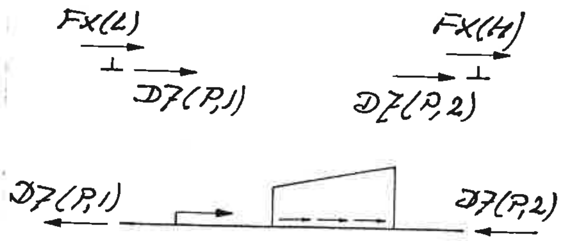



Fig.2b

```
'2b. Primary forces due to member
'loads along the member axis.
'staafas.
For P=1 TO P9 : L=LL(P) : H=HH(P)
EA=EAA(P)
D1=X1(H)-X1(L)
L1=Sqr(D1^2) : L11(P)=L1
C=D1/L1

MEMBER     (reactions N1 and N2)

D7(P,1)=N1*C : D7(P,2)=N2*C
Next P
```

With this subroutine the main calculation is carried out for axial loaded members with coinciding axes.

The first step is the subroutine

1. CONSTRMATCCAXMEMBER  (see page 20)
with which construction stiffness matrix CC is formed by using the member stiffness matrices S5.

2. The elements of force vector FF.
2a. The joint load forces.

Fig.1.
There are N9 joints. There is one possible displacement UH(I) for each joint I, thus the number of equations is N=1*N9.

First for each joint are put in
The joint load forces FX(I),
PH(I)=1 if the displacement is prescribed,
PH(I)=0 if that is not the case,
the prescribed displacement UH(I), and
UH(I)=0 if it is not prescribed, and
the spring constants with SH(I) and SH(I)=0 if that does not apply.
They are placed in total vectors FF, PP, UU and SS with A=1*I.

(Trusses can have two joint load forces, FX(I) and FY(I), and two displacements per joint, UH(I) and UV(I), etc. In that case A=2*I.

Here FF and FX etc. have the same size.

2b. Primary forces due to loads parallel to the member axis.

Fig.2a.
For each member P=1 To P9 the strain stiffness is EA=EAA(P). With D1=X1(H)-X1(L) are first calculated L1=Sqr(D1^2) and C=D1/L1.

With the subroutine
MEMBER  (see page 15)  the reactions N1 and N2 are calculated.

Fig.2b.
The assumptions for the directions of the on the joints acting primary forces D7(P,1) on joint L and D7(P,2) on joint H is to the right, the same as that of the joint load forces FX(I).

On the member ends act forces as large as but opposite directed forces, so to the left.
The directions of the on the ember ends acting forces N1 and D7(P,1), and N2 and D7(P,2) are the same. So one can write

D7(P,1)=N1*C : D7(P,2)=N2*C.

And then similar for each member.

Next P


Fig.3.

```
'2c.Alteration of force vector FF.
For I=1 To N9
A=1*I
For P=1 To P9 : L=LL(P) : H=HH(P)
If I=L Then
FF(A)=FF(A)+D7(P,1)
ElseIf I=H Then
FF(A)=FF(A)+D7(P,2)
End If
Next P
Next I
```


```
    1  2  3    4     5

1 [ .  .  . CC(1,4) . ]  [   .   ]  [ FF(1) ]
2 [ .  .  .    .    . ]  [   .   ]  [   .   ]
3 [ .  .  . CC(K,I) . ]  [   .   ]  [ FF(K) ]
4 [ .  .  .    .    . ]  [ UU(4) ]  [   .   ]
5 [ .  .  . CC(5,4) . ]  [   .   ]  [   .   ]

          CC                UU          FF
```

Fig.4.

```
'3. Alteration of force vector FF
'and construction matrix CC.
'3a. Of FF in case of prescribed
'displacements <>0.
For I=1 To N
If UU(I)<>0 Then
For K=1 To N
FF(K)=FF(K)-CC(K,I)*UU(I)
Next K
End If
Next I
```

```
1 [ .  .  .  0  . ]  [   .   ]  [   .   ]
2 [ .  .  .  0  . ]  [   .   ]  [   .   ]
3 [ .  .  .  0  . ]  [   .   ]  [   .   ]
4 [ 0  0  0  1  0 ]  [ UU(4) ]  [ FF(4) ]
5 [ .  .  .  0  . ]  [   .   ]  [   .   ]
```

Fig.5.

```
'3b. Of FF and CC in case of pres-
'cribed displacements.
For I=1 To N
If PP(I)=1 Then
For J=1 To N
CC(I,J)=0 : CC(J,I)=0
Next J
CC(I,I)=1 : FF(I)=UU(I)
End If
Next I
'3c. Of CC in case of elastic/
'springy supports.
For I=1 To N
If SS(I)>0 Then
CC(I,I)=CC(I,I)+SS(I)
Next I
```

2c. Alteration of force vector FF.

Fig.3.
For each joint I the primary forces D7(P,1) and D7(P,2) are added to an element of FF. The element number is A=1*I.
For each joint I all members are checked if they deliver a primary force on the joint. (All members, not necessary, but of more importance when dealing with trusses.)
If I=L then becomes
FF(A)=FF(A)+D7(P,1) and
if I=H then becomes
FF(A)=FF(A)+D7(P,2).
After the last member P9 is checked follows the next joint with
Next I.

3. Alteration of force vector FF and construction matrix CC.
3a. Of FF in case of prescribed displacements unequal to zero, <>0.

Fig.4.
For that the total vectors UU and FF are used... because it will be done the same way with other constructions.
If for I=4 displacement UU(4)<>0 is prescribed, then PP(4)=1 was put in, then each element FF(K) must be lessened with
CC(K,I)*UU(I), so here with CC(K,4)*UU(4) for K=1 To N.

```
K=1    FF(1)=FF(1)-CC(1,4)*UU(4)
K=2    FF(2)=FF(2)-CC(2,4)*UU(4)
K=3    FF(3)=FF(3)-CC(3,4)*UU(4)
K=4    FF(4)=FF(4)-CC(4,4)*UU(4)
K=5    FF(5)=FF(5)-CC(5,4)*UU(4)
```

3b. Of FF and CC in case of prescribed displacements.
Fig.5.
If PP(I)=1 then displacement UU(I) is prescribed.
If PP(4)=1 then the fourth row and the fourth column of matrix CC are filled with zeros.
For I=1 To N and For J=1 To N then follow
CC(I,J)=0 : CC(J,I)=0.

```
I=4   J=1   CC(4,1)=0   :   CC(1,4)=0
      J=2   CC(4,2)=0   :   CC(2,4)=0
      J=3   CC(4,3)=0   :   CC(3,4)=0
      J=4   CC(4,4)=0   :   CC(4,4)=0 (that's
      two times, but does not matter)
      J=5   CC(4,5)=0   :   CC(5,4)=0
```

After that the element on the main diagonal is made CC(I,I)=1, so for I=4 with CC(4,4)=1, and the fourth element FF(4)m of FF gets the value of the prescribed displacement, zero or not zero,
FF(I)=UU(I), here FF(4)=UU(4).

3c. Of CC in case of elastic/springy supports.
If SS(I)>0 then SH(I) is the spring constant put in which must be added to the concerning element CC(I,I) of the main diagonal.
If SS(I)>0 Then CC(I,I)=CC(I,I)+SS(I)

Now the set of N=1*N9 equations got ready to be solved.

```
'4. Calculation of the unknown
'displacements UH(I).
For I=1 To N : BB(I)=FF(I)
For J=1 To N
AA(I,J)=CC(I,J)
Next J
Next I          A x = b  is  C u = f
```

```
1   .        .        .
2 UU(A)    UH(2)    XX(A)
3   .        .        .
4   .        .        .
5   .        .        .

   UU       UH       XX
```

```
'The solution of the N=1*N9
'equations.
GAUSS
For I=1 To N9
A=1*I
UH(I)=XX(A)
UU(A)=XX(A)
Next I

'5. Calculation of the memberend
'forces w.r.t. construction axis X.
'5a. Due to the displacements
'alone.
```


```
[FLHX]   [ R   -R ]   [UH(L)]
[FHLX]   [-R    R ]   [UH(H)]

  f          S5          u

[FK(P,1)]   [(1,1)  (1,2)]   [UU(A)]
[FK(P,2)]   [(2,1)  (2,2)]   [UU(A)]
```

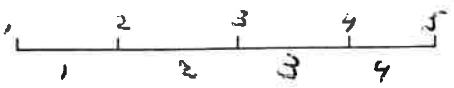



Fig.6a.

```
For P=1 To P9 : L=LL(P) : H=HH(P)
EA=EAA(P)
MEMBERMATS5AXMEMBER
TT(1)=1*L
TT(2)=1*H
For I=1 TO 2 : FK(P,I)=0
For J=1 TO 2 : A=TT(J)
FK(P,I)=FK(P,I)+S5(I,J)*UU(A)
Next J
Next I
'5b. Due to displacements and mem-
'ber loads along the member axis.
D5(P,1)=FK(P,1)-D7(P,1)
D5(P,2)=FK(P,2)-D7(P,2)
D1=X1(H)-X1(L)
L1=Sqr(D1^2)
C=D1/L1
NAA(P,1)=D5(P,1)*C
NAA(P,2)=D5(P,2)*C
Next P
```

4. Calculation of the unknown displacements UH(I).
In behalf of the vector BB and matrix AA used in the subroutine GAUSS
vector BB is filled with the elements of force vector FF, and
matrix AA with the elements of construction matrix CC.
4a. The solution of N equations.
With the subroune GAUSS the unknowns of vector XX, is x, solved and placed in UH(I) and UU(A).
{remark A ]
5. Calculation of the member end forces w.r.t. the construction axis X.
5a. Due to the displacements alone.
Fig.6a.
The relation between member end forces and displacements is f = S5 u.
For each member the stress stiffness is EA=EAA(P).
With the subroutine
MEMBERMATS5AXMEMBER (page 21 )
member matrix S5 is formed.
Member end force FK(P,I) is equal row I of S5 times column u.
With TT(1) and TT(2) the elements of u are gotten from total vector/column UU.

| | | |
|---|---|---|
| For member P=2 is | | L=2 and H=3. |
| TT(1)=1*L=2 and | | TT(2)=1*H=3. |
| I=1 | A=TT(J) | FK(P,I)=FK(2,1)=0 |
| J=1 | A=TT(1)=2 | FK(2,1)= 0 S5(1,1)*UU(2) |
| J=2 | A=TT(2)=3 | FK(2,1)=FK(2,1)+S5(1,2)*UU(3) |
| I=2 | | FK(P,I)=FK(2,2)=0 |
| J=1 | A=2 | FK(2,2)= 0 +S5(2,1)*UU(2) |
| J=2 | A=3 | FK(2,2)=FK(2,2)+S5(2,2)*UU(3) |
| (Or with R=EA/L1, | | FK(P,1)= R*UH(L)-R*UH(H) and |
| | | FK(P,2)=-R*UH(L)+R*UH(H).) |

5b. Due to displacements and member loads alog the member axis.

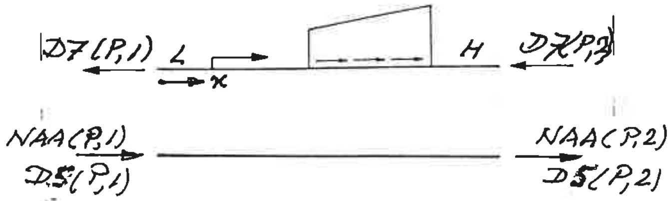



Fig.6b.

The on the joints acting primary forces D7(P,1) and D7(P,2) are assumed to the right; then on the member ends to the left directed.
The final member end forces
D5(P,1) at member end L and
D5(P,2) at member end H, are assume to be directed to the right. Then follow
D5(P,1)=FK(P,1)-D7(P,1) and
D5(P,2)=FK(P,2)-D7(P,2).

The final member end forces w.r.t. member axis x, NAA(P,1) and NAA(P,2) are directed according to the member axis,
With D1=X1(H)-X1(L) , L1=SQR(D1^2) and C=D1/L1
then follow
NAA(P,1)=D5(P,1)*C and NAA(P,2)=D5(P,2)*C.
And then the foloowing member with
Next P.

```
'6. Calculation of the joint for-
'ces KH(I).
'6a. Due to the displacements
'alone.
```

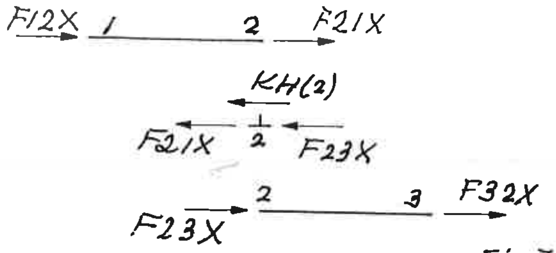



Fig.7a.

$$
\begin{matrix}1\\2\\3\\4\\5\end{matrix}
\begin{bmatrix} . \\ . \\ KH(3) \\ . \\ . \end{bmatrix}
=
\begin{bmatrix} . & . & . & . & . \\ . & . & . & . & . \\ . & .CC(A,J) & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix}
\cdot
\begin{bmatrix} UU(1) \\ UU(2) \\ . \\ . \\ . \end{bmatrix}
$$

CC UU

```
CONSTRMATCCAXMEMBER
For I=1 To N9
A=1*I
KH(I)=0
For J=1 To N
KH(I)=KH(I)+CC(A,J)*UU(J)
Next J
```

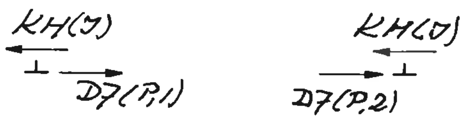



Fig.7b.

```
'6b. Due to the displacements and
'member loads along the member
'axis.
For P=1 To P9 : L=LL(P) : H=HH(P)
If I=L Then
KH(I)=KH(I)-D7(P,1)
ElseIf I=H Then
KH(I)=KH(I)-D7(P,2)
End If
Next P
Next I
```

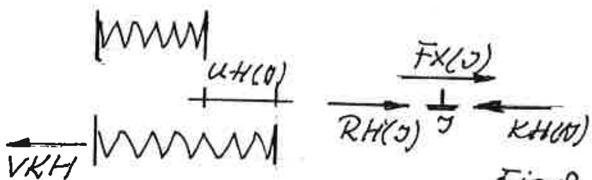



Fig.8.

```
'7. Calculation of the reactions.
For I=1 To N9
If SH(I)>0 Then
RH(I)=-SH(I)*UH(I)
Else
RH(I)=KH(I)-FX(I)
End If
Next I

End Sub
```

6. Calculation of the joint forces KH(I).
6a. Due to the displacements alone.

Fig.7a.
The on the joint acting 'joint force' KH(I), assumed to be directed to the left is equal one! force on the joint to the left acting member end force, or is equal
the sum of the on the joint to the left acting member end forces. (see page )
To calculate the joint forces the original, not altered construction matrix CC is used.
Therefore first the subroutine
CONSTRMATCCAXMEMBER

For I=1 To N9
Joint force KH(I) is equal a row A=1*I of matrix CC times column UU.
If I=3 Then A=1*3=3, before KH(3)=0.

```
J=1   KH(3)=  0    +CC(3,1)*UU(1)
J=2   KH(3)=KH(3)+CC(3,2)*UU(2)
J=3   KH(3)=KH(3)+CC(3,3)*UU(3)
J=4   KH(3)=KH(3)+CC(3,4)*UU(4)
J=5   KH(3)=KH(3)+CC(3,5)*UU(5)
```

6b. Due to the displacements and member loads along the member axis.
Fig.7b.
The member loads deliver the primary forces D7(P,1) at L and D7(P,2) at H.
For a joint I all (not necessary here, but see trusses page ,) the P=1 To P9 members are checked to see if a member delivers a primary force on that joint. That's the case if joint number I equals member end number L or member end number H.
The on the joints acting primary forces are as the joint load forces assumed to be directed to the right.
If I=L then becomes
KH(I)=KH(I)-D7(P,1) and
if I=H then becomes
KH(I)=KH(I)-D7(P,2).
After the last member follows the next joint, Next I.

7. Calculation of the reactions.
Fig.8.
The reactions RH(I) are as assumed for the joint load forces FX(I) directed to the right.
Is the joint elastic supported then the spring constant is SH(I)>0. Is the displacement UH(I) assumed to the right, the spring reaction becomes
VKH=SH(I)*UH(I) to the left, so that

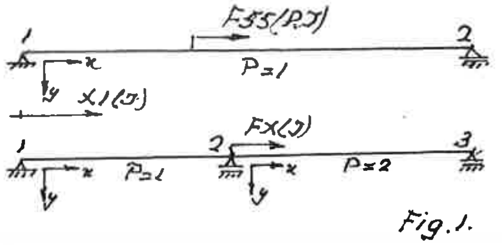
RH(I)=-SH(I)*UH(I).

In all other cases is

RH(I)=KH(I)-FX(I).

Finally the end of the main calculation with

End Sub.




The '<u>member axis system</u>' (x, y) is always placed at the lowest member end number

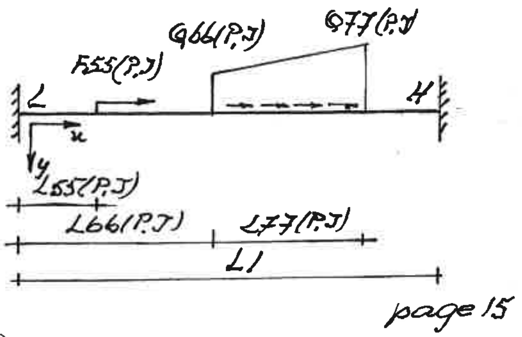



Fig.2.

The assumed direction of the load forces like the x-axis (just a name)of the member axis system.
Reactions N1 and N2.assumed in opposite direction.

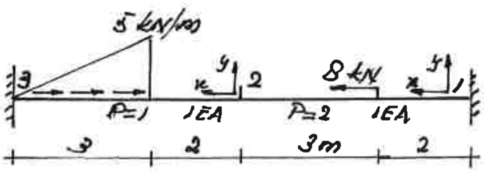



Fig.4.

Member axis system (x, y) at lowest!! member end number.

<u>N9=3 joints</u>

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 10 |
| 2 | 0 | 0 | 0 | 0 | 5 |
| 3 | 0 | 1 | 0 | 0 | 0 |

<u>P9=2 members</u>

| P | LL | HH | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 1 |
| I | Q6 | Q7 | L6 | L7 | |
| 1 | -5 | 0 | 2 | 3 | |
| | | | | | |
| 2 | 1 | 2 | 1 | 1 | 0 |
| I | F5 | L5 | | | |
| 1 | 8 | 2 | | | |

<u>Program **AXPROGRAM222** assumptions.</u>

Continuous members. Torsion not included.

Fig.1.

<u>Joint assumptions.</u>

<u>I FX PH UH SH X1</u>

<u>I</u> joint number
<u>FX(I)</u> horizontal joint load force

<u>PH(I)=0</u>
joint displacement UH(I) not prescribed
<u>PH(I)=1</u>
joint displacement UH(I) is prescribed
UH(I)=0 or <>0

<u>UH(I)</u> in EA (EA is strain stiffness)
a horizontal displacement
<u>SH(I)</u> horizontal spring constant in EA

<u>X1(I)</u> distance from left end in m

<u>RH(I)</u> reaction to the right

<u>Member assumptions.</u>

<u>P LL HH A1 NFA NQA</u>

<u>P</u> member number
<u>LL(P)</u> lowest member end number
<u>HH(P)</u> highest member end number

<u>NFA(P)</u> number of horizontal member load forces.

<u>NQA(P)</u> number of horizontal distributed member load forces.

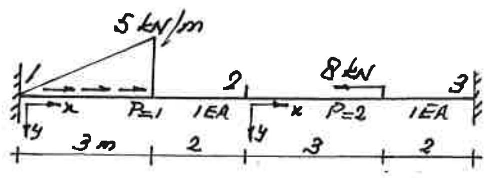



Fig.3.

With member loads, no joint loads.

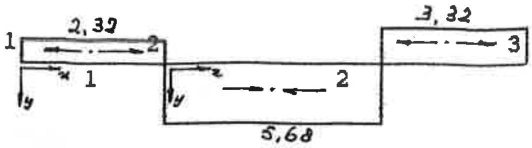
<u>N9=3 joints</u>

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 5 |
| 3 | 0 | 1 | 0 | 0 | 10 |

<u>P9=2 members</u>

| P | LL | HH | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 0 | 1 |
| I | Q6 | Q7 | L6 | L7 | |
| 1 | 0 | 5 | 0 | 3 | |
| | | | | | |
| 2 | 2 | 3 | 1 | 1 | 0 |
| I | F5 | L5 | | | |
| 1 | -8 | 3 | | | |

normal force diagram EX5




| | | | |
|---|---|---|---|
| NMAX= -2,32 kN | X= 0,00 m | P= 1 | |
| NMAX= 5,68 kN | X= 0,00 m | P= 2 | |
| N5MAXX= 5,68 kN | X= 0,00 m | P= 2 | |

| | | | |
|---|---|---|---|
| NAA(1,2)= | -2,32 kN | NAA(2,1)= | 2,32 kN |
| NAA(2,3)= | 5,68 kN | NAA(3,2)= | 3,32 kN |

| | | | |
|---|---|---|---|
| RH(1)= | -2,32 kN | UH(1)= | 0,00 /EA |
| RH(2)= | 0,00 kN | UH(2)= | 4,64 /EA |
| RH(3)= | 3,32 kN | UH(3)= | -0,55 /EA |

CSE=0 Results Reactions Show Again
N9= OK Cls
P9= G= .9 N5 N7 Step G All Over Again
PrF DRAWN5 Calculate STORE NR= ? GET
8EXAMPLES EX1 EX2 EX3 EX4 EX5 EX6 End

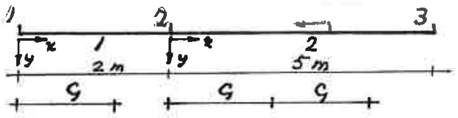



Fig. 4b.

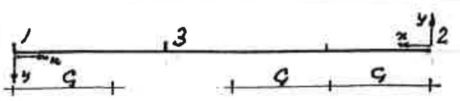
Type in TG 1.8 and click N5N7 Step G, results here below. See page 30.

member 1

| | | |
|---|---|---|
| X= 0,00 m | N5= -2,32 kN | N7= -2,32 kN |
| X= 1,80 m | N5= -2,32 kN | N7= -2,32 kN |
| X= 2,00 m | N5= -2,32 kN | N7= -2,32 kN |

member 2

| | | |
|---|---|---|
| X= 0,00 m | N5= 5,68 kN | N7= 5,68 kN |
| X= 1,80 m | N5= 5,68 kN | N7= 5,68 kN |
| X= 3,00 m | N5= 5,68 kN | N7= -3,32 kN |
| X= 3,60 m | N5= -3,32 kN | N7= -3,32 kN |
| X= 5,00 m | N5= -3,32 kN | N7= -3,32 kN |




Again All UH(I) set zero.

All Over Again To start position.

STORE NR= GET
Storing data put in, page 28.

CSE=0 click to CSE=1, to put in separate input values, page 31.

PrF to print the screen form.

AXPROGRAM222, the form controls.

Number of joints N9=, text box TN9.

Number of members P9=, text box TP9.

TSTRING is the large text box for input of joint and member data. After input of those data press Enter or click OK.
Click Show after all data put in to appear on the form, page 34, and disappear when clicking *Calculate to carry out the calculations.*
Click Cls to clear the form.

Next click DRAWN5 to print the normal force diagram, compression below the zero line and tension above the zero line.
NMAX for each member P.
N5MAXX is the largest of all NMAX.

Fig.4a.
Assumed direction of member end forces from lowest member end number L to highest member end number H.

NAA(P,1) and NAA(P,2), page 24, printed as NAA(H,L) and NAA(H,L).
*(NAA(P,1) determines 'compression' or 'tension'* according to made assumption.)

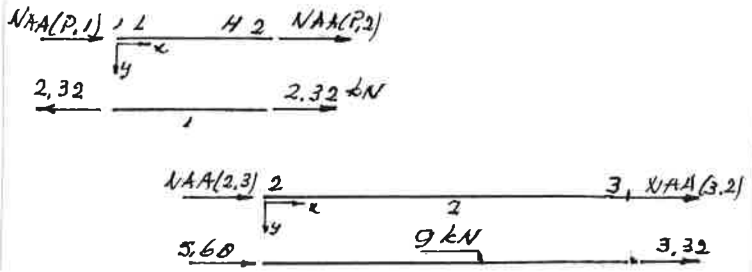



Fig. 4a.

Thus is assumed that NAA(P,1) pushes at member end L, and NAA(P,2) pulls at member end H.

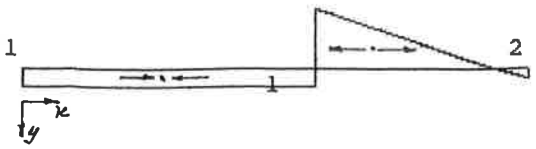
Member P=1.
NAA(1,2)= -2,32 kN, negative answer, so the real direction of member end force NAA(L,H) is not to the right as assumed, but to the left.
NAA(2,1)= 2,32 kN, positive answer, so the real direction of member end force NAA(H,L) is to the right as assumed.

*Member P=2.*
NAA(2,3)= 5,86 kN, positive answer, directed as assumed, NAA(3,2)= 3,32 kN, positive answer so directed as assumed.

Reactions assumed direction to the right.
RH(1)= -2,32 kN, real direction to the left.
RH(2)=0, no reaction.
HR(3)= 3,32 kN, real direction to the right.

Joint displacements, assumed to the right.
UH(1)= 0 /EA
UH(2)= 4,64/ EA, that's to the right.
Joint 3 is horizontally supported by a spring, SH(3)= 6,0 EA, page 34.
UH(3)= -0,55 /EA, that's to the left

**normal force diagram**




NMAX= -8,46 kN X= 3,51 m P= 1
N5MAXX= -8,46 kN X= 3,51 m P= 1

NAA(1,2)= 2,50 kN NAA(2,1)= -1,50 kN

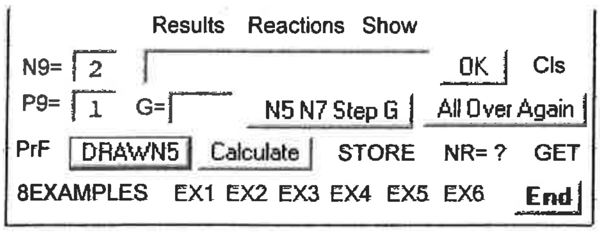
RH(1)= 2,50 kN UH(1)= 0,00 /EA
RH(2)= -1,50 kN UH(2)= 0,00 /EA

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 0 | 6,00 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 12,0 | 1 | 1 |

| I | F5 | L5 |
|---|---|---|
| 1 | -11,0 | 3,50 |

| I | Q6 | Q7 | L6 | L7 |
|---|---|---|---|---|
| 1 | 4,0 | 4,0 | 3,50 | 2,50 |



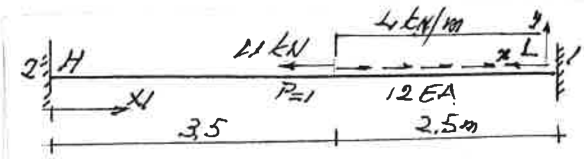
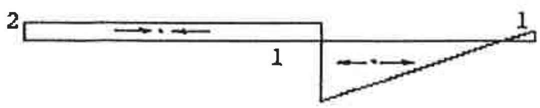




*Fig.6.*

**normal force diagram**




NMAX= -8,50 kN X= 2,50 m P= 1
N5MAXX= -8,50 kN X= 2,50 m P= 1

NAA(1,2)= 1,50 kN NAA(2,1)= -2,50 kN

RH(1)= -1,50 kN UH(1)= 0,00 /EA
RH(2)= 2,50 kN UH(2)= 0,00 /EA

Example.

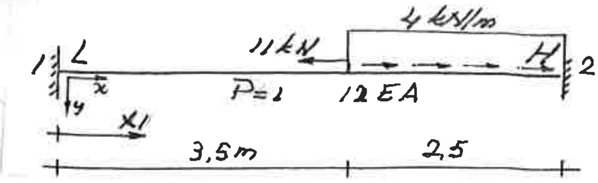



Fig.5.

The member axis system (x, y) at the lowest member end number.
The assumed direction of the member loads is that of x of the member axis system.

Input of joint, member and member load data, see them on the left.

N9=2 supports/joints.
Type 2 in text box TN9, Tab, cursor in TSTRING and type
1,0,1,0,0,0 Enter and 2,0,1,0,0,6 Enter, cursor appears in text box TP9.

P9=1 member.
Type 1 in text box TP9, Tab, cursor in TSTRING and type
1,1,2,12,1,1 Enter
followed by the load force 11 kN with
1,-11,3.5 Enter
and the distributed load with
1,4,4,3.5,2.5 Enter.

Click Show to see the data put in.
Click Calculate to carry out the calculation, first data shown disappears,
DRAWN5 to draw the normal force diagram, also appear maximum values,
Results for the member end forces NAA(L,H) and NAA(H,L), and
Reactions for support reactions RH(1) and RH(2) and joint displacements UH(1) and UH(2).

Fig.6.

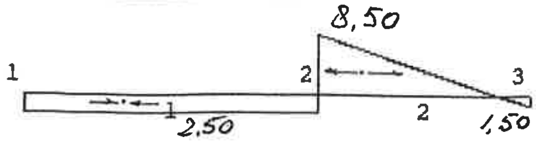
Member axis system at the other member end, must have the lowest member end number, so 1 at the right end and 2 at the left end.

N9=2 supports/joints.
Type 2 in text box TN9, Tab, cursor in TSTRING and type
1,0,1,0,0,**6** Enter and 2,0,1,0,0,**0** Enter, cursor appears in text box TP9.

P9=1 member.
Type 1 in text box TP9, Tab, cursor in TSTRING and type
1,1,2,12,1,1 Enter
followed by the load force 11 kN with
1,**11**,**2.5** Enter
and the distributed load with
1,**-4**,**-4**,**0**,2.5 Enter. Etc.

To compare with he data input here above.
The normal force diagram is mirrorred.
Now above the member compression and below the member line tension.

normal force diagram




| | | | | | |
|---|---|---|---|---|---|
| NMAX= | 2,50 kN | X= | 0,00 m | P= | 1 |
| NMAX= | -8,50 kN | X= | 0,00 m | P= | 2 |
| N5MAXX= | -8,50 kN | X= | 0,00 m | P= | 2 |

| | | | |
|---|---|---|---|
| NAA(1,2)= | 2,50 kN | NAA(2,1)= | -2,50 kN |
| NAA(2,3)= | -8,50 kN | NAA(3,2)= | -1,50 kN |

| | | | |
|---|---|---|---|
| RH(1)= | 2,50 kN | UH(1)= | 0,00 /EA |
| RH(2)= | 0,00 kN | UH(2)= | -0,73 /EA |
| RH(3)= | -1,50 kN | UH(3)= | 0,00 /EA |

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | -11,0 | 0 | -0,7 | 0 | 3,50 |
| 3 | 0 | 1 | 0 | 0 | 6,00 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 12,0 | 0 | 0 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 2 | 2 | 3 | 12,0 | 0 | 1 |

| I | Q6 | Q7 | L6 | L7 |
|---|---|---|---|---|
| 1 | 4,0 | 4,0 | 0 | 2,50 |

Hooke's law   UH(I)= F*L/'EA'

Here with 'EA' is 12EA

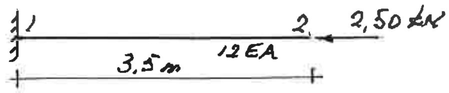



Fig.10.

UH(2)= (2,50*3,50)/12EA= 0,73/EA to the left, that is opposite to the assumed direction, so UH(2)=-0,73/EI.

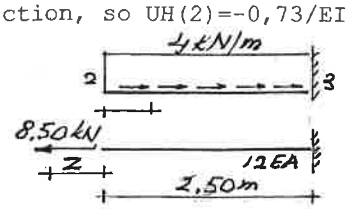



Fig.11.

See formula page 45, Z= Q*L^2/2EA.

(4*2,50^2)/(2*12EA)= 1,04/EA  and

(8,50*2,50)/12EA= 1,77/EA.

Assumed to the right,
UH(2)= 1,04/EA - 1,77/EA= -0,73/EA  ok

Example.

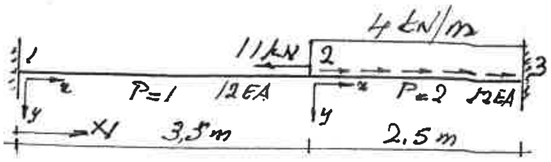



Fig.7.

N9=3 joints.  Supports 1 and 3.
Type 3 in text box TN9, Tab, cursor in TSTRING and type
1,0,1,0,0,0  Enter,  2,-11,0,0,0,3.5  Enter,
3,0,1,0,0,6  Enter,
cursor appears in text box TP9.

P9=2 members.
Type 2 in text box TP9, Tab, cursor in TSTRING and type
1,1,2,12,0,0  Enter  and  2,2,3,12,0,1  Enter,
and the distributed load with
2,4,4,0,2.5  Enter.
Click Show to check the input.

Storing the data.
Click NR=  to e.g. NR=2 if not underlined and click STORE gets STORE and NR=2 as well.
Numbering NR= up with left mouse button and numbering down with right mouse button, maximum R=10.
Wanting the stored data back, click to the underlined NR=2, click GET gets underlined and Show. Remove the data, click with right mouse button on GET, underlinig disappears, of NR=2 as well.

Click Calculate, DRAWN5, Results and Reactions to get the print shown on the left.

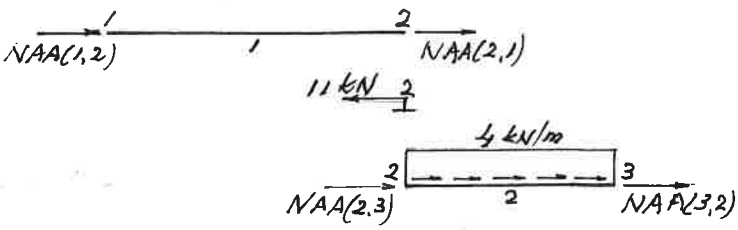



Fig.8.

Member end forces NAA(L,H) and NAA(H,L) have assumed directions from lowest to highest member end number.

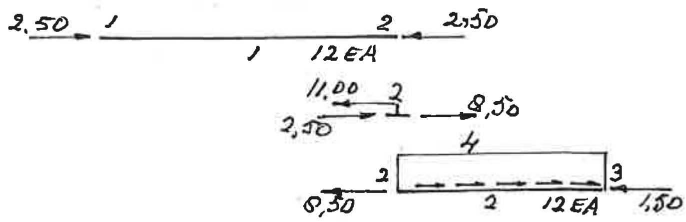
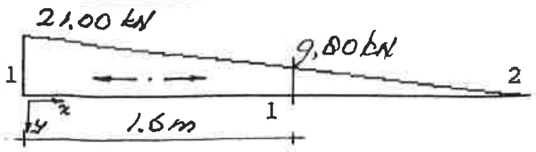



Fig.9.

The member end forces drawn with their real directions.
At joint 2 act the member end forces as large as but opposite directed.
Members and joints in equilibrium.

**normal force diagram**




NMAX= -21,00 kN X= 0,00 m P= 1
N5MAXX= -21,00 kN X= 0,00 m P= 1

NAA(1,2)= -21,00 kN NAA(2,1)= 0 kN

RH(1)= -21,00 kN UH(1)= 0,00 /EA
RH(2)= 0,00 kN UH(2)= 31,50 /EA

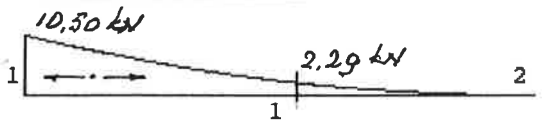
| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 31,5 | 0 | 3,00 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1,0 | 0 | 1 |

| I | Q6 | Q7 | L6 | L7 |
|---|---|---|---|---|
| 1 | 7,0 | 7,0 | 0 | 3,00 |

=

**normal force diagram**



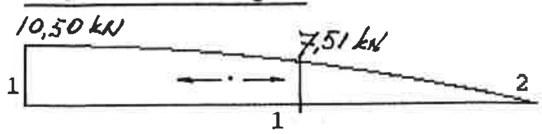

NMAX= -10,50 kN X= 0,00 m P= 1
N5MAXX= -10,50 kN X= 0,00 m P= 1

NAA(1,2)= -10,50 kN NAA(2,1)= 0 kN

RH(1)= -10,50 kN UH(1)= 0,00 /EA
RH(2)= 0,00 kN UH(2)= 10,50 /EA

+

**normal force diagram**

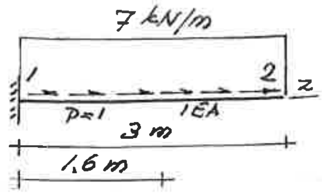



NMAX= -10,50 kN X= 0,00 m P= 1
N5MAXX= -10,50 kN X= 0,00 m P= 1

NAA(1,2)= -10,50 kN NAA(2,1)= 0 kN

RH(1)= -10,50 kN UH(1)= 0,00 /EA
RH(2)= 0,00 kN UH(2)= 21,00 /EA

Example.




Z = Q*L^2/2EA

Fig.12a.

N9=2 joints.

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 3 |

P9=1 member.

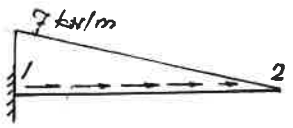
| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 0 | 1 |

| I | Q6 | Q7 | L6 | L7 |
|---|---|---|---|---|
| 1 | 7 | 7 | 0 | 3 |

Type 2 in TN9, Tab, type in TSTRING
1,0,1,0,0,0 Enter and 2,0,0,0,0,3 Enter.
Type 1 in TP9, Tab and type in TSTRING
1,1,2,1,0,1 Enter
and next in TSTRING 1,7,7,0,3 Enter.
Click Calculate, DRAWN5, Results, Reactions and Show to get the results shown on the left.
Normal force at 1.6 m from the left.
Click in TG to get the cursor there and type 1.6 Enter, and click N5 N7 Step G.

**member 1**

| | | |
|---|---|---|
| X= 0 m | N5= -21 kN | N7= -21 kN |
| X= 1,60 m | N5= -9,80 kN | N7= -9,80 kN |
| X= 3,00 m | N5= 0 kN | N7= 0 kN |



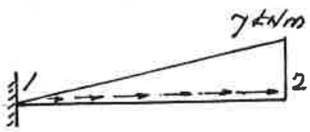

Z = Q*L^2/6EA

Fig.12b.

Joint data and member data the same.
The member load data 1,7,0,0,3 Enter.
In TG 1.6 Enter, and click N5 N7 Step G.

**member 1**

| | | |
|---|---|---|
| X= 0,00 m | N5=-10,50 kN | N7=-10,50 kN |
| X= 1,60 m | N5= -2,29 kN | N7= -2,29 kN |
| X= 3,00 m | N5= 0,00 kN | N7= 0,00 kN |




Z = Q*L^2/3EA

Fig.12c.

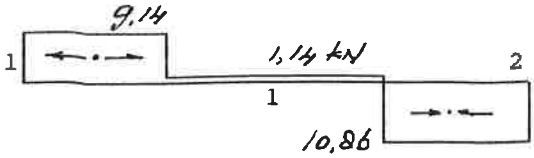
Joint data and member data the same.
The member load data 1,7,0,0,3 Enter.
In TG 1.6 Enter.

**member 1**

| | | |
|---|---|---|
| X= 0,00 m | N5=-10,50 kN | N7=-10,50 kN |
| X= 1,60 m | N5= -7,51 kN | N7= -7,51 kN |
| X= 3,00 m | N5= 0,00 kN | N7= 0,00 kN |

a) is b) + c), for N5 -9,80= -2,29 +(-7,51)
That's correct, the figures don't show that.
Each time a diagram is drawn the larges normal force determines how the diagram looks like.
Here the largest normal force in the diagram is about 0,75 cm.

**normal force diagram**




NMAX= 10,86 kN X= 5,01 m P= 1
N5MAXX= 10,86 kN X= 5,01 m P= 1

NAA(1,2)= -9,14 kN NAA(2,1)= -10,86 kN

RH(1)= -9,14 kN UH(1)= 0,00 /EA
RH(2)= -10,86 kN UH(2)= 0,00 /EA

member 1

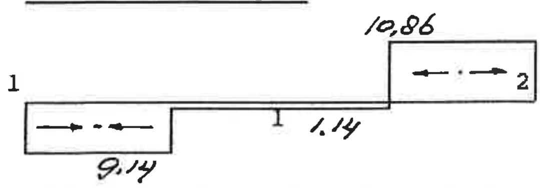
| | | |
|---|---|---|
| X= 0,00 m | N5= -9,14 kN | N7= -9,14 kN |
| X= 1,60 m | N5= -9,14 kN | N7= -9,14 kN |
| X= 2,00 m | N5= -9,14 kN | N7= -1,14 kN |
| X= 3,20 m | N5= -1,14 kN | N7= -1,14 kN |
| X= 4,80 m | N5= -1,14 kN | N7= -1,14 kN |
| X= 5,00 m | N5= -1,14 kN | N7= 10,86 kN |
| X= 6,40 m | N5= 10,86 kN | N7= 10,86 kN |
| X= 7,00 m | N5= 10,86 kN | N7= 10,86 kN |

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 0 | 7,00 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1,0 | 2 | 0 |

| I | F5 | L5 |
|---|---|---|
| 1 | 8,0 | 2,00 |
| 2 | 12,0 | 5,00 |

**normal force diagram**




NMAX= -10,86 kN X= 5,01 m P= 1
N5MAXX= -10,86 kN X= 5,01 m P= 1

NAA(1,2)= 9,14 kN NAA(2,1)= 10,86 kN

RH(1)= 9,14 kN UH(1)= 0,00 /EA
RH(2)= 10,86 kN UH(2)= 0,00 /EA

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 0 | 7,00 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1,0 | 2 | 0 |

| I | F5 | L5 |
|---|---|---|
| 1 | -8,0 | 2,00 |
| 2 | -12,0 | 5,00 |

Example.

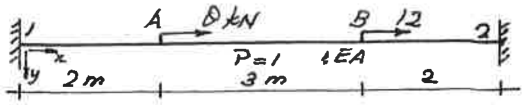



Fig.13.

N9=2 joints.

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 0 | 7 |

P9=1 member.

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 2 | 0 |

| I | F5 | L5 |
|---|---|---|
| 1 | 8 | 2 |
| 2 | 12 | 5 |

In TN9 type 2, Tab, in TSTRING 1,0,1,0,0,0 Enter, and 2,0,1,0,0,7 Enter, in TP9 type 1, Tab, and in TSTRING

1,1,2,1,2,0 Enter, the member loads with 1,8,2 Enter and 2,12,5 Enter.

Click Calculate, DRAWN5, Results and Reactions.

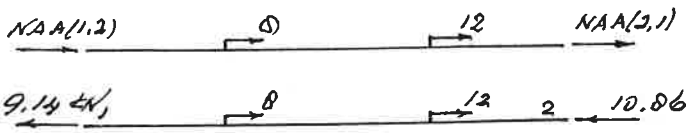



Fig.14.

The member with assumed direction of the member end forces NAA(1,2) and NAA(2,1).
NAA(1,2)= -9,14 kN, negative answer so not to the left as assumed but directed to the right.
NAA(2,1)= -10,86 kN, that's to the left.

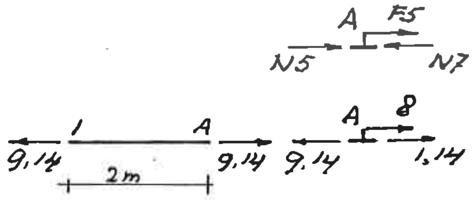


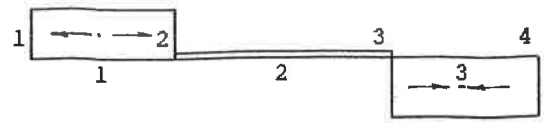

Fig.15.

Joint 2 with a member load force, not a joint load force, separated with the assumed directions of N5 and N7.
N5= -9,14 kN, a negative answer so not directed as assumed but opposite directed that's to the left. Acting at member end A as large as but opposite directed, that's to the right. Member part 1-A in equilibrium.
Joint 2 is A, N7= -1,14 kN, not directed to the left as assumed but to the right. The joint is in equilibrium.

Suppose the member load forces to the left. Type 1 in TP9, next 1,1,2,1,2,0 Enter and 1,-8,2 Enter and 2,-12,5 Enter. Calculate, DRAWN5 etc. on the left.

NMAX is the largest normal force of member 1, N5MAXX is the largest normal force of all members, here only one member. See next page with three members.

**normal force diagram**



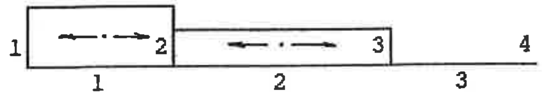

```
  NMAX=   -9,14 kN    X= 0,00 m   P= 1
  NMAX=   -1,14 kN    X= 0,00 m   P= 2
  NMAX=   10,86 kN    X= 0,00 m   P= 3
N5MAXX=   10,86 kN    X= 0,00 m   P= 3

NAA(1,2)=  -9,14 kN  NAA(2,1)=   9,14 kN
NAA(2,3)=  -1,14 kN  NAA(3,2)=   1,14 kN
NAA(3,4)=  10,86 kN  NAA(4,3)= -10,86 kN

 RH(1)=  -9,14 kN     UH(1)=   0,00 /EA
 RH(2)=   0,00 kN     UH(2)=  18,29 /EA
 RH(3)=   0,00 kN     UH(3)=  21,71 /EA
 RH(4)= -10,86 kN     UH(4)=   0,00 /EA

member 1
X= 0,00 m  N5= -9,14 kN  N7= -9,14 kN
X= 1,60 m  N5= -9,14 kN  N7= -9,14 kN
X= 2,00 m  N5= -9,14 kN  N7= -9,14 kN
member 2
X= 0,00 m  N5= -1,14 kN  N7= -1,14 kN
X= 1,60 m  N5= -1,14 kN  N7= -1,14 kN
X= 3,00 m  N5= -1,14 kN  N7= -1,14 kN
member 3
X= 0,00 m  N5= 10,86 kN  N7= 10,86 kN
X= 1,60 m  N5= 10,86 kN  N7= 10,86 kN
X= 2,00 m  N5= 10,86 kN  N7= 10,86 kN
```

**normal force diagram**




```
  NMAX= -20,00 kN    X= 0,00 m   P= 1
  NMAX= -12,00 kN    X= 0,00 m   P= 2
  NMAX=   0,00 kN    X= 0,00 m   P= 3
N5MAXX= -20,00 kN    X= 0,00 m   P= 1

NAA(1,2)= -20,00 kN  NAA(2,1)=  20,00 kN
NAA(2,3)= -12,00 kN  NAA(3,2)=  12,00 kN
NAA(3,4)=      0 kN  NAA(4,3)=      0 kN

 RH(1)= -20,00 kN     UH(1)=   0,00 /EA
 RH(2)=   0,00 kN     UH(2)=  40,00 /EA
 RH(3)=   0,00 kN     UH(3)=  76,00 /EA
 RH(4)=   0,00 kN     UH(4)=  76,00 /EA
```

Example.

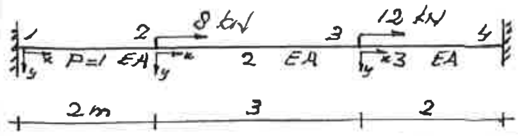



Fig.16.

N9=4 joints.

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 8 | 0 | 0 | 0 | 2 |
| 3 | 12 | 0 | 0 | 0 | 5 |
| 4 | 0 | 1 | 0 | 0 | 7 |

P9=3 members.

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 0 | 0 |
| 2 | 2 | 3 | 1 | 0 | 0 |
| 3 | 3 | 4 | 1 | 0 | 0 |

Type 4 in TN9, next joint data in TSTRING,
type 3 in TP9, next member data in TSTRING.

Click Calculate, DRAWN5, Results and Reactions.

Type 1.6 in TG for G=1.6 m, click N5N7 Step G.

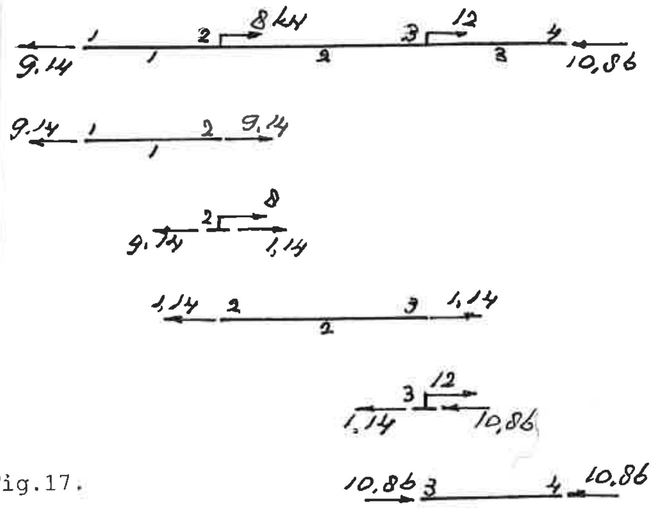



Fig.17.

Members and joints are separated from each other. The member end forces are drawn with their real directions. On the joints act forces as large as but opposite directed, Members and joints are in equilibrium.

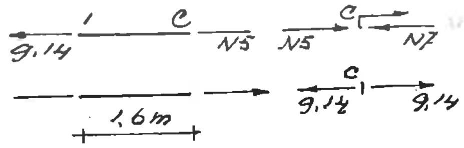
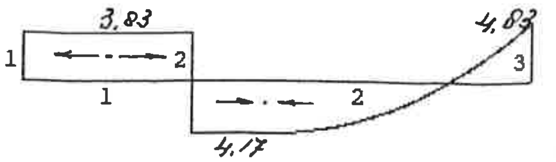



Fig.18.

Member 1, drawn a part with length 1.6 m. And member end C seen as a 'joint' without joint load force, with assumed N5 and N7. Next N5 and N7 are drawn with their real direction. At C act a force as large as but opposit directed. Equilibrium.
In such case N7 can be omitted and not printed because 'joint' load force does not exist. See the preceding page.

Suppose joint 4 can move freely horizontally, displacement not prescribed, so PH(4)=0. Click CSE=0 to CSE=1, and type in TSTRING PH(4)=0 Enter. Etc. See the results shown on the left.

normal force diagram EX1



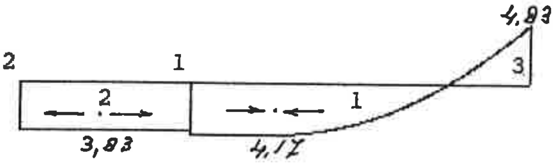

| | | | | | |
|---|---|---|---|---|---|
| NMAX= | -3,83 kN | X= | 0,00 m | P= | 1 |
| NMAX= | -4,83 kN | X= | 4,01 m | P= | 2 |
| N5MAXX= | -4,83 kN | X= | 4,01 m | P= | 2 |

| | | | |
|---|---|---|---|
| NAA(1,2)= | -3,83 kN | NAA(2,1)= | 3,83 kN |
| NAA(2,3)= | 4,17 kN | NAA(3,2)= | 4,83 kN |

| | | | |
|---|---|---|---|
| RH(1)= | -3,83 kN | UH(1)= | 0,00 /EA |
| RH(2)= | 0,00 kN | UH(2)= | 7,67 /EA |
| RH(3)= | 4,83 kN | UH(3)= | 0,00 /EA |

CSE=0 Results Reactions Show Again
N9= OK Cls
P9= G= N5 N7 Step G All Over Again
PrF DRAWN5 Calculate STORE NR=? GET
EXAMPLES81 EX1 EX2 EX3 EX4 EX5 EX6 End

normal force diagram EX2

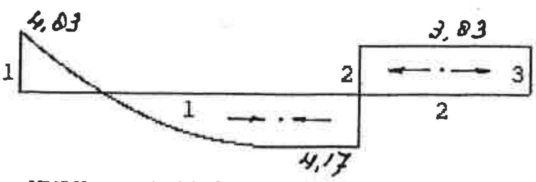



| | | | | | |
|---|---|---|---|---|---|
| NMAX= | -4,83 kN | X= | 4,01 m | P= | 1 |
| NMAX= | -3,83 kN | X= | 0,00 m | P= | 2 |
| N5MAXX= | -4,83 kN | X= | 4,01 m | P= | 1 |

| | | | |
|---|---|---|---|
| NAA(1,3)= | 4,17 kN | NAA(3,1)= | 4,83 kN |
| NAA(1,2)= | -3,83 kN | NAA(2,1)= | 3,83 kN |

| | | | |
|---|---|---|---|
| RH(1)= | 0,00 kN | UH(1)= | 7,67 /EA |
| RH(2)= | -3,83 kN | UH(2)= | 0,00 /EA |
| RH(3)= | 4,83 kN | UH(3)= | 0,00 /EA |

normal force diagram EX3




| | | | | | |
|---|---|---|---|---|---|
| NMAX= | -4,83 kN | X= | 0,00 m | P= | 1 |
| NMAX= | -3,83 kN | X= | 0,00 m | P= | 2 |
| N5MAXX= | -4,83 kN | X= | 0,00 m | P= | 1 |

| | | | |
|---|---|---|---|
| NAA(1,2)= | -4,83 kN | NAA(2,1)= | -4,17 kN |
| NAA(2,3)= | -3,83 kN | NAA(3,2)= | 3,83 kN |

| | | | |
|---|---|---|---|
| RH(1)= | -4,83 kN | UH(1)= | 0,00 /EA |
| RH(2)= | 0,00 kN | UH(2)= | -7,67 /EA |
| RH(3)= | 3,83 kN | UH(3)= | 0,00 /EA |

Example EX1.

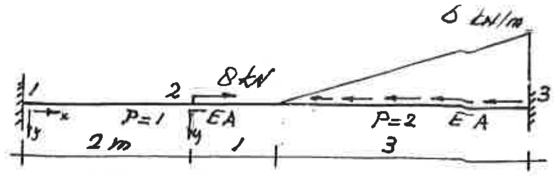



Fig.19.

Joints and members are regularly numbered from left to right, 1-2-3 and 1-2.

N9=3 joints.

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 8 | 0 | 0 | 0 | 2 |
| 3 | 0 | 1 | 0 | 0 | 6 |

P9=2 members.

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 0 | 0 |
| 2 | 2 | 3 | 1 | 0 | 1 |
| I | Q6 | Q7 | L6 | L7 | |
| 1 | 0 | -6 | 1 | 3 | |

Type 3 in TN9, next joint data in TSTRING, 1,0,1,0,0,0 Enter, 2,8,0,0,0,2 Enter and 3,0,1,0,0,6 Enter.
Type 2 in TP9, next membrer data in TSTRING, 1,1,2,1,0,0 Enter, 2,2,3,1,0,1 Enter and 1,0,-6,1,3 Enter. Click Calculate etc.

UH(2)= 7,67/EA

Example EX2.

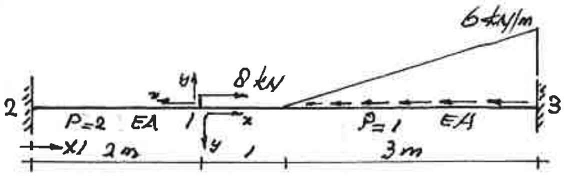



Fig.20

Joints and members are irregularly numbered from left to right, 2-1-3 and 2-1.

N9=3 joints.

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 8 | 0 | 0 | 0 | 2 |
| 2 | 0 | 1 | 0 | 0 | 0 |
| 3 | 0 | 1 | 0 | 0 | 6 |

P9=2 members.

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 3 | 1 | 0 | 1 |
| I | Q6 | Q7 | L6 | L7 | |
| 1 | 0 | -6 | 1 | 3 | |
| 2 | 1 | 2 | 1 | 0 | 0 |

The member axis system of member 2 is placed at the lowest member end number according to the assumption! UH(1)= 7,67/EA

Example EX3.

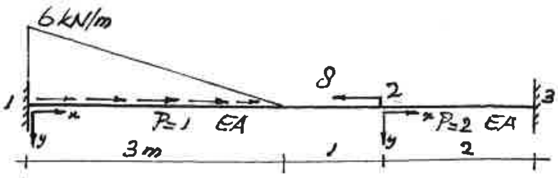



Fig.21.

N9=3 joints.

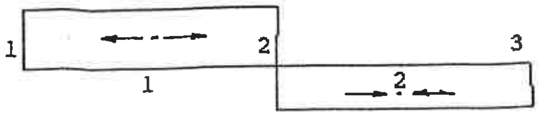
| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | -8 | 0 | 0 | 0 | 4 |
| 3 | 0 | 1 | 0 | 0 | 6 |

P9=2 members.

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 0 | 1 |
| I | Q6 | Q7 | L6 | L7 | |
| 1 | 6 | 0 | 0 | 3 | |
| 2 | 2 | 3 | 1 | 0 | 0 |

UH(2)= -7,67/EA

normal force diagram EX4




| | | | | | |
|---|---|---|---|---|---|
| NMAX= | -3,43 kN | X= | 0,00 m | P= | 1 |
| NMAX= | 2,57 kN | X= | 0,00 m | P= | 2 |
| N5MAXX= | -3,43 kN | X= | 0,00 m | P= | 1 |

| | | | |
|---|---|---|---|
| NAA(1,2)= | -3,43 kN | NAA(2,1)= | 3,43 kN |
| NAA(2,3)= | 2,57 kN | NAA(3,2)= | -2,57 kN |

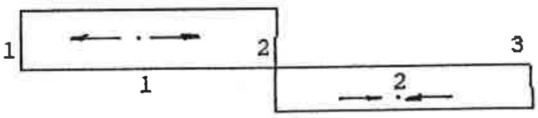
| | | | |
|---|---|---|---|
| RH(1)= | -3,43 kN | UH(1)= | 0,49 /EA |
| RH(2)= | 0,00 kN | UH(2)= | 6,21 /EA |
| RH(3)= | -2,57 kN | UH(3)= | 0,37 /EA |

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0,5 | 7,0 | 0 |
| 2 | 6,0 | 0 | 6,2 | 0 | 2,50 |
| 3 | 0 | 0 | 0,4 | 7,0 | 5,00 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1,5 | 0 | 0 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 2 | 2 | 3 | 1,1 | 0 | 0 |

normal force diagram




| | | | | | |
|---|---|---|---|---|---|
| NMAX= | -3,44 kN | X= | 0,00 m | P= | 1 |
| NMAX= | 2,56 kN | X= | 0,00 m | P= | 2 |
| N5MAXX= | -3,44 kN | X= | 0,00 m | P= | 1 |

When going on after the last results with prescribed UH(1) and UH(3) and no horizontal hinges, as follows.

Click CSE=0 to CSE=1 red.
First click Again to make all displacements UH(I)=0 for the next main calculation.

Next in to type in TSTRING
PH(1)=1 Enter, PH(3)=1 Enter,

no horizontal hinges,
SH(1)=0 Enter, SH(3)=0 Enter,

values of prescribed displacements,
UH(1)=.49 Enter, UH(3)=.37 Enter.

Click Cls and Show to check the new data and click Calculate, DRDRAWN5 etc. Finding the same results as above.

Example EX4.

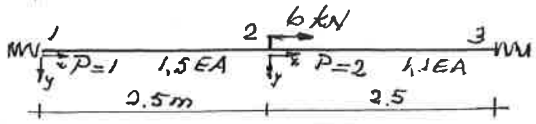



Fig.22

Joint 1 and 3 are horizontal hinges with hinge constant 7 EA, the horizontal displacements of joint/support 1 and 3 are not prescribed so PH(1)=0 and PH(3)=0.
Member 1 and 2 with different strain stiffness A11(1)= 1.5 EA and A11(2)= 1.1 EA.

N9=3 joints.

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 7 | 0 |
| 2 | 6 | 0 | 0 | 0 | 2.5 |
| 3 | 0 | 0 | 0 | 7 | 5.0 |

P9=2 members.

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1.5 | 0 | 0 |
| 2 | 2 | 3 | 1.1 | 0 | 0 |

The calculation horizontal diaplacements are UH(1)= 0,49 UH(2)= 6,21 UH(3)=0,37 /EA. positive answers so as assumed to the rignt.

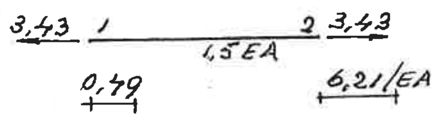



Fig.23.

Member 1 becomes longer by the member end force of 3,43 kN. With ΔL= F*L/'EA" follows

ΔL= (3,43*2,50)/1,5EA= 5,72/EA longer.
The figure shows
ΔL= UH(2)-UH(I)= 6,21/EA -5,72/EA= 0,49/EA ok

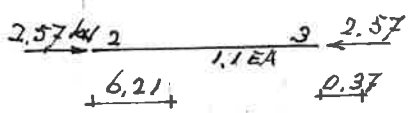



Fig.24.

Member 2 becomes shorter by the member end force of 2,57 kN.

ΔL= (2,57*2,50)/1,1EA= 5,84/EA shorter.
The figure shows
ΔL= UH(2)-UH(3)= 6,21/EA -0,37/EA= 5,84/EA ok

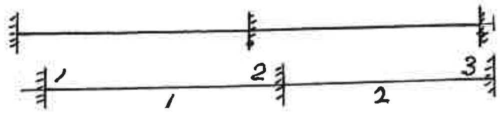



Fig.25.

N9=3 joints.

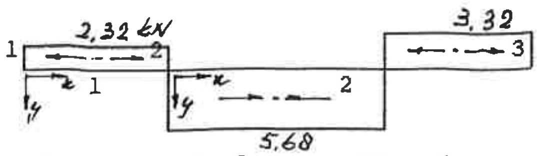
| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | .49 | 0 |
| 2 | 6 | 0 | 0 | 0 | 2.5 |
| 3 | 0 | 1 | 0 | .37 | 5 |

P9=2 members.

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1.5 | 0 | 0 |
| 2 | 2 | 3 | 1.1 | 0 | 0 |

Displcements UH(1) and UH(3) are prescribed, PH(1)=1 and PH(3)=1, no hinges at member end 1 and 3, SH(1)=0 and SH(3)=0.
Final results are like found above, ofcourse.
(Or with prescribed UH(2)=6.21 and PH(2)=1, final results the same.)

**normal force diagram** EX5




| | | | | | |
|---|---|---|---|---|---|
| NMAX= | -2,32 kN | X= 0,00 m | P= 1 | | |
| NMAX= | 5,68 kN | X= 0,00 m | P= 2 | | |
| N5MAXX= | 5,68 kN | X= 0,00 m | P= 2 | | |

| | | | |
|---|---|---|---|
| NAA(1,2)= | -2,32 kN | NAA(2,1)= | 2,32 kN |
| NAA(2,3)= | 5,68 kN | NAA(3,2)= | 3,32 kN |

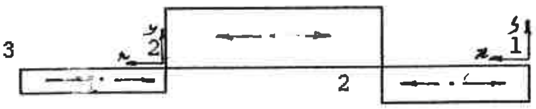
| | | | |
|---|---|---|---|
| RH(1)= | -2,32 kN | UH(1)= | 0,00 /EA |
| RH(2)= | 0,00 kN | UH(2)= | 4,64 /EA |
| RH(3)= | 3,32 kN | UH(3)= | -0,55 /EA |

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 8,0 | 0 | 4,6 | 0 | 2,00 |
| 3 | 0 | 0 | -0,6 | 6,0 | 7,00 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1,0 | 0 | 0 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 2 | 2 | 3 | 2,0 | 1 | 0 |
| I | F5 | L5 | | | |
| 1 | -9,0 | 3,00 | | | |

**normal force diagram** EX6




| | | | | | |
|---|---|---|---|---|---|
| NMAX= | -2,32 kN | X= 0,00 m | P= 1 | | |
| NMAX= | 5,68 kN | X= 2,01 m | P= 2 | | |
| N5MAXX= | 5,68 kN | X= 2,01 m | P= 2 | | |

| | | | |
|---|---|---|---|
| NAA(2,3)= | -2,32 kN | NAA(3,2)= | 2,32 kN |
| NAA(1,2)= | -3,32 kN | NAA(2,1)= | -5,68 kN |

| | | | |
|---|---|---|---|
| RH(1)= | 3,32 kN | UH(1)= | -0,55 /EA |
| RH(2)= | 0,00 kN | UH(2)= | 4,64 /EA |
| RH(3)= | -2,32 kN | UH(3)= | 0,00 /EA |

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 0 | -0,6 | 6,0 | 7,00 |
| 2 | 8,0 | 0 | 4,6 | 0 | 2,00 |
| 3 | 0 | 1 | 0 | 0 | 0 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 1,0 | 0 | 0 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 2 | 1 | 2 | 2,0 | 1 | 0 |
| I | F5 | L5 | | | |
| 1 | 9,0 | 2,00 | | | |

Example EX5.

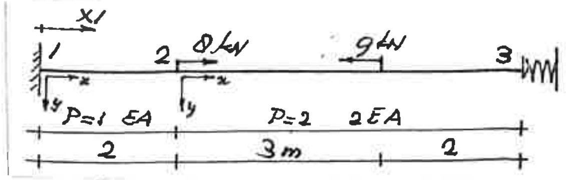



Fig.26

Member 1 and 2 with different strain stiffness A11(1)= 1 EA and A11(2)= 2 EA. At joint 3 a horizontal hinge with constant SH(3)= 6 EA.

N9=3 joints.

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 8 | 0 | 0 | 0 | 2 |
| 3 | 0 | 0 | 0 | 6 | 7 |

P9=2 members.

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 0 | 0 |
| 2 | 2 | 3 | 2 | 0 | 0 |
| I | F5 | L5 | | | |
| 1 | -9 | 3 | | | |

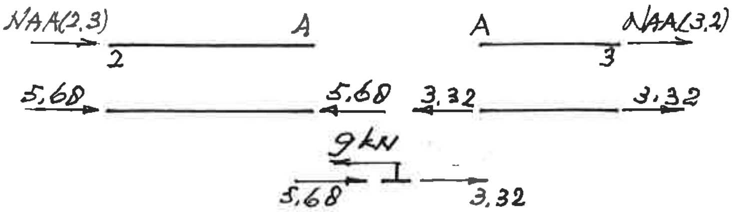



Fig.27.

Member 2 divided into two parts at A. Two member end forces NAA(2,3) and NAA(3,2) are known and drawn with their real directions. Both parts are in equilibrium, the forces at A are drawn with their real directions. At the separated joint act forces as large as but opposite directed. The joint is in equilibrium. ok

Example EX6.

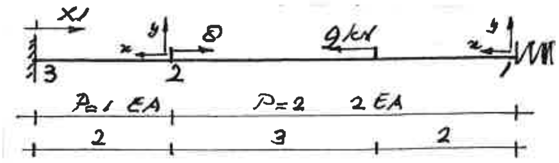



Fig.28.

Like fig. 26 but with different joint numbering, 3-2-1 in stead of 1-2-3.
The member axis system ⌐x,y always at lowest member end number (becoming of importance wiyh trusses).
Assumed direction of joint load forces like X1, also here 8 kN, not -8 kN, but 9 kN is a member load force with assumed direction of x-axis of the member axis system.

N9=3 joints.

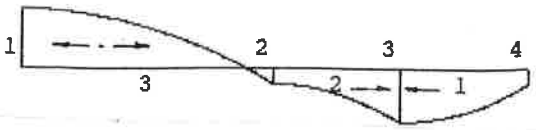
| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 8 | 0 | 0 | 0 | 2 |
| 3 | 0 | 0 | 0 | 6 | 7 |

P9=2 members.

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 0 | 0 |
| 2 | 2 | 3 | 2 | 0 | 0 |
| 1 | F5 | L5 | | | |
| 1 | 9 | 2 | | | |

Hinge force at member end 1 on the right, is UH(1)*6EA= (0,55/EA)*6EA= 3,30 kN is 3,32 kN ok

**normal force diagram** EX81




```
    NMAX=    5,67 kN    X= 0,00 m   P= 1
    NMAX=    5,67 kN    X= 1,00 m   P= 2
    NMAX=   -6,33 kN    X= 0,00 m   P= 3
 N5MAXX=   -6,33 kN    X= 0,00 m   P= 3

NAA(3,4)=   5,67 kN  NAA(4,3)=  -1,67 kN
NAA(2,3)=   1,67 kN  NAA(3,2)=  -5,67 kN
NAA(1,2)=  -6,33 kN  NAA(2,1)=  -1,67 kN

  RH(1)=  -6,33 kN     UH(1)=   0,00 /EA
  RH(2)=   0,00 kN     UH(2)=   7,33 /EA
  RH(3)=   0,00 kN     UH(3)=   4,33 /EA
  RH(4)=  -1,67 kN     UH(4)=   0,00 /EA
```

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 2,00 |
| 3 | 0 | 0 | 0 | 0 | 3,00 |
| 4 | 0 | 1 | 0 | 0 | 4,00 |

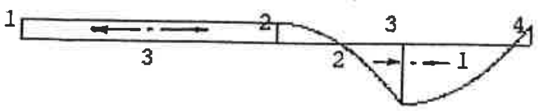
| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 3 | 4 | 1,0 | 0 | 1 |
| I | | Q6 | Q7 | L6 | L7 |
| 1 | | 0 | -8,0 | 0 | 1,00 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 2 | 2 | 3 | 1,0 | 0 | 1 |
| I | | Q6 | Q7 | L6 | L7 |
| 1 | | 0 | 8,0 | 0 | 1,00 |

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 3 | 1 | 2 | 1,0 | 0 | 1 |
| I | | Q6 | Q7 | L6 | L7 |
| 1 | | 0 | 8,0 | 0 | 2,00 |

PrF

**normal force diagram**




```
    NMAX=    3,00 kN    X= 0,00 m   P= 1
    NMAX=    3,00 kN    X= 1,00 m   P= 2
    NMAX=   -1,00 kN    X= 0,00 m   P= 3
 N5MAXX=    3,00 kN    X= 0,00 m   P= 1
```

Examples with EXAMPLES8 to click on for eight different possible normal force diagrams from EXAMPLES8 to EXAMPLES88.

Click EXAMPLES8 to EXAMPLES81

All examples with N9=4 joints and P9=3 members, all members with A1=1.
Joint numbering in various ways, member numbering regular for all eight cases from right to left with 1-2-3-4.

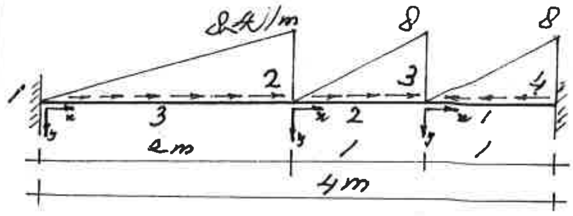



Fig.29

N9=4 joints.

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 2 |
| 3 | 0 | 0 | 0 | 0 | 3 |
| 4 | 0 | 1 | 0 | 0 | 4 |

P9=3 members.

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 3 | 4 | 1 | 0 | 1 |
| I | Q6 | Q7 | L6 | L7 | |
| 1 | 0 | -8 | 0 | 1 | |
| 2 | 2 | 3 | 1 | 0 | 1 |
| I | Q6 | Q7 | L6 | L7 | |
| 1 | 0 | 8 | 0 | 1 | |
| 3 | 1 | 2 | 1 | 0 | 1 |
| I | Q6 | Q7 | L6 | L7 | |
| 1 | 0 | 8 | 0 | 2 | |

After input of joint and member data click Calculate, DRAWN5, Results and Reactions.

On the right of the reactions the horizontal displacements UH(I) are printed, values UH(1)=0 UH(2)= 7,33 UH(3)= 4,33 UH(4)= 0 /EA

Before clicking Show click first Again to make all UH(I) zero, next click Show.

The bottom of the data 'disappear' on the controls, click somewhere on the form to make the controls invisible, with another click they become visible again.

The member axis system (x, y) always placed at the lowest member end number determines which side of the member axis indicates pressure or tension. Assumed is pressure at the side of the axis system. For this example pressure below and tension above the zero line.

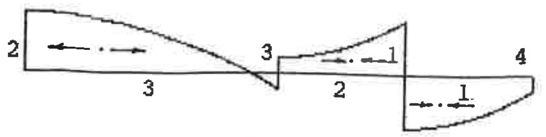
Suppose the distributed load forces of member 3 to be removed, as follows.
Double click TP9 if necessary, type 3 in TP9, Tab, and for P L H A1 NFA NQA in TSTRING

3,1,2,1,0,**0** Enter. Show gives the line below

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 3 | 1 | 2 | 1,0 | 0 | 0 |

Next Calculate, DRAWN5, Results and Reactions, a beautiful normal diagram appears.

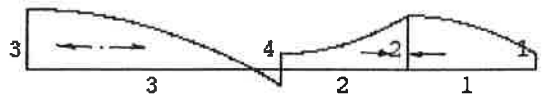
**normal force diagram** EX82




| | | | |
|---|---|---|---|
| NAA(1,4)= | 5,67 kN | NAA(4,1)= | -1,67 kN |
| NAA(1,3)= | 5,67 kN | NAA(3,1)= | -1,67 kN |
| NAA(2,3)= | -6,33 kN | NAA(3,2)= | -1,67 kN |

| | | | |
|---|---|---|---|
| RH(1)= | 0,00 kN | UH(1)= | 4,33 /EA |
| RH(2)= | -6,33 kN | UH(2)= | 0,00 /EA |
| RH(3)= | 0,00 kN | UH(3)= | 7,33 /EA |
| RH(4)= | -1,67 kN | UH(4)= | 0,00 /EA |

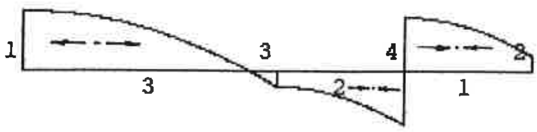
**normal force diagram** EX83




| | | | |
|---|---|---|---|
| NAA(1,2)= | 1,67 kN | NAA(2,1)= | -5,67 kN |
| NAA(2,4)= | 5,67 kN | NAA(4,2)= | -1,67 kN |
| NAA(3,4)= | -6,33 kN | NAA(4,3)= | -1,67 kN |

| | | | |
|---|---|---|---|
| RH(1)= | -1,67 kN | UH(1)= | 0,00 /EA |
| RH(2)= | 0,00 kN | UH(2)= | 4,33 /EA |
| RH(3)= | -6,33 kN | UH(3)= | 0,00 /EA |
| RH(4)= | 0,00 kN | UH(4)= | 7,33 /EA |

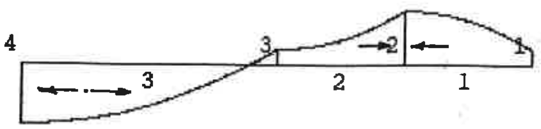
**normal force diagram** EX84




| | | | |
|---|---|---|---|
| NAA(2,4)= | 1,67 kN | NAA(4,2)= | -5,67 kN |
| NAA(3,4)= | 1,67 kN | NAA(4,3)= | -5,67 kN |
| NAA(1,3)= | -6,33 kN | NAA(3,1)= | -1,67 kN |

| | | | |
|---|---|---|---|
| RH(1)= | -6,33 kN | UH(1)= | 0,00 /EA |
| RH(2)= | -1,67 kN | UH(2)= | 0,00 /EA |

**normal force diagram** EX85




| | | | |
|---|---|---|---|
| NAA(1,2)= | 1,67 kN | NAA(2,1)= | -5,67 kN |
| NAA(2,3)= | 5,67 kN | NAA(3,2)= | -1,67 kN |
| NAA(3,4)= | 1,67 kN | NAA(4,3)= | 6,33 kN |

Click EXAMPLES81 to EXAMPLES82

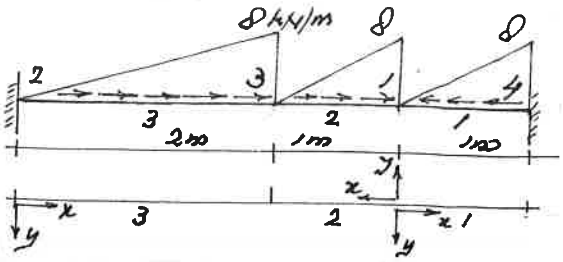


Fig.30.

N9=4 joints.

| I | FX | PH | UH | SH | X1 |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 3 |
| 2 | 0 | 1 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 2 |
| 4 | 0 | 1 | 0 | 0 | 4 |

P9=3 members.

| P | L | H | A1 | NFA | NQA |
|---|---|---|---|---|---|
| 1 | 1 | 5 | 1 | 0 | 1 |
| I | Q6 | Q7 | L6 | L7 | |
| 1 | 0 | -8 | 0 | 1 | |
| 2 | 1 | 3 | 1 | 0 | 1 |
| I | Q6 | Q7 | L6 | L7 | |
| 1 | -8 | 0 | 0 | 1 | |
| 3 | 2 | 3 | 1 | 0 | 1 |
| I | Q6 | Q7 | L6 | L7 | |
| 1 | 0 | 8 | 0 | 2 | |

Click EXAMPLES82 to EXAMPLES83

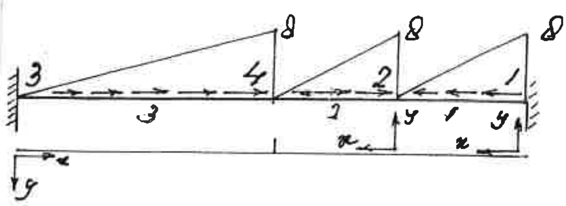


Fig.31.

Click EXAMPLES83 to EXAMPLES84

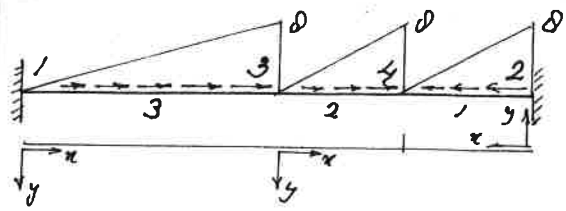


Fig.32.

Click EXAMPLES84 to EXAMPLES85

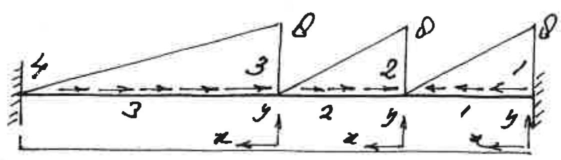


Fig.33.

The examples show various possibilities of joint numbering with correspondind drawings of the normal force diagrams. There are various possibilities of member numbering as well.

Three other cases EXAMPLES86 to EXAMPLES88 left to compare.

```
I=1   X1(1)=0
I=2   X1(2)=0,5
I=3   X1(3)=1
P=1   LL(1)=1   HH(1)=2   A11(1)=2
P=2   LL(2)=2   HH(2)=3   A11(2)=3
```

```
N9= 3    [          ]  OK   Show CC
P9= 2    EX1  EX2  EX3  EX4  Again  End
         STORE  NR= ?  GET   Cls    PrF
```

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 4,00 | -4,00 | |
| 2 | -4,00 | 4,00 | |
| 3 | | | |

S5

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 4,00 | -4,00 | |
| 2 | -4,00 | 4,00 | |
| 3 | | | |

CC

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | | | |
| 2 | | 6,00 | -6,00 |
| 3 | | -6,00 | 6,00 |

S5

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 4,00 | -4,00 | |
| 2 | -4,00 | 10,00 | -6,00 |
| 3 | | -6,00 | 6,00 |

CC

```
I=1   X1(1)=1
I=2   X1(2)=0,5
I=3   X1(3)=0
P=1   LL(1)=2   HH(1)=3   A11(1)=2
P=2   LL(2)=1   HH(2)=2   A11(2)=3
```

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | | | |
| 2 | | 4,00 | -4,00 |
| 3 | | -4,00 | 4,00 |

S5

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | | | |
| 2 | | 4,00 | -4,00 |
| 3 | | -4,00 | 4,00 |

CC

AXCC111

Member stiffness matrices S5 and construction stiffness matrix CC.

Exmaple, see page 10. (A1 is EAA(P))

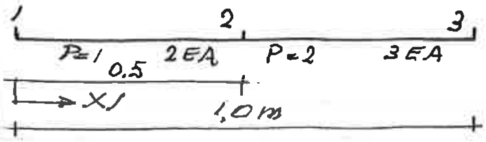


Fig.1.

| N9=3 joints | | | |
|---|---|---|---|
| I | 1 | 2 | 3 |
| X1(I) | 0 | .5 | 1 |

| P9=2 members | | |
|---|---|---|
| P | 1 | 2 |
| L | 1 | 2 |
| H | 2 | 3 |
| A1 | 2 | 3 |

When puting in data line after line appears on the screen .
Type 3 in TN9, Tab and type in TSTRING 1,0, Enter, 2,.5 Enter and 3,1 Enter,

cursor in TP9, Tab, and type in TSTRING 1,1,2,2 Enter and 2,2,3,3 Enter.

First click on Show CC, prints first stiffness matrix S5 of member 1 two times with L=1 and H=2, row and column 1 and 2.

Second click on Show CC, prints second stiffness matrix S5 of member 2,
first time with L=2 and H=3, row and column 2 and 3,
second time it is added to stiffness matrix CC, with L=2 and H=3, row and column 2 and 3.

Joint numbering 3-2-1 in stead of 1-2-3.

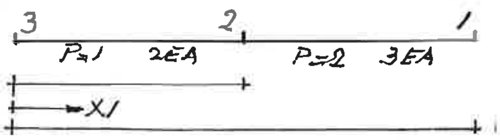


Fig.2.

| N9=3 joints | | | |
|---|---|---|---|
| I | 1 | 2 | 3 |
| X1(I) | 1 | .5 | 0 |

| P9=2 members | | |
|---|---|---|
| P | 1 | 2 |
| L | 2 | 1 |
| H | 3 | 2 |
| A1 | 2 | 3 |

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 6,00 | -6,00 | |
| 2 | -6,00 | 6,00 | |
| 3 | | | |

S5

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 6,00 | -6,00 | |
| 2 | -6,00 | 10,00 | -4,00 |
| 3 | | -4,00 | 4,00 |

CC

```
I=1   X1(1)=0
I=2   X1(2)=2
I=3   X1(3)=4
I=4   X1(4)=6
P=1   LL(1)=1   HH(1)=2   A11(1)=1
P=2   LL(2)=2   HH(2)=3   A11(2)=2
P=3   LL(3)=3   HH(3)=4   A11(3)=3
```

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | 0,50 | -0,50 |
| 4 | | | -0,50 | 0,50 |

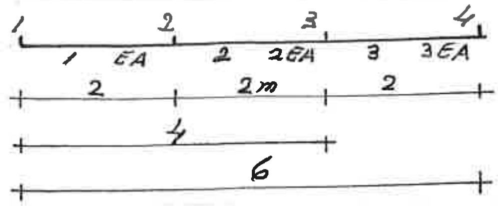
| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0,50 | -0,50 | | |
| 2 | -0,50 | 1,00 | -0,50 | |
| 3 | | -0,50 | 1,00 | -0,50 |
| 4 | | | -0,50 | 0,50 |

```
I=1   X1(1)=0
I=2   X1(2)=0,5
I=3   X1(3)=1,1
I=4   X1(4)=1,8
I=5   X1(5)=2,6
P=1   LL(1)=1   HH(1)=2   A11(1)=1
P=2   LL(2)=2   HH(2)=3   A11(2)=1
P=3   LL(3)=3   HH(3)=4   A11(3)=1
P=4   LL(4)=4   HH(4)=5   A11(4)=1
```

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | 1,25 | -1,25 |
| 5 | | | | -1,25 | 1,25 |

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 2,00 | -2,00 | | | |
| 2 | -2,00 | 3,67 | -1,67 | | |
| 3 | | -1,67 | 3,10 | -1,43 | |
| 4 | | | -1,43 | 2,68 | -1,25 |
| 5 | | | | -1,25 | 1,25 |

Example.   EX1   N9=4   P9=3



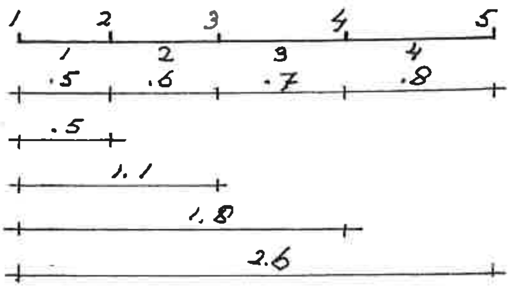

| N9=4 joints | | | | |
|---|---|---|---|---|
| I | 1 | 2 | 3 | 4 |
| X1(I) | 0 | 2 | 4 | 6 |

| P9=3 members | | | |
|---|---|---|---|
| P | 1 | 2 | 3 |
| L | 1 | 2 | 3 |
| H | 2 | 3 | 4 |
| A1 | 1 | 2 | 3 |

Fig.3.

Click EX1, data are printed. Click three times to find the results shown on the left.

Example.   EX2   N9=5   P9=4



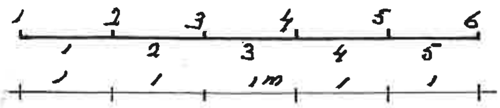

| N9=5 joints | | | | | |
|---|---|---|---|---|---|
| I | 1 | 2 | 3 | 4 | 5 |
| X1(I) | 0 | .5 | 1.1 | 1.8 | 2.6 |

| P9=3 members | | | | |
|---|---|---|---|---|
| P | 1 | 2 | 3 | 4 |
| L | 1 | 2 | 3 | 4 |
| H | 2 | 3 | 4 | 5 |
| A1 | 1 | 1 | 1 | 1 |

Fig.4.

Example.   EX3   N9=6   P9=5

Results below after fifth click.




| N9=6 joints | | | | | | |
|---|---|---|---|---|---|---|
| I | 1 | 2 | 3 | 4 | 5 | 6 |
| X1(I) | 0 | 1 | 2 | 3 | 4 | 5 |

| P9=5 members | | | | | |
|---|---|---|---|---|---|
| P | 1 | 2 | 3 | 4 | 5 |
| L | 3 | 4 | 5 | 1 | 2 |
| H | 4 | 5 | 6 | 2 | 3 |
| A1 | all 1.6 | | | | |

Fig.5.

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1,60 | -1,60 | | | | |
| 2 | -1,60 | 3,20 | -1,60 | | | |
| 3 | | -1,60 | 3,20 | -1,60 | | |
| 4 | | | -1,60 | 3,20 | -1,60 | |
| 5 | | | | -1,60 | 3,20 | -1,60 |
| 6 | | | | | -1,60 | 1,60 |

Input joint and member data.

Type 6 in TN9, Tab, and in TSTRING

1,0 Enter, 2,1.1 Enter, 3,2.4 Enter,
4,3.9 Enter, 5,5.6 Enter, 6,7.5 Enter.

Type 5 in TP9, Tab, and in TSTRING

1,1,2,1 Enter, 2,2,3,1 Enter,
3,3,4,1 Enter, 4,4,5,1 Enter,
4,4,5,1 Enter, 5,5,6,1 Enter.

| | | | |
|---|---|---|---|
| I=1 | X1(1)=0 | | |
| I=2 | X1(2)=1,1 | | |
| I=3 | X1(3)=2,4 | | |
| I=4 | X1(4)=3,9 | | |
| I=5 | X1(5)=5,6 | | |
| I=6 | X1(6)=7,5 | | |
| P=1 | LL(1)=1 | HH(1)=2 | A11(1)=1 |
| P=2 | LL(2)=2 | HH(2)=3 | A11(2)=1 |
| P=3 | LL(3)=3 | HH(3)=4 | A11(3)=1 |
| P=4 | LL(4)=4 | HH(4)=5 | A11(4)=1 |
| P=5 | LL(5)=5 | HH(5)=6 | A11(5)=1 |

Example.

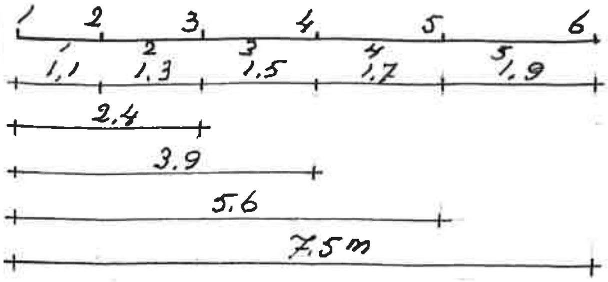



Fig.6.

N9=6 joints

| I | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| X1(I) | 0 | 1.1 | 2.4 | 3.9 | 5.6 | 7.5 |

P9=5 members

| P | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| L | 1 | 2 | 3 | 4 | 5 |
| H | 2 | 3 | 4 | 5 | 6 |
| A1 | 1 | 1 | 1 | 1 | 1 |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,91 | -0,91 | | | | |
| 2 | -0,91 | 0,91 | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

SS

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,91 | -0,91 | | | | |
| 2 | -0,91 | 0,91 | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

CC

1

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | 0,77 | -0,77 | | | |
| 3 | | -0,77 | 0,77 | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,91 | -0,91 | | | | |
| 2 | -0,91 | 1,68 | -0,77 | | | |
| 3 | | -0,77 | 0,77 | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

2

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | 0,67 | -0,67 | | |
| 4 | | | -0,67 | 0,67 | | |
| 5 | | | | | | |
| 6 | | | | | | |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,91 | -0,91 | | | | |
| 2 | -0,91 | 1,68 | -0,77 | | | |
| 3 | | -0,77 | 1,44 | -0,67 | | |
| 4 | | | -0,67 | 0,67 | | |
| 5 | | | | | | |
| 6 | | | | | | |

3

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | 0,59 | -0,59 | |
| 5 | | | | -0,59 | 0,59 | |
| 6 | | | | | | |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,91 | -0,91 | | | | |
| 2 | -0,91 | 1,68 | -0,77 | | | |
| 3 | | -0,77 | 1,44 | -0,67 | | |
| 4 | | | -0,67 | 1,25 | -0,59 | |
| 5 | | | | -0,59 | 0,59 | |
| 6 | | | | | | |

4

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | 0,53 | -0,53 |
| 6 | | | | | -0,53 | 0,53 |

SS

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,91 | -0,91 | | | | |
| 2 | -0,91 | 1,68 | -0,77 | | | |
| 3 | | -0,77 | 1,44 | -0,67 | | |
| 4 | | | -0,67 | 1,25 | -0,59 | |
| 5 | | | | -0,59 | 1,11 | -0,53 |
| 6 | | | | | -0,53 | 0,53 |

CC

5

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,53 | -0,53 | | | | |
| 2 | -0,53 | 0,53 | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

SS

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,53 | -0,53 | | | | |
| 2 | -0,53 | 1,11 | -0,59 | | | |
| 3 | | -0,59 | 1,25 | -0,67 | | |
| 4 | | | -0,67 | 1,44 | -0,77 | |
| 5 | | | | -0,77 | 1,68 | -0,91 |
| 6 | | | | | -0,91 | 0,91 |

CC

Member numbering in reversed order, 5-4-3-2-1 i.s.o. 1-2-3-4-5.

EX4

| | | | | |
|---|---|---|---|---|
| I=1 | X1(1)=7,5 | | |
| I=2 | X1(2)=5,6 | | |
| I=3 | X1(3)=3,9 | | |
| I=4 | X1(4)=0 | | |
| I=5 | X1(5)=1,1 | | |
| I=6 | X1(6)=2,4 | | |
| P=1 | LL(1)=4 | HH(1)=5 | A11(1)=1 |
| P=2 | LL(2)=5 | HH(2)=6 | A11(2)=1 |
| P=3 | LL(3)=3 | HH(3)=6 | A11(3)=1 |
| P=4 | LL(4)=1 | HH(4)=2 | A11(4)=1 |
| P=5 | LL(5)=2 | HH(5)=3 | A11(5)=1 |

| | | | |
|---|---|---|---|
| I=1 | X1(1)=2,4 | | |
| I=2 | X1(2)=3,9 | | |
| I=3 | X1(3)=1,1 | | |
| I=4 | X1(4)=7,5 | | |
| I=5 | X1(5)=0 | | |
| I=6 | X1(6)=5,6 | | |
| P=1 | LL(1)=1 | HH(1)=3 | A11(1)=1 |
| P=2 | LL(2)=1 | HH(2)=2 | A11(2)=1 |
| P=3 | LL(3)=4 | HH(3)=6 | A11(3)=1 |
| P=4 | LL(4)=2 | HH(4)=6 | A11(4)=1 |
| P=5 | LL(5)=3 | HH(5)=5 | A11(5)=1 |

Example.

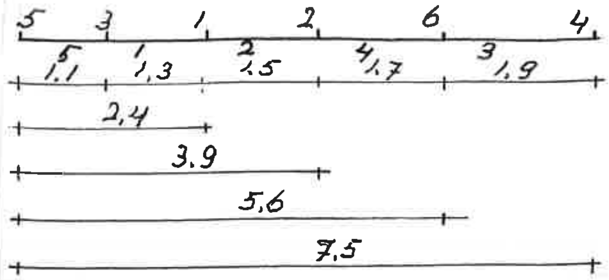



Fig.7.

N9=6 joints

| I | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| X1(I) | 2.4 | 3.9 | 1.1 | 7.5 | 0 | 5.6 |

P9=5 members

| P | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| L | 1 | 1 | 4 | 2 | 3 |
| H | 3 | 2 | 6 | 6 | 5 |
| A1 | 1 | 1 | 1 | 1 | 1 |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,77 | | -0,77 | | | |
| 2 | | | | | | |
| 3 | -0,77 | | 0,77 | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,77 | | -0,77 | | | |
| 2 | | | | | | |
| 3 | -0,77 | | 0,77 | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,67 | -0,67 | | | | |
| 2 | -0,67 | 0,67 | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1,44 | -0,67 | -0,77 | | | |
| 2 | -0,67 | 0,67 | | | | |
| 3 | -0,77 | | 0,77 | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | 0,53 | | -0,53 |
| 5 | | | | | | |
| 6 | | | | -0,53 | | 0,53 |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1,44 | -0,67 | -0,77 | | | |
| 2 | -0,67 | 0,67 | | | | |
| 3 | -0,77 | | 0,77 | | | |
| 4 | | | | 0,53 | | -0,53 |
| 5 | | | | | | |
| 6 | | | | -0,53 | | 0,53 |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | 0,59 | | | | -0,59 |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | -0,59 | | | | 0,59 |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1,44 | -0,67 | -0,77 | | | |
| 2 | -0,67 | 1,25 | | | | -0,59 |
| 3 | -0,77 | | 0,77 | | | |
| 4 | | | | 0,53 | | -0,53 |
| 5 | | | | | | |
| 6 | | -0,59 | | -0,53 | | 1,11 |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | 0,91 | | -0,91 | |
| 4 | | | | | | |
| 5 | | | -0,91 | | 0,91 | |
| 6 | | | | | | |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1,44 | -0,67 | -0,77 | | | |
| 2 | -0,67 | 1,25 | | | | -0,59 |
| 3 | -0,77 | | 1,68 | | -0,91 | |
| 4 | | | | 0,53 | | -0,53 |
| 5 | | | -0,91 | | 0,91 | |
| 6 | | -0,59 | | -0,53 | | 1,11 |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | 0,59 | -0,59 | | | |
| 3 | | -0,59 | 0,59 | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

EX4

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,53 | -0,53 | | | | |
| 2 | -0,53 | 1,11 | -0,59 | | | |
| 3 | | -0,59 | 1,25 | | | -0,67 |
| 4 | | | | 0,91 | -0,91 | |
| 5 | | | | -0,91 | 1,68 | -0,77 |
| 6 | | | -0,67 | | -0,77 | 1,44 |

after 5th click for EX4

Example.

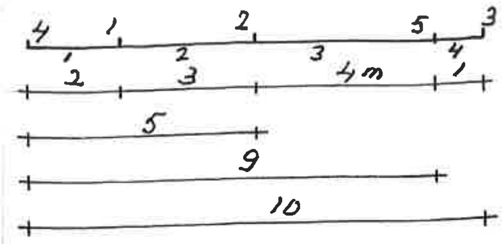



Fig.8.

N9=6 joints

| I | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| X1(I) | 2 | 5 | 10 | 0 | 9 |

P9=5 members

| P | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| L | 1 | 1 | 2 | 3 |
| H | 4 | 2 | 5 | 5 |
| A1 | 1 | 1 | 1 | 1 |

```
I=1   X1(1)=2
I=2   X1(2)=5
I=3   X1(3)=10
I=4   X1(4)=0
I=5   X1(5)=9
P=1   LL(1)=1   HH(1)=4   A11(1)=1
P=2   LL(2)=1   HH(2)=2   A11(2)=1
P=3   LL(3)=2   HH(3)=5   A11(3)=I
P=4   LL(4)=3   HH(4)=5   A11(4)=1
```

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0,50 | | | -0,50 | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | -0,50 | | | 0,50 | |
| 5 | | | | | |

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0,50 | | | -0,50 | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | -0,50 | | | 0,50 | |
| 5 | | | | | |

1

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0,33 | -0,33 | | | |
| 2 | -0,33 | 0,33 | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0,83 | -0,33 | | -0,50 | |
| 2 | -0,33 | 0,33 | | | |
| 3 | | | | | |
| 4 | -0,50 | | | 0,50 | |
| 5 | | | | | |

2

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | 0,25 | | | -0,25 |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | -0,25 | | | 0,25 |

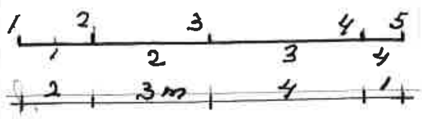
| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0,83 | -0,33 | | -0,50 | |
| 2 | -0,33 | 0,58 | | | -0,25 |
| 3 | | | | | |
| 4 | -0,50 | | | 0,50 | |
| 5 | | -0,25 | | | 0,25 |

3

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | 1,00 | | -1,00 |
| 4 | | | | | |
| 5 | | | -1,00 | | 1,00 |

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0,83 | -0,33 | | -0,50 | |
| 2 | -0,33 | 0,58 | | | -0,25 |
| 3 | | | 1,00 | | -1,00 |
| 4 | -0,50 | | | 0,50 | |
| 5 | | -0,25 | -1,00 | | 1,25 |

4




```
I=1   X1(1)=0
I=2   X1(2)=2
I=3   X1(3)=5
I=4   X1(4)=9
I=5   X1(5)=10
P=1   LL(1)=1   HH(1)=2   A11(1)=1
P=2   LL(2)=2   HH(2)=3   A11(2)=1
P=3   LL(3)=3   HH(3)=4   A11(3)=1
P=4   LL(4)=4   HH(4)=5   A11(4)=1
```

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | 1,00 | -1,00 |
| 5 | | | | -1,00 | 1,00 |

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0,50 | -0,50 | | | |
| 2 | -0,50 | 0,83 | -0,33 | | |
| 3 | | -0,33 | 0,58 | -0,25 | |
| 4 | | | -0,25 | 1,25 | -1,00 |
| 5 | | | | -1,00 | 1,00 |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,63 | -0,63 | | | | |
| 2 | -0,63 | 2,05 | -1,43 | | | |
| 3 | | -1,43 | 1,93 | -0,50 | | |
| 4 | | | -0,50 | 1,68 | -1,18 | |
| 5 | | | | -1,18 | 1,61 | -0,43 |
| 6 | | | | | -0,43 | 0,43 |

1.

N9= [ ] [ ] **OK** Show CC
P9= [ ] EX1 EX2 EX3 EX4 Again End
Show STORE NR=3 GET Cls PrF

Storing data.
Click NR= to e.g. not underlined NR=3. Click STORE, gets underlined, NR=3 as well, NR=3. See STORE NR=3 GET

Click NR=3 underlining disappears.

If later clicking NR=... to NR=3 then click GET gets underlined GET to get the stored data. First click Again !! and Show shows those data.

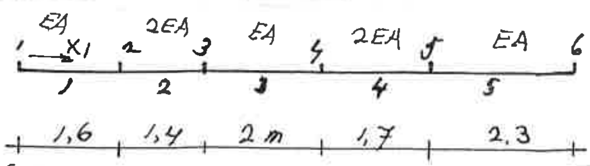
To remove data of NR=3, click GET (or GET if wanted) with right mouse button and underlining of NR=3 and GET disappears.

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0,43 | -0,43 | | | | |
| 2 | -0,43 | 1,61 | -1,18 | | | |
| 3 | | -1,18 | 1,68 | -0,50 | | |
| 4 | | | -0,50 | 1,93 | -1,43 | |
| 5 | | | | -1,43 | 2,05 | -0,63 |
| 6 | | | | | -0,63 | 0,63 |

2. 3.

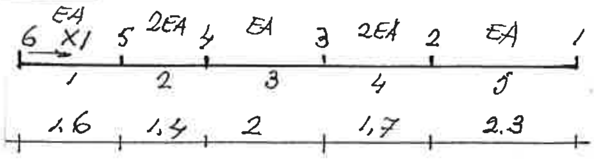
| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1,93 | | | | -1,43 | -0,50 |
| 2 | | 0,43 | | -0,43 | | |
| 3 | | | 0,63 | | -0,63 | |
| 4 | | -0,43 | | 1,61 | | -1,18 |
| 5 | -1,43 | | -0,63 | | 2,05 | |
| 6 | -0,50 | | | -1,18 | | 1,68 |

Example 1. AXCC222



| | | | | | | |
|---|---|---|---|---|---|---|
| N9=6 | | | | | | |
| I | 1 | 2 | 3 | 4 | 5 | 6 |
| X1(I) | 0 | 1.6 | 3 | 5 | 6.7 | 9 |
| P9=5 | | | | | | |
| P | 1 | 2 | 3 | 4 | 5 | |
| LL(P) | 1 | 2 | 3 | 4 | 5 | |
| HH(P) | 2 | 3 | 4 | 5 | 6 | |
| A11(P) | 1 | 2 | 1 | 2 | 1 | |

Example 2.

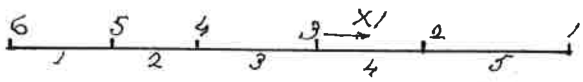


The members are numbered from left to right and the joints from right to left, in nice order.

| | | | | | | |
|---|---|---|---|---|---|---|
| N9=6 | | | | | | |
| I | 1 | 2 | 3 | 4 | 5 | 6 |
| X1(I) | 9 | 6.7 | 5 | 3 | 1.6 | 0 |
| P9=5 | | | | | | |
| P | 1 | 2 | 3 | 4 | 5 | |
| LL(P) | 5 | 4 | 3 | 2 | 1 | |
| HH(P) | 6 | 5 | 4 | 3 | 2 | |
| A11(P) | 1 | 2 | 1 | 2 | 1 | |

Again matrix CC with elements around the main axis with values mirrored w.r.t. the other axis comparing with the results just found before.

Example 3.

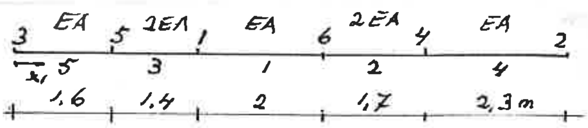


Like the last example but now the origin of the X1 axis is placed at joint 3, so only the joint coordinates must be changed.

| | | | | | | |
|---|---|---|---|---|---|---|
| N9=6 | | | | | | |
| I | 1 | 2 | 3 | 4 | 5 | 6 |
| X1(I) | 4 | 1.7 | 0 | -2 | -3.4 | -5 |

P9=5 data like above.
The results like those of the previous example.

Example 4. With irregular numbering.



| | | | | | | |
|---|---|---|---|---|---|---|
| N9=6 | | | | | | |
| I | 1 | 2 | 3 | 4 | 5 | 6 |
| X1(I) | 3 | 9 | 0 | 6.7 | 1.6 | 5 |
| P9=5 | | | | | | |
| P | 1 | 2 | 3 | 4 | 5 | |
| LL(P) | 1 | 4 | 1 | 2 | 3 | |
| HH(P) | 6 | 6 | 5 | 4 | 5 | |
| A11(P) | 1 | 2 | 2 | 1 | 1 | |

And compare the places and values with the examples 2 and 3.

```
Private Sub AXMAINCALC()   page 22
'1. Composition of construction ma-
'trix CC with member matrices S5.
CONSTRMATCCAXMEMBER     page  20

'2. Elements of force vector FF,
'2a. Joint load forces FX(I).
N=1*N9
For I=1 To N9
A=1*I
FF(A)=FX(I)
PP(A)=PH(I)
UU(A)=UH(I)
SS(A)=SH(I)
Next I

'2b. Primary forces due to member
'loads along the member axis.
'staafas.
For P=1 TO P9 : L=LL(P) : H=HH(P)
EA=EAA(P)
D1=X1(H)-X1(L)
L1=Sqr(D1^2) : L11(P)=L1
C=D1/L1

MEMBER    page  15

D7(P,1)=N1*C : D7(P,2)=N2*C
Next P

'2c. Alteration of force vector FF.
For I=1 To N9
A=1*I
For P=1 To P9 : L=LL(P) : H=HH(P)
If I=L Then
FF(A)=FF(A)+D7(P,1)
ElseIf I=H Then
FF(A)=FF(A)+D7(P,2)
End If
Next P
Next I

'3. Alteration of force vector FF
'and construction matrix CC.
'3a. Of FF in case of prescribed
'displacements <>0.
For I=1 To N
If UU(I)<>0 Then
For K=1 To N
FF(K)=FF(K)-CC(K,I)*UU(I)
Next K
End If
Next I
'3b. Of FF and CC in case of pres-
'cribed displacements.
For I=1 To N
If PP(I)=1 Then
For J=1 To N
CC(I,J)=0 : CC(J,I)=0
Next J
CC(I,I)=1 : FF(I)=UU(I)
End If
Next I
'3c. Of CC in case of elastic/
'springy supports.
For I=1 To N
If SS(I)>0 Then
CC(I,I)=CC(I,I)+SS(I)
Next I
```

```
'4. Calculation of the unknown
'displacements UH(I).
For I=1 To N : BB(I)=FF(I)
For J=1 To N
AA(I,J)=CC(I,J)
Next J
Next I
'The solution of the N=1*N9
'equations.
GAUSS     part  12
For I=1 To N9
A=1*I
UH(I)=XX(A)
UU(A)=XX(A)
Next I
'5. Calculation of the memberend
'forces w.r.t. construction axis X.
'5a. Due to the displacements
'alone.
For P=1 To P9 : L=LL(P) : H=HH(P)
EA=EAA(P)
MEMBERMATS5AXMEMBER     page  21
TT(1)=1*L
TT(2)=1*H
For I=1 TO 2 : FK(P,I)=0
For J=1 TO 2 : A=TT(J)
FK(P,I)=FK(P,I)+S5(I,J)*UU(A)
Next J
Next I
'5b. Due to displacements and mem-
'ber loads along the member axis.
D5(P,1)=FK(P,1)-D7(P,1)
D5(P,2)=FK(P,2)-D7(P,2)
D1=X1(H)-X1(L) : L1=Sqr(D1^2) : C=D1/L1
NAA(P,1)=D5(P,1)*C
NAA(P,2)=D5(P,2)*C
Next P
'6. Calculation of the joint for-
'ces KH(I).
'6a. Due to the displacements
'alone.
CONSTRMATCCAXMEMBER     page  20
For I=1 To N9
A=1*I
KH(I)=0
For J=1 To N
KH(I)=KH(I)+CC(A,J)*UU(J)
Next J
'6b. Due to the displacements and
'member loads along the member
'axis.
For P=1 To P9 : L=LL(P) : H=HH(P)
If I=L Then
KH(I)=KH(I)-D7(P,1)
ElseIf I=H Then
KH(I)=KH(I)-D7(P,2)
End If
Next P
Next I
'7. Calculation of the reactions.
For I=1 To N9
If SH(I)>0 Then
RH(I)=-SH(I)*UH(I)
Else
RH(I)=KH(I)-FX(I)
End If
Next I
End Sub
```

```
Private Sub CONSTRMATCCAXMEMBER()
N=N9                                        page 20
For I=1 To N : For J=1 To N
CC(I,J)=0 : Next J : Next I
FOR P=1 To P9 : L=LL(P) : H=HH(P)
EA=EAA(P)
MEMBERMATS5AXMEMBER
TT(1)=L : TT(2)=H
For I=1 To 2 : I1=TT(I)
For J=1 To 2 : J1=TT(2)
CC(I1,J1)=CC(I1,J1)+S5(I,J)
Next J
Next I
Next P
End Sub
```

Subroutine CONSTRMATCCAXMEMBER will be extended in coming programmes for trusses beams and frames, getting their suitable names.
Same for subroutine MEMBERMATS5AXMEMBER.

Subroutine MEMBER will be used in other programmes. N5G and N5XX can be applied there if wanted.

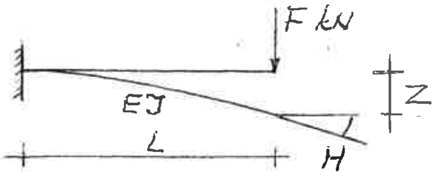
```
Private Sub MEMBERMATS5AXMEMBER()
D1=X1(H)-X1(L)                              page 21
L1=SQR(D1^2)
R=EA/L1
S5(1,1)=R : S5(1,2)=-R
S5(2,1)=-R : S5(2,2)=R
End Sub
```

```
Private Sub MEMBER()                        page 15
'Calculation of the reactions due
'to member loads along the member.
N1=0 : N2=0
'The concentrated loads.
For I=1 To NFA(P)
F5=F55(P,I) : L5=L55(P,I)
N4=F5*L5/L1 : N3=N5-N4
N1=N1+N3 : N2=N2+N4
Next I
'The distributed loads.
For I=1 To NQA(P)
Q6=Q66(P,I) : L6=L66(P,I)
Q7=Q77(P,I) : L7=L77(P,I)
F=.5*(Q6+Q7)*L7 : V3=F*L6/EA
V5=Q7*L7^2/(2*EA)
V6=(Q7-Q6)*L7^2/(6*EA)
V1=V3+V5-V6
N4=V1*EA/L1 : N3=F-N4
N1=N1+N3 : N2=N2+N4
Next I
End Sub
```

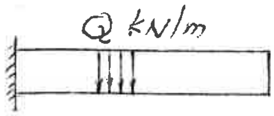
```
Private Sub N5G()                           page 19
'Calculation of the normal forces
'every G meter.
NA=0 : L1=L11(P)
For XG=0 To L1+G Step G
If XG=0 Then
X=XG : N5XX
NA=NA+1 : LA(P,NA)=X
NAL(P,NA)=N5 : NAR(P,NA)=N7
ElseIf XG>0 And XG<=L1 Then
C2=1
For I1=1 To NFA(P)
L5=L55(P,I1)
If L5>XG-G And L5<=XG Then
X=L5 : N5XX
NA=NA+1 : LA(P,NA)=X
NAL(P,NA)=N5 : NAR(P,NA)=N7
If L5=XG Then C2=0
End If
Next I1
If C2=1 Then
X=XG : N5XX
NA=NA+1 : LA(P,NA)=X
NAL(P,NA)=N5 : NAR(P,NA)=N7
End If
ElseIf XG>L1 Then
For I1=1 To NFA(P)
L5=L55(P,I1)
If L5>XG-G And L5<L1 Then
X=L5 : N5XX
NA=NA+1 : LA(P,NA)=X
NAL(P,NA)=N5 : NAR(P,NA)=N7
End If
Next I1

If XG-G<L1 Then
X=L1 : N5XX
NA=NA+1 : LA(P,NA)=X
NAL(P,NA)=N5 : NAR(P,NA)=N7
End If
End If
Next XG : NAC(P)=NA
End Sub
```

```
Private Sub N5XX()                          page 17
'Calculoation of the normal force
'at X meter from member end L.
N5=BN : N7=BN
'The concentrated loads.
For I=1 To NFA(P)
F5=F55(P,I) : L5=L55(P,I)
If X>L5 Then
N5=N5+F5 : N7=N7+F5
ElseIf X=L5 Then
N7=N7+F5
End If
Next I
'The distributed loads.
For I=1 To NQA(P)
Q6=Q66(P,I) : L6=L66(P,I)
Q7=Q77(P,I) : L7=L77(P,I)
If X>L6 Then
If X>L6 And X<=L6+L7 Then
Q8=Q6+(Q7-Q6)*(X-L6)/L7
T=.5*(Q6+Q8)*(X-L6)
N5=N5+T : N7=N7+T
ElseIf X>L6+L7
T=.5*(Q6+Q7)*L7
N5=N5+T : N7=N7+T
End If
End If
Next I
End Sub
```

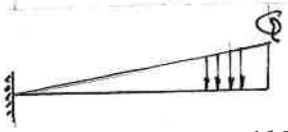



H= F*L^2/(2*EI)        Z= F*L^3/(3*EI)



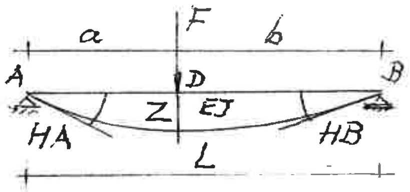

H= Q*L^3/(6*EI)        Z= Q*L^4/(8*EI)


H= Q*L^3/(24*EI)        Z= Q*L^4/(30*EI)




H= Q*L^3/(8EI)        Z= 11Q*L^3/(120EI)

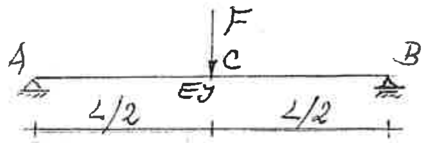

H= M*L/EI        Z= M*L^2/(2*EI)




HA= F*a*b*(L+b)/(6*L*EI)

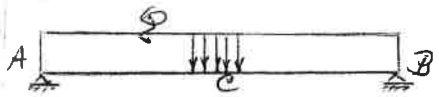
HB= F*a*b*(L+a)/(6*L*EI)

ZD= F*a^2*b^2/(3*L*EI)



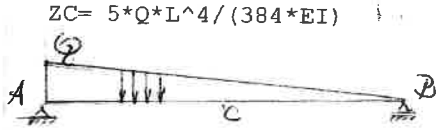

HA= HB= F*L^2/(16*EI)

ZC= F*L^3/(48*EI)




HA= HB= Q*L^3/(24*EI)

ZC= 5*Q*L^4/(384*EI)




HA= Q*L^3/(45*EI)

HB= 7*Q*L^3/(360*EI)

ZC= (5*Q*L^4/(384*EI))/2

Standard formulas for simple beams.

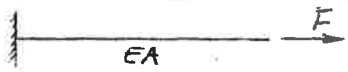
E is modulus of elasticity in kN/m^2
EI is bending stiffness, EI is E*I with
I is moment of inertia in m^4

EI is (kN/m^2)*m^4 is kNm^2

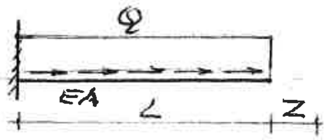
EA is strain stiffness, EA is E*A with
A is cross sectial area in m^2

EA is (kN/m^2)*m^2 is kN

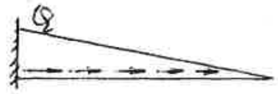
Displcement Z in m, angle H in radians




Z=F*L/EA

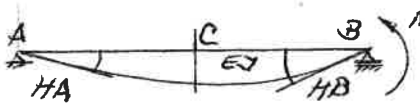



Z=Q*L^2/(2*EA)



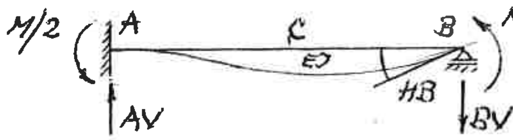

Z=Q*L^2/(6*EA)


Z=Q*L^2/(3EI)



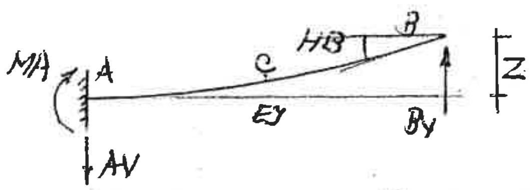

HA= M*L/(6*EI)        HB= M*L/(3*EI)

ZC= M*L^2/(16*EI)



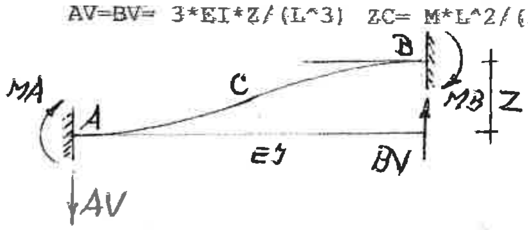

HB= M*L/(4*EI)    ZC= M*L^2/(32*EI)

AV=BV= 3*M/(2*EI)




MA=3*EI*Z/(L^2)        HB= 3*Z/(2*L)

AV=BV= 3*EI*Z/(L^3)   ZC= M*L^2/(32*EI)




MA=MB= 6*EI*Z/(L^2)     ZC= Z/2

AV=BV= 12*EI*Z/(L^3)