

Part 15

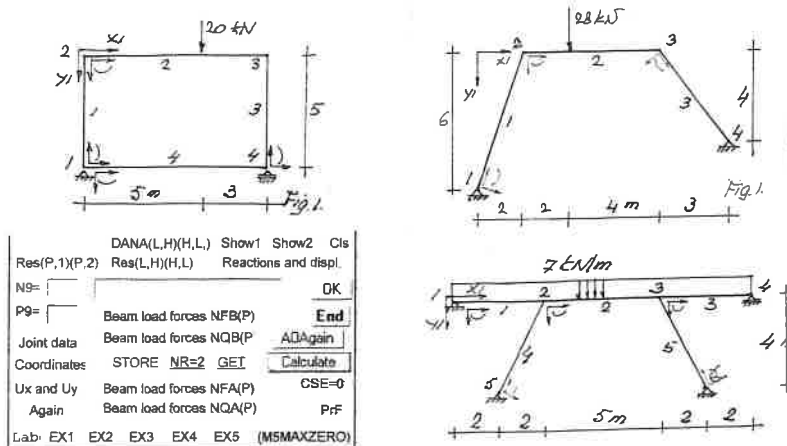
Program **FRAMEPROGRAM111**

1 -24

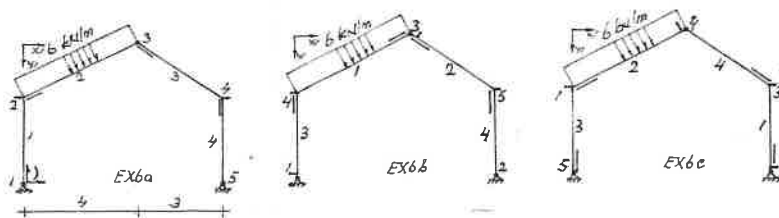
Joint and member data, assumptions!!!

Calculation of the horizontal joint displacements $UH()$ and vertical joint displacements $UV()$, and the joint rotations $UR()$.

What the labels to click on mean, examples EX1 to EX6. All worked out in detail.



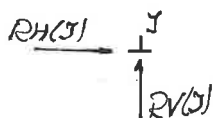
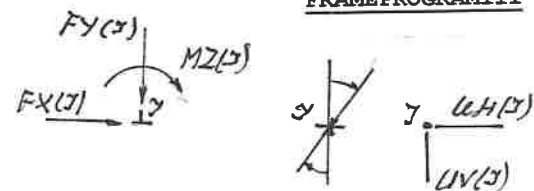
Calculation of member end forces $DA(,)$ perpendicular to the member axis, and of the member end forces $NA(,)$ along the member axis. Calculation of horizontal member end forces $D5(,)$ and vertical member end forces $D6(,)$ And the member end moments $MF(,)$.



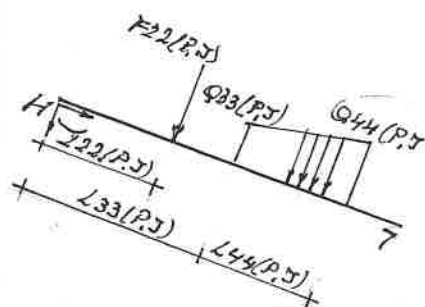
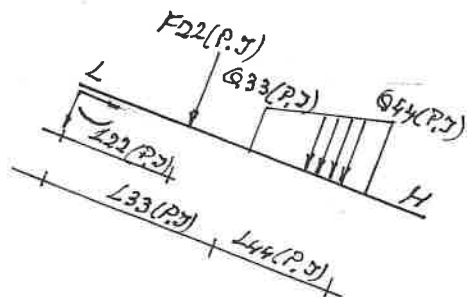
Some '3 hinge arch' examples to show the arbitrarily joint and member numbering delivering same results. See Part14 page 8.

15A

FRAMEPROGRAM111

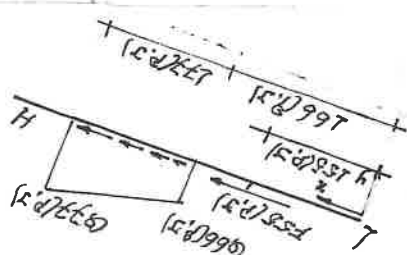
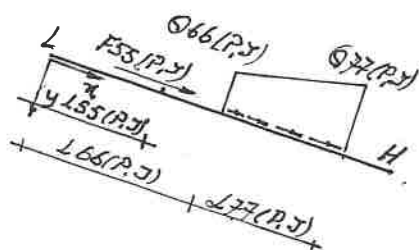


Assumptions.



Beam load forces NFB(P)

Beam load forces NQB(P)



Beam load forces NFA(P)

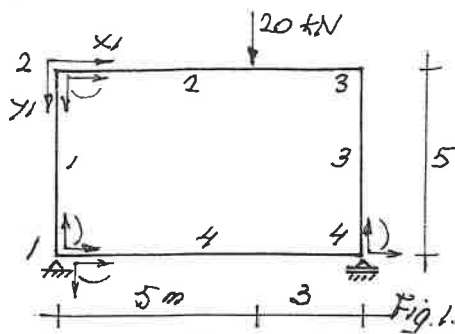
Beam load forces NQA(P)

The data to be put in for N9 joints are

- I the joint number,
- X1(I) the horizontal coordinate of the joint,
- Y1(I) the vertical coordinate of the joint.
- FY(I) the vertical joint load force, assumption directed downward.
- PV(I) for whether or not the vertical displacement of the joint is prescribed.
PV(I)=1 the displacement is prescribed, PV(I)=0 then not prescribed.
- UV(I) the vertical displacement of the joint, assumption downward.
PV(I)=1 then the prescribed value of the displacement is put in, and if PV(I)=0 then a 'start' value zero is put in.
- SV(I) a constant for a vertical spring, a zero or a value >0 is put in.
- FX(I) the horizontal joint load force, assumption directed to the right.
- PH(I) for whether or not the horizontal displacement of the joint is prescribed.
PH(I)=1 the displacement is prescribed, PH(I)=0 then not prescribed.
- UH(I) the horizontal displacement of the joint, assumption to the right
PH(I)=1 then the prescribed value of the displacement is put in, and if PH(I)=0 then a 'start' value zero is put in.
- SH(I) a constant for a horizontal spring, a zero or a value >0 is put in.
- MZ(I) the joint load moment, assumption to the right.
- PR(I) for whether or not the joint rotation of the joint is prescribed.
PR(I)=1 the rotation is prescribed, PR(I)=0 then not prescribed.
- UR(I) the joint rotation of the joint, assumption to the right.
PR(I)=1 then the prescribed value of the rotation is put in, and if PR(I)=0 then a 'start' value zero is put in.
- SR(I) The constant for a rotation spring, a zero or a value >0 is put in.

The data to be put in for P9 members are

- P the member number,
- LL(P) the lowest member end number,
- HH(P) the highest member end number.
- NL(P) to indicate if member end L is connected fixed or hinged to joint L,
NL(P)=0 then fixed, NL(P)=1 then hinged.
- NH(P) to indicate if member end H is connected fixed or hinged to joint H,
NH(P)=0 then fixed, NH(P)=1 then hinged.
- BA(P) to indicate at what member end the beam axis system is placed,
BA(P)=L then at member end L,
BA(P)=H then at member end H.
- EII(P) the bending stiffness of the beam, a proportional value, 1'EI', 2'EI', etc.
- EAA(P) the strain stiffness, a value expressed bending stiffness EI. See the examples.



DANA(L,H)(H,L) Show1 Show2 Cls
 Res(P,1)(P,2) Res(L,H)(H,L) Reactions and displ.
 N9= OK
 P9= End
 Joint data Beam load forces NQB(P) A0Again
 Coordinates STORE NR=2 GET Calculate
 Ux and Uy Beam load forces NFA(P) CSE=0
 Again Beam load forces NQA(P) PrF
 EX1 EX2 EX3 EX4 EX5 (M5MAXZERO)

N9= 4

I	FY	PV	UV	SV	X1	MZ	PR	UR	SR
1	0	1	0	0	0,00	0	0	0	0
2	0	0	0	0	0,00	0	0	0	0
3	0	0	0	0	8,00	0	0	0	0
4	0	1	0	0	8,00	0	0	0	0

I FX FH UB SH Y1

1	0	1	0	0	5,00
2	0	0	0	0	0,00
3	0	0	0	0	0,00
4	0	0	0	0	5,00

P9= 4

P	LL	BB	ML	MB	BA	EI	EA	L1
1	1	2	0	0	1	1	100	5,00
2	2	3	0	0	2	5	300	8,00

NFB(2)=1 I F2 L2

1	20	5,00
---	----	------

P LL BB ML MB BA EI EA L1

3	3	4	0	0	4	1	100	5,00
4	1	4	0	0	1	5	300	8,00

D5(1,1)= 1,83 kN	D5(1,2)= -1,83 kN
D6(1,1)= -7,44 kN	D6(1,2)= 7,44 kN
MF(1,1)= 2,53 kNm	MF(1,2)= 6,64 kNm
D5(2,1)= 1,83 kN	D5(2,2)= -1,83 kN
D6(2,1)= -7,44 kN	D6(2,2)= -12,56 kN
MF(2,1)= -6,64 kNm	MF(2,2)= 7,08 kNm
D5(3,1)= 1,83 kN	D5(3,2)= -1,83 kN
D6(3,1)= 12,56 kN	D6(3,2)= -12,56 kN
MF(3,1)= -7,08 kNm	MF(3,2)= -2,09 kNm
D5(4,1)= -1,83 kN	D5(4,2)= 1,83 kN
D6(4,1)= -0,06 kN	D6(4,2)= 0,06 kN
MF(4,1)= -2,53 kNm	MF(4,2)= 2,09 kNm

Example. EX1

Fig.1.

N9=4 joints and P9=4 members.

At the start all joint variables are set zero.

Type 4 in TN9 and Tab, cursor in TSTRING.

For support 1 PV(1)=1 Enter, PH(!)=1 Enter, and support 4 PV(4)=1 Enter.

The joint coordinates are

I	X1(I)	Y1(I)	I	X1(I)	Y1(I)
1	0	5	3	8	0
2	0	0	4	8	5

Next click Coordinates and type in TSTRING

1,0,5 Enter, 2,0,0 Enter, 3,8,0 Enter and 4,8,5 Enter.

Click Show1 to check the data put in.

Then type in TP9 4 ann Tab, cursor in TSTRING

P(9)=4

P	LL	HH	NL	NH	BA	EI	EA
1	1	2	0	0	1	1	100
2	2	3	0	0	2	5	300
3	3	4	0	0	4	1	100
4	1	4	0	0	1	5	300

To type in TSTRING

1,1,2,0,0,1,1,100 Enter, etc. Click Cls, Show1 and Show2 to see the data put in, not shown here.

Member 2 with member load force 20 kN.

P	NFB(P)	I	F2	L2
2	1	1	20	5

Click Beam load forces NFB(P) and

type in TSTRING 2,1,1,20,5 Enter.

NB Data put in can be stored, STORE NR=3 GET.

Later, with NR=3 click GET to GET.

Click first Show1 and Show2 and after that Calculate!!

Click Cls, Show1 and Show2 to get the print on the left, after data of member 2 follow the member load data.

Click Calculate and Res(P,1)(P,2) to get the member end forces D5(,), D6(,) and member end moments MF(,).

Click Cls and Reactions and displ.

UV(1)= 0,00 /EI	UB(1)= 0,00 /EI
UR(1)= -1,91 /EI	
UV(2)= 0,37 /EI	UB(2)= -3,00 /EI
UR(2)= 8,35 /EI	
UV(3)= 0,63 /EI	UB(3)= -3,05 /EI
UR(3)= -10,67 /EI	
UV(4)= 0,00 /EI	UB(4)= 0,05 /EI
UR(4)= 1,79 /EI	
RB(1)= 0,00 kN	RV(1)= 7,50 kN
	RV(4)= 12,50 kN

$$\begin{aligned} D5(1,2) &= 1,83 \text{ kN} & D5(2,1) &= -1,83 \text{ kN} \\ D6(1,2) &= -7,44 \text{ kN} & D6(2,1) &= 7,44 \text{ kN} \\ MF(1,2) &= 2,53 \text{ kNm} & MF(2,1) &= 6,64 \text{ kNm} \end{aligned}$$

$$\begin{aligned} D5(2,3) &= 1,83 \text{ kN} & D5(3,2) &= -1,83 \text{ kN} \\ D6(2,3) &= -7,44 \text{ kN} & D6(3,2) &= -12,56 \text{ kN} \\ MF(2,3) &= -6,64 \text{ kNm} & MF(3,2) &= 7,08 \text{ kNm} \end{aligned}$$

$$\begin{aligned} D5(3,4) &= 1,83 \text{ kN} & D5(4,3) &= -1,83 \text{ kN} \\ D6(3,4) &= 12,56 \text{ kN} & D6(4,3) &= -12,56 \text{ kN} \\ MF(3,4) &= -7,08 \text{ kNm} & MF(4,3) &= -2,09 \text{ kNm} \end{aligned}$$

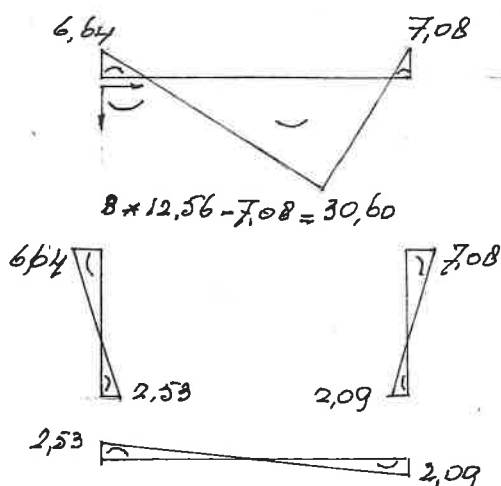
$$\begin{aligned} D5(1,4) &= -1,83 \text{ kN} & D5(4,1) &= 1,83 \text{ kN} \\ D6(1,4) &= -0,06 \text{ kN} & D6(4,1) &= 0,06 \text{ kN} \\ MF(1,4) &= -2,53 \text{ kNm} & MF(4,1) &= 2,09 \text{ kNm} \end{aligned}$$

Click Clc and click RES(L,H) (H,L) to get the member end forces and moments acting at the member ends.

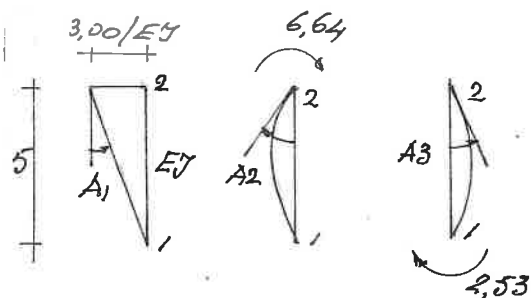
D5(L,H) and D5(H,L) assumed to the right $\rightarrow$,
D6(L,H) and D6(H,L) assumed downward $\downarrow$ and
MF(L,H) and MF(H,L) assumed to the right $\curvearrowright$.

member 1

D5(1,2) = 1,83 kN, positive answer, so as assumed directed to the right.
D5(2,1) = -1,83 kN, negative answer, so not as assumed to the right but directed to the left.
D6(1,2) = -7,44 kN, not downward but upward.
D6(2,1) = 7,44 kN, downward.
MF(1,2) = 2,53 kNm, to the right.
MF(2,1) = 6,64 kNm, to the right.



Calculation of the rotation of member end 2 of member 1, is joint rotation UR(2), using the results.



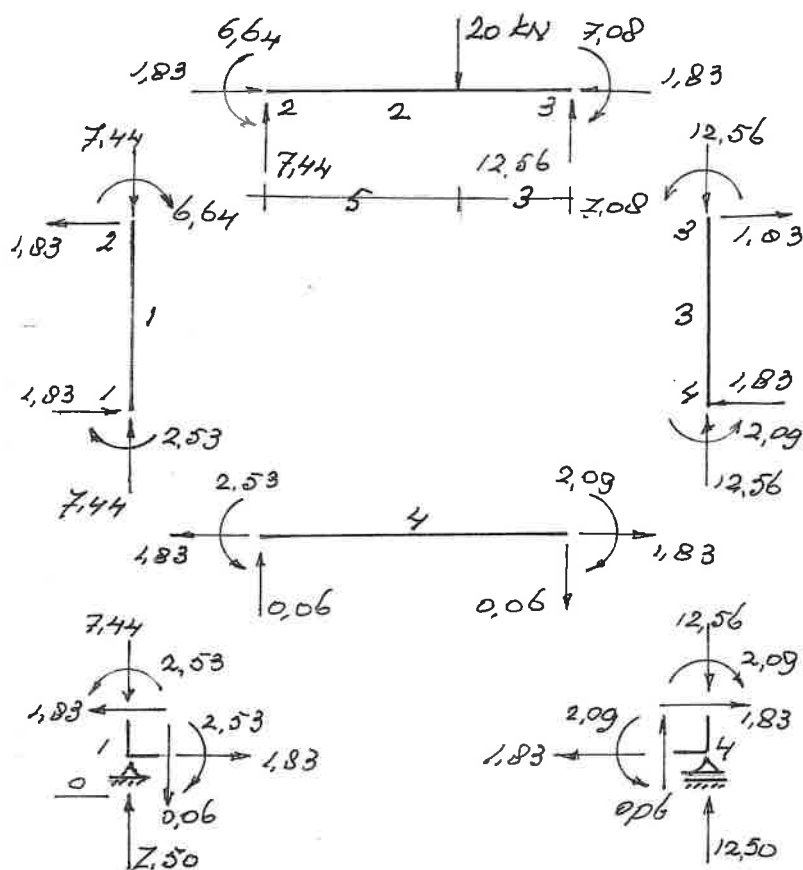
UH(2) = -3,00/EI, prec. page, joint 2 displaces to the left, angle A1 becomes A1 = (3,00/EI) / 5 = 0,60/EI.
With formula page 24 follow

$$A2 = (6,64 * 5) / 3EI = 11,07/EI \text{ and}$$

$$A3 = (2,53 * 5) / 6EI = 2,11/EI.$$

To the right minus to the left,
A2 - A1 - A3 = (11,07 - 0,60 - 2,11) / EI = 8,36/EI,
as assumed to the right.

UR(2) = 8,36/EI, positive answer, so as assumed to the right.

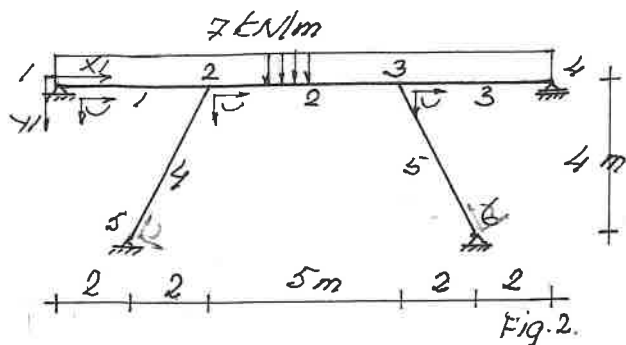


Member end forces and moments are drawn with their real directions. At the joints act forces and moments as large as but opposite directed, see joint/support 1 and 4. The reactions are added, RH(1) = 0 kN,
RV(1) = 7,50 kN, as assumed upward and
RV(4) = 12,50 kN, as assumed upward.
Equilibrium, yes.

member 2 Equilibrium?

$\sum$ mom. w.r.t. member end 2 = 0?
To the right - to the left = 0
20 * 5 + 7,08 - 6,64 - 12,56 * 8 =
100,00 + 0,44 - 100,48 = -0,04 ok

$\sum$ hor. = 0 1,83 - 1,83 = 0
 $\sum$ vert. = 0 7,44 + 12,56 - 20,00 = 0 ok



Example. EX2.

Fig.2.

N9= 6 joints and P9=5 members.

The construction is four fold statically indeterminate.

Type 6 in TN9 and Tab, and type in TSTRING

PV(1)=1 Enter, PH(1)=1 Enter, PV(4)=1 Enter,

PV(5)=1 Enter, PH(5)=1 Enter, and

PV(6)=1 Enter, PH(6)=1 Enter.

The joint coordinates are

I	X1(I)	Y1(I)	I	X1(I)	Y1(I)
1	0	0	4	13	0
2	4	0	5	2	4
3	9	0	6	11	4

Next click Coordinates and type in TSTRING

1,0,0 Enter, 2,4,0 Enter, 3,9,0 Enter and 4,13,0 Enter, 5,2,4 Enter, 6,11,4 Enter.

Click Show1 to check these data put in.

The member data are

P	LL	HH	NL	NH	BA	EI	EA
1	1	2	1	0	1	1	100
2	2	3	0	0	2	1	100
3	3	4	0	1	3	1	100
4	2	5	0	1	5	1	100
5	3	6	0	1	6	1	100

Then type in TP9 5 and Tab, cursor in TSTRING

and type in TSTRING, row after row,

1,1,2,1,0,1,1,100 Enter,

2,2,3,0,0,2,1,100 Enter, etc.

Next the distributed member loads for the members 1, 2 and 3.

P	NQB(P)	I	Q3	Q4	L3	L4
1	1	1	7	7	0	4
2	1	1	7	7	0	5
3	1	1	7	7	0	4

Member 1, 2 and 3.

Click label caption 'Beam load forces NQB(P)'

and type in TSTRING 1,1,1,7,7,0,4 Enter,

2,1,1,7,7,0,5 Enter, 3,1,1,7,7,0,4 Enter.

Click Calculate, Cls and 'Reactions and displ.' and see the results on the left and below.

N9= 6									
I	FX	PV	UV	SV	X1	Y1	PR	UR	SR
1	0	1	0	0	0,00	0	0	0	0
2	0	0	0	0	4,00	0	0	0	0
3	0	0	0	0	8,00	0	0	0	0
4	0	1	0	0	13,00	0	0	0	0
5	0	1	0	0	2,00	0	0	0	0
6	0	1	0	0	11,00	0	0	0	0
P5= 5									
I	FX	PH	UB	SB	Y1				
1	0	1	0	0	0,00				
2	0	0	0	0	0,00				
3	0	0	0	0	0,00				
4	0	0	0	0	0,00				
5	0	1	0	0	4,00				
6	0	1	0	0	4,00				
P									
1	1	2	1	0	1	1	100	4,00	
NQB(1)=1									
	I	Q3	Q4	L3	L4				
	1	7	7	0,00	4,00				
P									
2	2	3	0	0	2	1	100	5,00	
NQB(2)=1									
	I	Q3	Q4	L3	L4				
	1	7	7	0,00	5,00				
P									
3	3	4	0	1	3	1	100	4,00	
NQB(3)=1									
	I	Q3	Q4	L3	L4				
	1	7	7	0,00	4,00				
P									
4	2	5	0	1	5	1	100	4,47	
5	3	6	0	1	6	1	100	4,47	

DANA(L,H)(H,L) Show1 Show2 Cls	
Res(P,1)(P,2)	Res(L,H)(H,L) Reactions and displ.
N9=	OK
P9=	Beam load forces NFB(P) A0Again
Joint data	Beam load forces NQB(P) CSE=0
Coordinates	STORE NR=? GET Calculate
Ux and Uy	Beam load forces NFA(P) End
Again	Beam load forces NQA(P) PrF
EX1 EX2 EX3 EX4 EX5	(M5MAXZERO)

RB(1)=	-0,09 kN	RV(1)=	10,47 kN
RB(5)=	17,61 kN	RV(4)=	10,49 kN
RB(6)=	-17,53 kN	RV(5)=	35,07 kN
		RV(6)=	34,97 kN

UV(1)=	0,00 /EI	UB(1)=	0,00 /EI
UV(2)=	1,96 /EI	UB(2)=	0,00 /EI
UR(2)=	0,65 /EI		
UV(3)=	2,39 /EI	UB(3)=	-0,87 /EI
UR(3)=	-0,66 /EI		
UV(4)=	0,00 /EI	UB(4)=	-0,87 /EI
UV(5)=	0,00 /EI	UB(5)=	0,00 /EI
UV(6)=	0,00 /EI	UB(6)=	0,00 /EI
BE(1,2)=	9,74 /EI		
BE(4,3)=	-9,90 /EI		
BE(5,2)=	-0,03 /EI		
BE(6,3)=	-0,23 /EI		

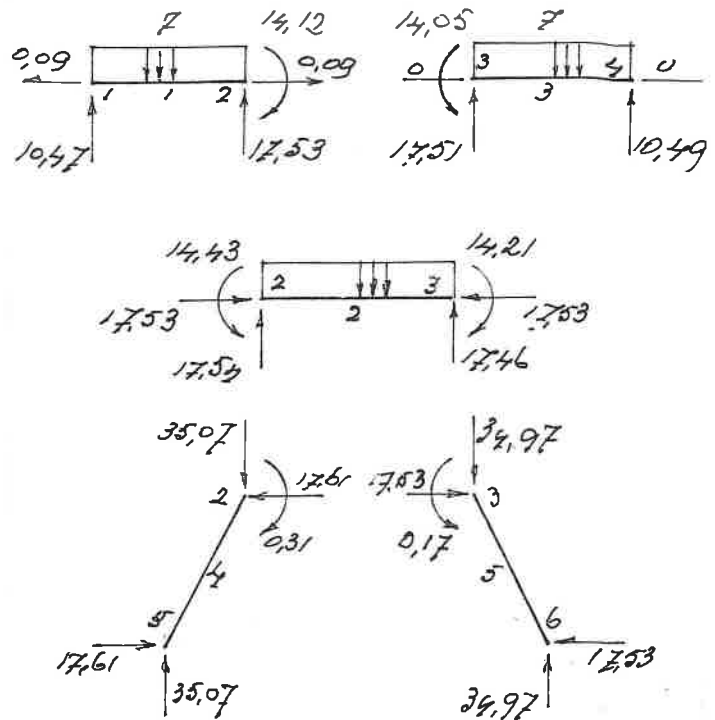
D5(1,2)= -0,09 kN	D5(2,1)= 0,09 kN
D6(1,2)= -10,47 kN	D6(2,1)= -17,53 kN
MF(1,2)= 0,00 kNm	MF(2,1)= 14,12 kNm
D5(2,3)= 17,53 kN	D5(3,2)= -17,53 kN
D6(2,3)= -17,54 kN	D6(3,2)= -17,46 kN
MF(2,3)= -14,43 kNm	MF(3,2)= 14,21 kNm
D5(3,4)= 0,00 kN	D5(4,3)= 0,00 kN
D6(3,4)= -17,51 kN	D6(4,3)= -10,49 kN
MF(3,4)= -14,05 kNm	MF(4,3)= 0,00 kNm
D5(2,5)= -17,61 kN	D5(5,2)= 17,61 kN
D6(2,5)= 35,07 kN	D6(5,2)= -35,07 kN
MF(2,5)= 0,31 kNm	MF(5,2)= 0,00 kNm
D5(3,6)= 17,53 kN	D5(6,3)= -17,53 kN
D6(3,6)= 34,97 kN	D6(6,3)= -34,97 kN
MF(3,6)= -0,17 kNm	MF(6,3)= 0,00 kNm

Making support 1 a roller support.
Click CSE=0 to CSE=1 and
type in TSTRING PH(1)=0 Enter.
Before calculation make all 'displacements' UH(I), UV(I) and UR(I) zero,
click Again next Calculate etc.
The values of the results show that
there is symmetry.

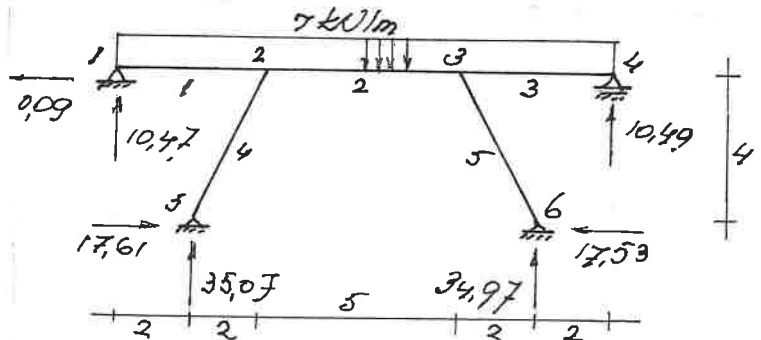
D5(1,2)= 0,00 kN	D5(2,1)= 0,00 kN
D6(1,2)= -10,43 kN	D6(2,1)= -17,52 kN
MF(1,2)= 0,00 kNm	MF(2,1)= 14,08 kNm
D5(2,3)= 17,57 kN	D5(3,2)= -17,57 kN
D6(2,3)= -17,50 kN	D6(3,2)= -17,50 kN
MF(2,3)= -14,32 kNm	MF(3,2)= 14,32 kNm
D5(3,4)= 0,00 kN	D5(4,3)= 0,00 kN
D6(3,4)= -17,52 kN	D6(4,3)= -10,48 kN
MF(3,4)= -14,08 kNm	MF(4,3)= 0,00 kNm
D5(2,5)= -17,57 kN	D5(5,2)= 17,57 kN
D6(2,5)= 35,02 kN	D6(5,2)= -35,02 kN
MF(2,5)= 0,24 kNm	MF(5,2)= 0,00 kNm
D5(3,6)= 17,57 kN	D5(6,3)= -17,57 kN
D6(3,6)= 35,02 kN	D6(6,3)= -35,02 kN
MF(3,6)= -0,24 kNm	MF(6,3)= 0,00 kNm
UV(1)= 0,00 /EI	UH(1)= 0,44 /EI
UV(2)= 2,18 /EI	UH(2)= 0,44 /EI
UR(2)= 0,66 /EI	
UV(3)= 2,18 /EI	UH(3)= -0,44 /EI
UR(3)= -0,66 /EI	
UV(4)= 0,00 /EI	UH(4)= -0,44 /EI
UV(5)= 0,00 /EI	UH(5)= 0,00 /EI
UV(6)= 0,00 /EI	UH(6)= 0,00 /EI

HE(1,2)= 8,82 /EI
HE(4,3)= -8,82 /EI
HE(5,2)= 0,13 /EI
HE(6,3)= -0,13 /EI

Click Cls and Res(L,H) (H,1) for the member end forces and moments indicated with the member end numbers L and H shown on the left.
Here below drawn at the member ends with their real directions.



No symmetric values because joint 1 is a hinge support and joint 4 a roller support. See also the reactions of the supports.

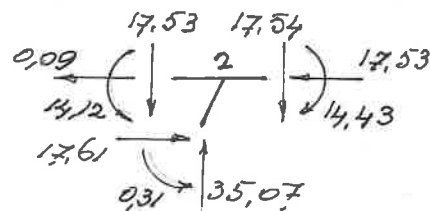


$$\sum \text{hor.} = 0 \quad 0,09 + 17,53 - 17,61 = 0,01 \quad \text{ok}$$

$$\sum \text{mom. w.r.t. joint/support } 5 = 0?$$

$$7 \cdot 13 \cdot 4,5 - 0,09 \cdot 4 - 34,97 \cdot 9 - 10,49 \cdot 11 + 10,47 \cdot 2 = 409,50 - 0,36 - 314,73 - 115,39 + 20,94 = 430,45 - 430,48 = -0,03 \quad \text{'is' } 0 \quad \text{ok}$$

Support 2



At joint 2 act member end forces and moments as large as but opposite directed
 $\sum \text{mom.} = 0, \quad \sum \text{hor.} = 0, \quad \sum \text{vert.} = 0, \quad \text{yes!}$

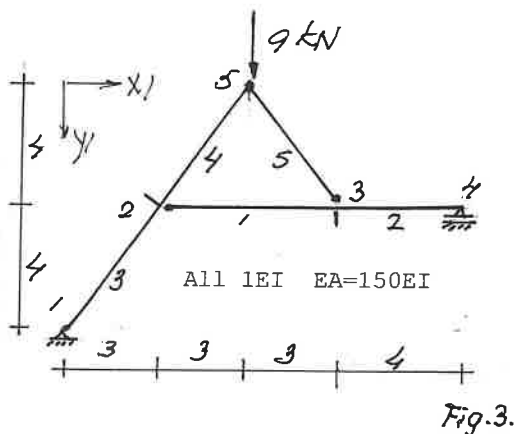


Fig.3.

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Private Sub LDANA_Click()
    FMT="0.00"
    For P=1 To P: L=LL(P): H=HH(P)
    Print " DA(" & L & "," & H & ")=";
    CX=2100
    T1=Format(DAA(P,1),FMT) & " kN "
    TW=TextWidth(T1)
    CurrentX=CX-TW : Print T1;
    CurrentX=2205
    Print " DA(" & H & "," & L & ")=";
    CX=4305
    T1=Format(DAA(P,2),FMT) & " kN "
    TW=TextWidth(T1)
    CurrentX=CX-TW : Print T1
    Print " NA(" & L & "," & H & ")=";
    CX=2100
    T1=Format(NAA(P,1),FMT) & " kN "
    TW=TextWidth(T1)
    CurrentX=CX-TW : Print T1;
    CurrentX=2205
    Print " NA(" & H & "," & L & ")=";
    CX=4305
    T1=Format(NAA(P,2),FMT) & " kN "
    TW=TextWidth(T1)
    CurrentX=CX-TW: Print T1
    Print " MF(" & L & "," & H & ")=";
    CX=2100
    T1=Format(MF(P,1),FMT) & " kN "
    TW=TextWidth(T1)
    CurrentX=CX-TW : Print T1;
    CurrentX=2205
    Print " MF(" & H & "," & L & ")=";
    CX=4305
    T1=Format(MF(P,2),FMT) & " kN "
    TW=TextWidth(T1)
    CurrentX=CX-TW: Print T1: Print
    Next P
End Sub
```

A click on DANA(L,H) (H,L) prints the results shown below.

DA(2,3)= -2.77 kN	DA(3,2)= 2.77 kN
NA(2,3)= -3.15 kN	NA(3,2)= 3.15 kN
MF(2,3)= 0.00 kNm	MF(3,2)= -14.52 kNm
DA(3,4)= 4.15 kN	DA(4,3)= -4.15 kN
NA(3,4)= 0.00 kN	NA(4,3)= 0.00 kN
MF(3,4)= 14.52 kNm	MF(4,3)= 0.00 kNm
DA(1,2)= -2.51 kN	DA(2,1)= 2.51 kN
NA(1,2)= 3.88 kN	NA(2,1)= -3.88 kN
MF(1,2)= 0.00 kNm	MF(2,1)= -14.54 kNm

Example. EX3

Fig.3.

The construction is statically determinate, externally and internally.

There are $N_9=5$ joints, 5 Tab, click on Joint data to set the data all zero. Then click in TSTRING to put in the support data, PV(1)=1 Enter, PH(1)=1 Enter, PV(4)=1 Enter.

The following joint coordinates must be put in, see preceding page, first click Coordinates.

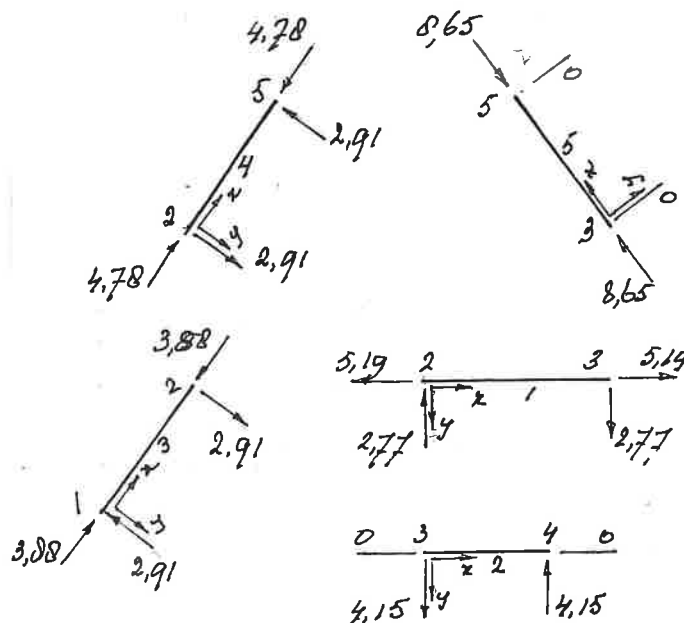
I	X1(I)	Y1(I)	I	X1(I)	Y1(I)
1	0	8	4	13	4
2	3	4	5	6	0
3	9	4			

The member data. In TP9 type 5 Tab, etc.

P9=5	P	LL	HH	NL	NH	BA	EI	EA
	1	2	3	1	0	2	1	150
	2	3	4	0	1	3	1	150
	3	1	2	1	0	1	1	150
	4	2	5	0	1	2	1	150
	5	3	5	1	1	3	1	150

There is one vertical joint load force of 9 kN. Click CSE=0 to CSE=1, then type in TSTRING FY(5)=-9 Enter, next Calculate etc.

With the results printed on the left the member end forces are drawn with their real directions.



E.g.

DA(2,5) is directed as assumed, y direction, DA(5,2) is not directed as assumed. not y dir., NA(2,5) is directed as assumed, x direction, NA(5,2) is not directed as assumed, not x dir.

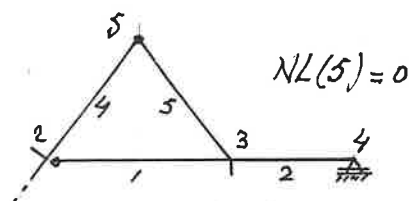
DA(2,5)= 2.51 kN	DA(5,2)= -2.51 kN
NA(2,5)= 4.78 kN	NA(5,2)= -4.78 kN
MF(2,5)= 14.54 kNm	MF(5,2)= 0.00 kNm
DA(3,5)= 0.00 kN	DA(5,3)= 0.00 kN
NA(3,5)= 3.88 kN	NA(5,3)= -3.88 kN
MF(3,5)= 0.00 kNm	MF(5,3)= 0.00 kNm

D5(2,3)= -5,19 kN	D5(3,2)= 5,19 kN
D6(2,3)= -2,77 kN	D6(3,2)= 2,77 kN
MF(2,3)= 0,00 kNm	MF(3,2)= -16,62 kNm
D5(3,4)= 0,00 kN	D5(4,3)= 0,00 kN
D6(3,4)= 4,15 kN	D6(4,3)= -4,15 kN
MF(3,4)= 16,62 kNm	MF(4,3)= 0,00 kNm
D5(1,2)= 0,00 kN	D5(2,1)= 0,00 kN
D6(1,2)= -4,85 kN	D6(2,1)= 4,85 kN
MF(1,2)= 0,00 kNm	MF(2,1)= -14,54 kNm
D5(2,5)= 5,19 kN	D5(5,2)= -5,19 kN
D6(2,5)= -2,08 kN	D6(5,2)= 2,08 kN
MF(2,5)= 14,54 kNm	MF(5,2)= 0,00 kNm
D5(3,5)= -5,19 kN	D5(5,3)= 5,19 kN
D6(3,5)= -6,92 kN	D6(5,3)= 6,92 kN
MF(3,5)= 0,00 kNm	MF(5,3)= 0,00 kNm

UV(1)= 0,00 /EI	UB(1)= 0,00 /EI
UV(2)= 163,22 /EI	UB(2)= 217,40 /EI
UR(2)= 30,14 /EI	
UV(3)= 158,21 /EI	UB(3)= 217,61 /EI
UR(3)= -27,40 /EI	
UV(4)= 0,00 /EI	UB(4)= 217,61 /EI
UV(5)= 181,07 /EI	UB(5)= 240,53 /EI
UR(5)= 5,79 /EI	

BE(2,3)= 22,45 /EI
BE(4,3)= -60,63 /EI
BE(1,2)= 66,49 /EI
BE(5,2)= -6,21 /EI
BE(3,5)= 5,79 /EI

RE(1)= 0,00 kN	RV(1)= 4,85 kN
	RV(4)= 4,15 kN

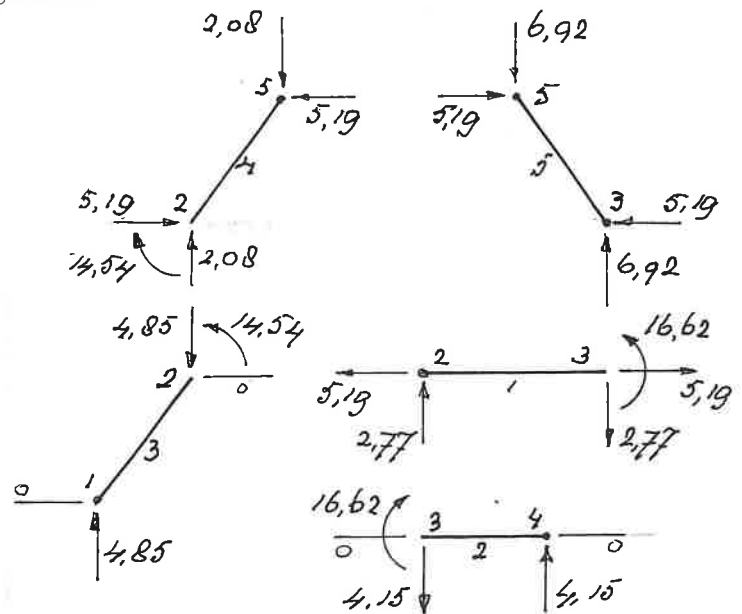


Click CSE=0 to CSE=1 and type in TSTRING NL(5)=0 Enter.
Click Again, Calculate etc.

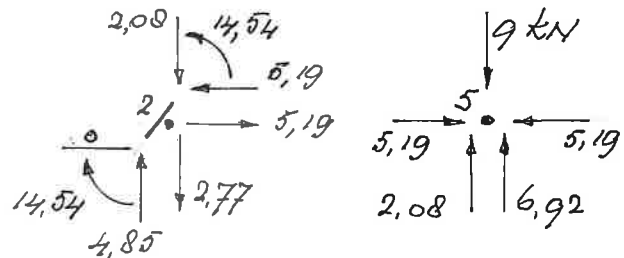
UV(1)= 0,00 /EI	UH(1)= 0,00 /EI
UV(2)= 146,51 /EI	UH(2)= 195,13 /EI
UR(2)= 24,57 /EI	
UV(3)= 148,11 /EI	UH(3)= 195,39 /EI
UR(3)= -14,87 /EI	
UV(4)= 0,00 /EI	UH(4)= 195,39 /EI
UV(5)= 147,71 /EI	UH(5)= 196,36 /EI

HE(2,3)= 7,84 /EI
HE(4,3)= -48,10 /EI
HE(1,2)= 60,92 /EI
HE(5,2)= -11,77 /EI
HE(5,3)= 7,74 /EI

And with another click the results are printed as shown on the left. The member end forces and moments indicated with their member end numbers. Here below they are drawn with their real directions. The members are separated from the joints.



Joints 2 and 5 are separated from the members. On the joints act forces and moments as large as but opposite directed.

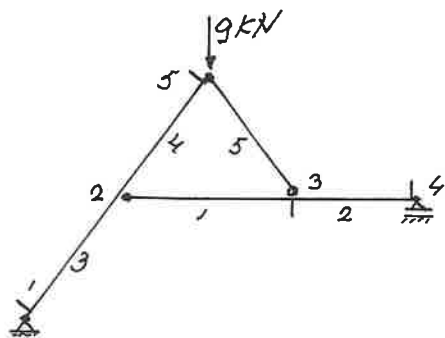


At member end 2 of member 4 acts 2,08 kN upward thus downward on the joint.

At member end 2 of member 3 acts 4,85 kN downward, then on the joint upward. $\sum \text{vert.} = 0$. Etc. Considering joint 5, it depends on the form of the construction how the load of 9 kN is divided. Suppose member 5 is at member end 3 rigidly connected, then type NL(5)=0 Enter. The results are printed on the left and below.

D5(2,5)= 6,32 kN	D5(5,2)= -6,32 kN
D6(2,5)= -3,58 kN	D6(5,2)= 3,58 kN
MF(2,5)= 14,54 kNm	MF(5,2)= 0,00 kNm
D5(3,5)= -6,32 kN	D5(5,3)= 6,32 kN
D6(3,5)= -5,42 kN	D6(5,3)= 5,42 kN
MF(3,5)= -9,05 kNm	MF(5,3)= 0,00 kNm

Now there is a bending moment MF(3,5) and the horizontal and vertical member end forces at member ends 5 are different from those above. The deformation of the construction differs also, see the results on the left. Now UR(3)= -14,87/EI instead of -27,40/EI, and slope HE(3,5) has disappeared. With NL(1)=0 slope HE(2,3) will disappear because then the hinge has disappeared.



IS= 5									
P	EL	EB	NL	NB	EA	EI	EA	LI	
1	2	3	1	0	2	1	150	6,00	
2	3	4	0	0	3	1	150	4,00	
3	1	2	0	0	1	1	150	5,00	
4	2	5	0	0	2	1	150	5,00	
5	3	5	1	1	3	1	150	5,00	

UV(1)=	0,00 /EI	UH(1)=	0,00 /EI
UR(1)=	66,49 /EI		
UV(2)=	163,22 /EI	UH(2)=	217,40 /EI
UR(2)=	30,14 /EI		
UV(3)=	198,21 /EI	UH(3)=	217,61 /EI
UR(3)=	-27,40 /EI		
UV(4)=	0,00 /EI	UH(4)=	217,61 /EI
UR(4)=	-60,63 /EI		
UV(5)=	181,07 /EI	UH(5)=	240,95 /EI
UR(5)=	-6,21 /EI		

HE(2,3)=	22,45 /EI
HE(3,5)=	5,79 /EI
HE(5,3)=	5,79 /EI

RE(1)=	0,00 kN	RV(1)=	4,85 kN
		RV(4)=	4,15 kN

D5(2,5)=	6,09 kN	D5(5,2)=	-6,09 kN
D6(2,5)=	-2,08 kN	D6(5,2)=	2,08 kN
MF(2,5)=	14,54 kNm	MF(5,2)=	3,59 kNm

D5(3,5)=	-6,09 kN	D5(5,3)=	6,09 kN
D6(3,5)=	-6,92 kN	D6(5,3)=	6,92 kN
MF(3,5)=	0,00 kNm	MF(5,3)=	-3,59 kNm

UV(1)=	0,00 /EI	UH(1)=	0,00 /EI
UR(1)=	156,32 /EI	UH(2)=	208,21 /EI
UV(2)=	27,84 /EI		
UV(3)=	195,45 /EI	UH(3)=	208,45 /EI
UR(3)=	-26,71 /EI		
UV(4)=	0,00 /EI	UH(4)=	208,45 /EI
UR(4)=	176,28 /EI	UH(5)=	234,53 /EI
UR(5)=	0,48 /EI		

HE(2,3)=	23,14 /EI
HE(4,3)=	-59,94 /EI
HE(1,2)=	64,19 /EI
HE(3,5)=	9,47 /EI

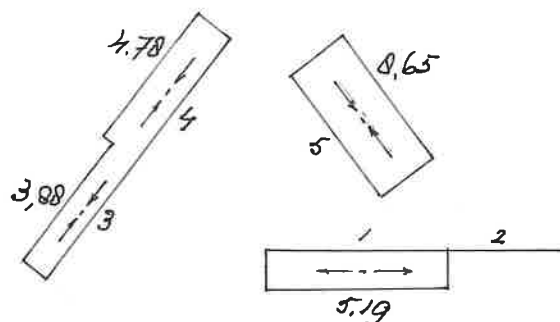
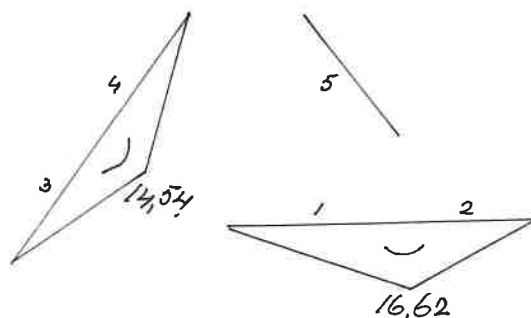
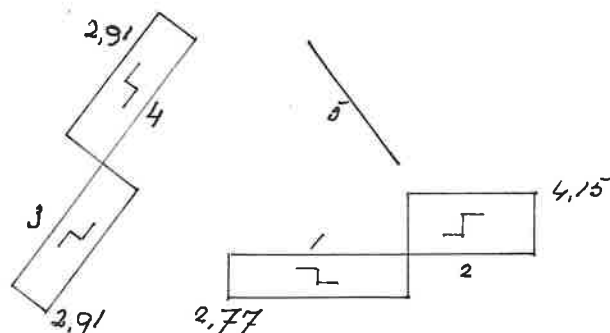
Changing some member end indications NL(P) and NH(P). With CSE=1 in TSTRING, NL(5)=1 Enter, NH(5)=1 Enter, NL(3)=0 Enter, NH(2)=0 Enter, NH(4)=0 Enter.

So real joints at member ends 3, 4 and 2.

Next Again, Calculate. See the displacements printed on the left to compare with the first results. Now joint rotations,

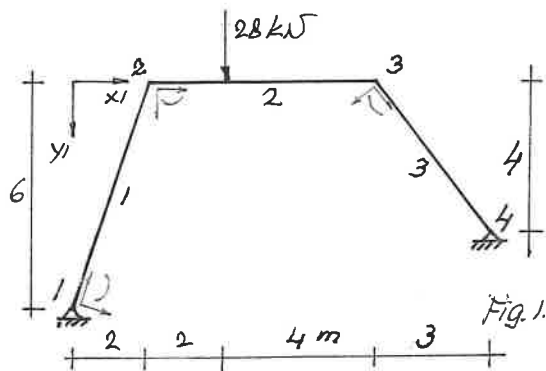
UR(1)= 66,49/EI instead of HE(1,2)= 66,49/EI,
UR(4)= -60,63/EI instead of HE(4,3)= -60,63/EI,
UR(5)= -6,21/EI instead of HE(5,2)= -6,21/EI.

Below the shear force, bending moment and normal force diagrams. For sloping beams shear force and normal force diagrams are faster to draw with the member end forces printed w.r.t. the member axis system x-y.



Some changes, with data from the beginning, click AOagain, EX3, and CSE=0 to CSE=1, next NH(4)=0 Enter and NH(5)=0 Enter.

Hinge 5 has disappeared, both member ends seen as a real joint. One finds UR(5) instead of separately calculated slopes HE(5,2) and HE(5,3).



Example. EX4

Fig.1.

There are $N9=4$ joints, input 9 in text box TN9, a click on Joint data, 8 joint data are set zero. With four joints $8 \times 4=40$ data do not need to be put in, except a few alterations. Then click in TSTRING to put in the support data, $PV(1)=1$ Enter, $PH(1)=1$ Enter, and $PV(4)=1$ Enter, $PH(4)=1$ Enter.

Click on Coordinates, the cursor appears in TSTRING, then type each time a row of I, $X1(I)$, $Y1(I)$, $1,0,6$ Enter, $2,2,0$ Enter, $3,8,0$ Enter and $4,11,4$ Enter.

The member data.

P9=3 Type 3 in TP9, Tab etc.

P	LL	HH	NL	NH	BA	EI	EA
1	1	2	1	0	1	1	1000
2	2	3	0	0	2	1	1000
3	3	4	0	1	3	1	10000

Then a click on Beam load forces NFB(P) and then type in TSTRING

P NFB(P) I F2 L2
2 1 1 28 2

A click on Calculate and then on one or more, eventually clearing the form with a click on Cls. A click on DANA gives the results on the left. Below member end forces are drawn with their real directions

P9= 3							
P	LL	HH	NL	NH	BA	EI	EA
1	1	2	1	0	1	1	1000
2	2	3	0	0	2	1	1000
NFB(2)=1 I F2 L2							
1 28 2,00							
P	LL	HH	NL	NH	BA	EI	EA
3	3	4	0	1	3	1	1000

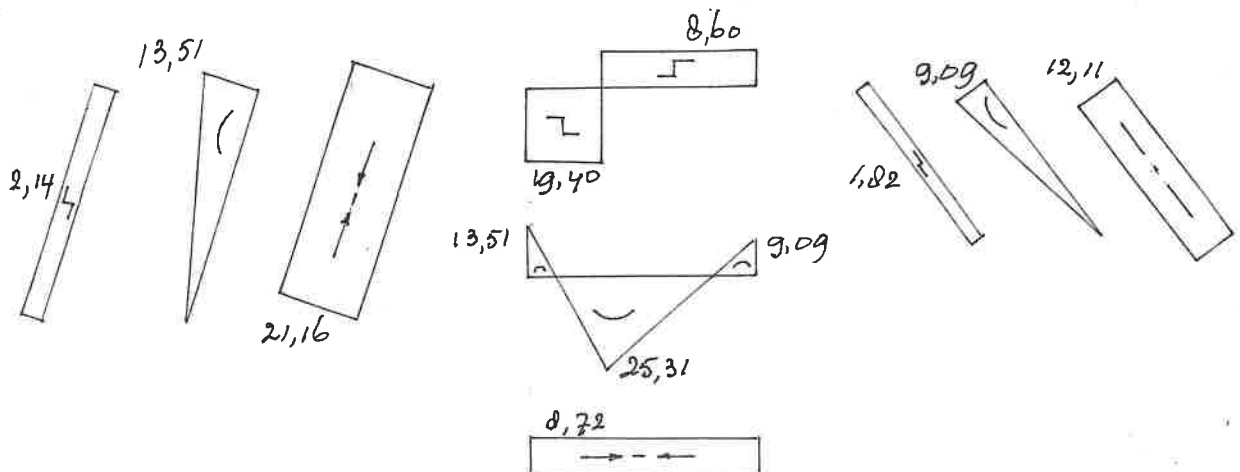
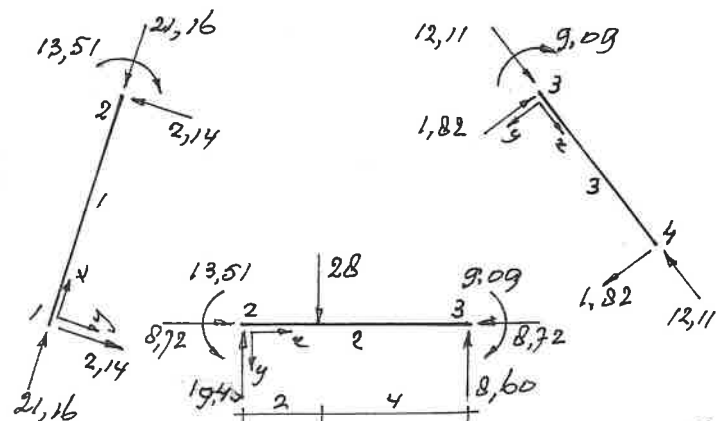
DA(1,2)= 2,14 kN DA(2,1)= -2,14 kN
NA(1,2)= 21,16 kN NA(2,1)= -21,16 kN
MF(1,2)= 0,00 kNm MF(2,1)= 13,51 kNm

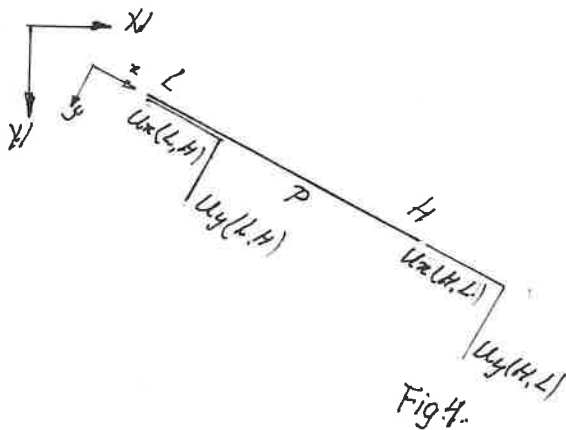
DA(2,3)= -19,40 kN DA(3,2)= -8,60 kN
NA(2,3)= 8,72 kN NA(3,2)= -8,72 kN
MF(2,3)= -13,51 kNm MF(3,2)= 9,09 kNm

DA(3,4)= -1,82 kN DA(4,3)= 1,82 kN
NA(3,4)= 12,11 kN NA(4,3)= -12,11 kN
MF(3,4)= -9,09 kNm MF(4,3)= 0,00 kNm

UV(1)= 0,00 /EI UH(1)= 0,00 /EI
UV(2)= -2,13 /EI UH(2)= -6,82 /EI
UR(2)= 27,35 /EI UH(3)= -6,87 /EI
UV(3)= 5,23 /EI UH(4)= 0,00 /EI
UR(3)= -16,87 /EI UH(4)= 0,00 /EI
UV(4)= 0,00 /EI UH(4)= 0,00 /EI

HE(1,2)= -15,37 /EI
HE(4,3)= 5,85 /EI



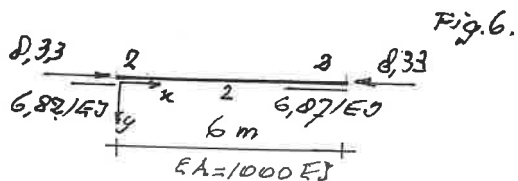
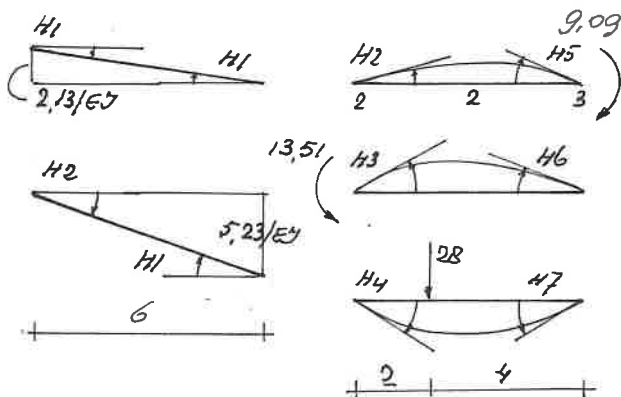
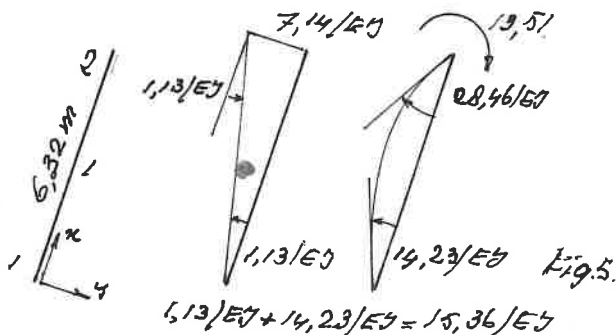


$$\begin{aligned} RH(1) &= 8,72 \text{ kN} & RV(1) &= 19,40 \text{ kNm} \\ RH(4) &= -8,72 \text{ kN} & RV(4) &= 8,60 \text{ kNm} \end{aligned}$$

$$\begin{aligned} Ux(1,2) &= 0,00 /EI & Ux(2,1) &= -0,13 /EI \\ Uy(1,2) &= 0,00 /EI & Uy(2,1) &= -7,14 /EI \end{aligned}$$

$$\begin{aligned} Ux(2,3) &= -6,82 /EI & Ux(3,2) &= -6,87 /EI \\ Uy(2,3) &= -2,13 /EI & Uy(3,2) &= 5,23 /EI \end{aligned}$$

$$\begin{aligned} Ux(3,4) &= 0,06 /EI & Ux(4,3) &= 0,00 /EI \\ Uy(3,4) &= 8,63 /EI & Uy(4,3) &= 0,00 /EI \end{aligned}$$



$$(8,72) * (6) / (1000EI) = 0,0523 /EI$$

Fig. 4.

The member end displacements according to the member axis system x-y. These are the assumed directions/distances derived on page .

Fig. 5.

Member 1 with $Ux(1,2)=0$ and $Uy(1,2)=0$, and $Ux(2,1)=-0,13/EI$ not as assumed directed as the x axis but opposite, and $Uy(2,1)=-7,14/EI$ not as assumed directed as the y axis but opposite, both drawn with their real directions (sketchy not same scale).

Now member end rotations can be calculated due to the displacement $Uy(2,1)$ and due to the member end moment of 13,51 kNm.

Member end 1.

Due to $7,14/EI$ alone a slope to the left of $(7,14/EI)/6,32=1,13/EI$ and due to 13,51 kNm (see formula page 2.4) $(13,51*6,32)/(6*EI)=14,23$ to the left, so

$1,13/EI + 14,23/EI = 15,36/EI$ to the left, and that was found on the preceding page, $HE(1,2)=-15,37/EI$.

Member end 2.

Also at this member end now $1,13/EI$ to the left and due to 13,51 kNm $(13,51*6,32)/(3*EI)=28,46/EI$ to the right, then follows $28,46/EI - 1,13/EI = 27,33/EI$ as was found, joint rotation $UR(2)=27,35/EI$, correct.

Fig. 6.

Member 2, on the left the slopes at the member ends due to the displacements, on the right due to member end moments and member load.

Member end 2.

Assume a slope to the right, then is written $H1+H2-H2-H3+H4=$ $(2,13/EI)/6 + (5,23/EI)/6 - (9,09*6)/(6*EI)$

$$-(13,51*6)/(3*EI) + ((28*2*48(6+4))/(6*6*EI))=$$

$$(0,36 + 0,87 - 9,09 - 27,02 + 62,22)/EI = 27,34/EI$$

as found, $UR(2)=27,35/EI$ to the right.

For member end 3 follows $H1+H2+H5+H6-H7,$

$$(0,36 + 0,87) + (9,09*6)/(3*EI) =$$

$$+(13,51*6)/(6*EI) - (28*2*4*(6+2))/(6*6*EI) =$$

$$(0,36 + 0,87 + 18,18 + 13,51 - 49,78)/EI = -16,86/EI$$

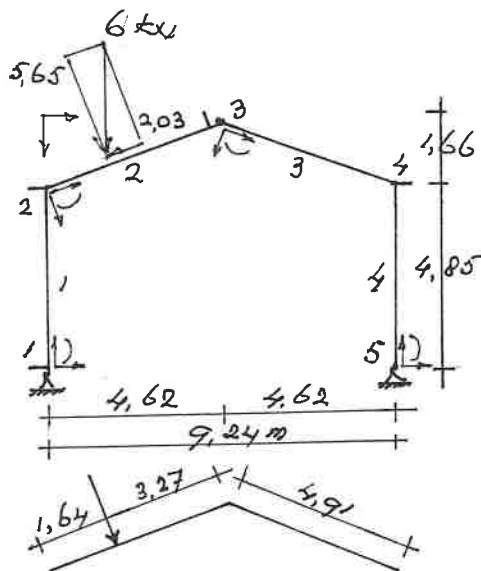
as was found, $UR(3)=-16,87/EI$.

Fig. 7.

$Ux(2,3)=-6,82/EI$, member end 2 of member 3 displaces $6,82/EI$ to the left, $Ux(3,2)=-6,87/EI$, member end 3 displaces to the left, the beam shortens $0,05/EI$. With Hooke's law is

$$F=(EA/6)*(0,05/EI)=(1000*EI/6)*(0,05/EI)$$

= $8,33$ kN (rounded value 0,05). Preceding page with $NA(2,3)=8,72$ and $NA(3,2)=-8,72$ kN.



$D5(1,2) = 0,71 \text{ kN}$ $D5(2,1) = -0,71 \text{ kN}$
 $D6(1,2) = -5,00 \text{ kN}$ $D6(2,1) = 5,00 \text{ kN}$
 $MF(1,2) = 0,00 \text{ kNm}$ $MF(2,1) = 3,45 \text{ kNm}$

$D5(2,3) = 0,71 \text{ kN}$ $D5(3,2) = -0,71 \text{ kN}$
 $D6(2,3) = -5,00 \text{ kN}$ $D6(3,2) = -1,00 \text{ kN}$
 $MF(2,3) = -3,45 \text{ kNm}$ $MF(3,2) = 0,00 \text{ kNm}$

$D5(3,4) = 0,71 \text{ kN}$ $D5(4,3) = -0,71 \text{ kN}$
 $D6(3,4) = 1,00 \text{ kN}$ $D6(4,3) = -1,00 \text{ kN}$
 $MF(3,4) = 0,00 \text{ kNm}$ $MF(4,3) = 3,45 \text{ kNm}$

$D5(4,5) = 0,71 \text{ kN}$ $D5(5,4) = -0,71 \text{ kN}$
 $D6(4,5) = 1,00 \text{ kN}$ $D6(5,4) = -1,00 \text{ kN}$
 $MF(4,5) = -3,45 \text{ kNm}$ $MF(5,4) = 0,00 \text{ kNm}$

$UV(1) = 0,00 /EI$ $UH(1) = 0,00 /EI$
 $UR(1) = -0,38 /EI$
 $UV(2) = 0,08 /EI$ $UH(2) = 11,69 /EI$
 $UR(2) = 7,99 /EI$
 $UV(3) = 24,24 /EI$ $UH(3) = 20,35 /EI$
 $UR(3) = 1,32 /EI$
 $UV(4) = 0,02 /EI$ $UH(4) = 29,04 /EI$
 $UR(4) = 0,41 /EI$
 $UV(5) = 0,00 /EI$ $UH(5) = 0,00 /EI$

$HE(3,4) = -8,07 /EI$
 $HE(5,4) = 8,78 /EI$

$RH(1) = 0,71 \text{ kN}$ $RV(1) = 5,00 \text{ kN}$
 $RH(5) = -0,71 \text{ kN}$ $RV(5) = 1,00 \text{ kN}$

Example. EX5a

Fig.1.

N9=5 joints and P9=4 beams/members. Cursor in TN9, type 5, Tab, click Joint data to set most of necessary joint data, see page . Next the data for the supports, PH(I) and PV(I) are set all zero. Now click in TSTRING and type PH(1)=1 Enter, PV(1)=1 Enter, PH(5)=1 Enter and PH(5)=1 Enter.

Next click Coordinates and type in TSTRING each row of three joint data followed by Enter.

I	X1(I)	Y1(I)	I	X1(I)	Y1(I)
1	0	6.51	4	9.24	1.66
2	0	1.66	5	9.24	6.51
3	4.62	0			

Click in TP9 and type 4, Tab (or click in TSTRING) and type each row of the following beam data followed by Enter.

With the little stripes are the 'real' joints indicated, then NL(P)=0 or NH(P)=0.

P9=4

P	LL	HH	NL	NH	BA	EI	EA
1	1	2	0	0	1	1	300
2	2	3	0	0	2	1	300
3	3	4	1	0	3	1	300
4	4	5	0	1	5	1	300

Fig.2.

Beam P=2 is loaded with a vertical load force of 6 kN. Until now this force must be resolved into forces perpendicular to and along the beam. Using similarity of triangles follow

$$(6/4,91) \cdot 4,62 = 5,65 \text{ kN}, \quad (6/4,91) \cdot 1,66 = 2,03 \text{ kN}.$$

Click on Beam load forces NFB(P) and type in TSTRING

P	NFB(P)	I	F22(P,I)	L22(P,I)
2	1	1	5.65	1.64

Enter

A load force F55(P,I) is assumed to be directed from lowest to highest beam end number at distance L55(P,I) from the lowest beam end number. Here 2,03 kN is directed in opposite direction therefore the minus sign -2,03.

Now click on Beam load forces NFA(P) and type in TSTRING needed row of data followed by Enter.

P	NFA(P)	I	F55(P,I)	L55(P,I)
2	1	1	-2,03	1.64

Enter

Click Again (to be sure unknowns UV(I), UH(I) and UR(I) are zero), Calculate to calculate the unknowns, and with them beam end forces, beam end moments reactions.

Next page, with Part14 page 5,

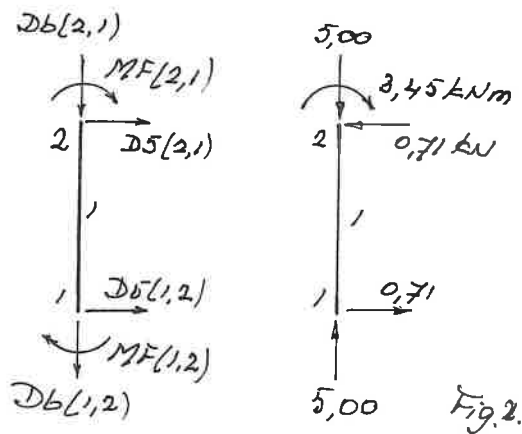
S5(1,1,1) Enter

S5(1,1,1)= 0,10518 55374 26877

S5(1,1,3)= 0,25507 49282 60176

S5(1,3,3)= 0,82474 22680 41237

S5(1,3,6)= 0,41237 11340 20619

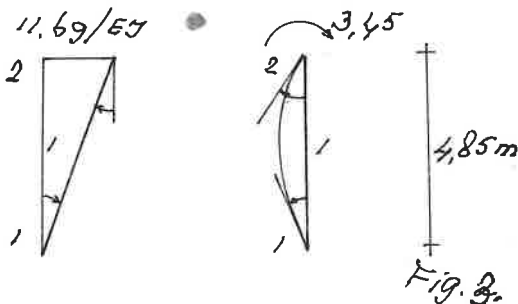


CH=4,85 m RH=1,66 m HW=9.24 m page

NL(1)=0 NH(4)=1 NH(2)=0 NL(3)=1

1	11	0	26	-11	0	26
2	0	6186	0	0	-6186	0
3	26	0	82	-26	0	41
4	-11	0	-26	11	0	-26
5	0	-6186	0	0	6186	0
6	26	0	41	-26	0	82

S5 of beam 1 EA= 300 *EI
to multiply by EI/100
See Part 14 page 9, there
to multiply by EI/1000,
there 105, here rounded 11, etc.



The joint rotations UR(1) and UR(2), both indicated with the little stripe.

The displacement of beam end 2 is UH(2)= 11,69/EI, that's to the right. At both ends arise a slope to the right (11,69/EI)/4,85= 2,41/EI.

Due to beam end moment at beam end 2 of 3,45 kNm arises at end 2 a slope to the right, 3,45*4,85/(3*EI)= 5,58/EI,

and at beam end 1 to the left 3,45*4,85/(6*EI)= 2,79/EI.

With assumption to the right is UR(1)= (2,41-2,79)/EI=-0,38/EI and UR(2)= (2,41+5,58)/EI= 7,99/EI.

Some check calculations

Beam 1.

Fig.1.

On the left beam end forces and beam end moments are drawn with their assumed directions, on the right with their real directions according to the calculated values. A minus sign, then opposite to the assumed direction. The relation between 'forces' and 'displacements' here below.

$$\begin{bmatrix} D5(1,2) \\ D6(1,2) \\ MF(1,2) \\ D5(2,1) \\ D6(2,1) \\ MF(2,1) \end{bmatrix} = S5 \begin{bmatrix} UH(1) \\ UV(1) \\ UR(1) \\ UH(2) \\ UV(2) \\ UR(2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -0,38 \\ 11,69 \\ 0,08 \\ 7,99 \end{bmatrix} \text{ or } \begin{bmatrix} 0 \\ 0 \\ -0,379 \\ 11,692 \\ 0,081 \\ 7,990 \end{bmatrix} / EI$$

On the left the beam stiffness matrix S5 found on page 9 of Part14.

A 'force' = row of S5 times column u. Zero multiplications are omitted.

$$D5(1,2) = 26(-0,38) - 11(11,69) + 26(7,99) = -9,88 - 128,59 + 207,74 = 69,27 \text{ times } (EI/100) (/EI) = 0,69 \text{ kN}$$

$$D6(1,2) = -6186(0,08) = -494,88 \text{ times } (EI/100) (/EI) = -4,95 \text{ kN}$$

$$MF(1,2) = 82(-0,38) - 26(11,69) + 41(7,99) = -31,16 - 303,94 + 327,59 = 7,51 \text{ times } (EI/100) (/EI) = 0,08 \text{ kNm}$$

With one more decimal for the displacements follow

$$D5(1,2) = 26(-0,379) - 11(11,692) + 26(7,990) = -9,854 - 128,612 + 207,740 = 69,274 \text{ * } (EI/100) (/EI) = 0,69 \text{ kN}$$

$$D6(1,2) = -6186(0,081) = -501,066 \text{ is } 5,01 \text{ kN}$$

$$MF(1,2) = 82(-0,379) - 26(11,692) + 41(7,990) = -31,078 - 303,992 + 327,590 = -7,480 \text{ is } 0,08 \text{ kNm}$$

See Part14 page 5 to get elements of S5 with decimals, with S5(1,1,4) Enter.

Two decimals? 11 is 10,52 82 is 82,47 26 is 25,51 41 is 41,24 Then follows

$$D5(1,2) = 25,51(-0,379) - 10,51(11,692) + 25,51(7,990) = -9,668 - 122,883 + 203,825 = 71,274 \text{ is } 0,71 \text{ kN}$$

$$D6(1,2)? \text{ a smaller value than } 0,081? \text{ Well, it is } 0,08085 \text{ and then is } D6(1,2) = -6186(0,08085) = -500,138 \text{ is } 5,00 \text{ kN}$$

$$MF(1,2) = 82,47(-0,379) - 25,51(11,692) + 41,24(7,990) = -31,256 - 298,262 + 329,508 = -0,010 \text{ is } 0,00! \text{ kNm}$$

Beam end forces and moments are calculated with numbers with a lot of decimals. The results with rounded values are sufficient accurate...

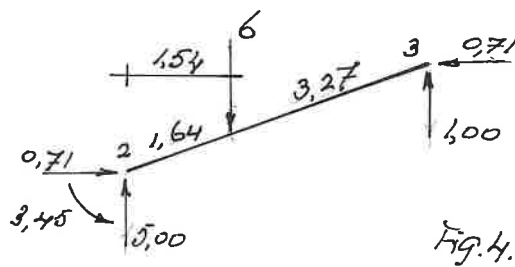


Fig. 4.

	1	2	3	4	5	6
1	5413	-1941	8	-5413	1941	8
2	-1941	708	23	1941	-708	23
3	8	23	81	-8	-23	41
4	-5413	1941	-8	5413	-1941	-8
5	1941	-708	-23	-1941	708	-23
6	8	23	41	-8	-23	81

S5 of beam 2 EA=300 *EI
to multiply by EI/100

* Part14 page 9

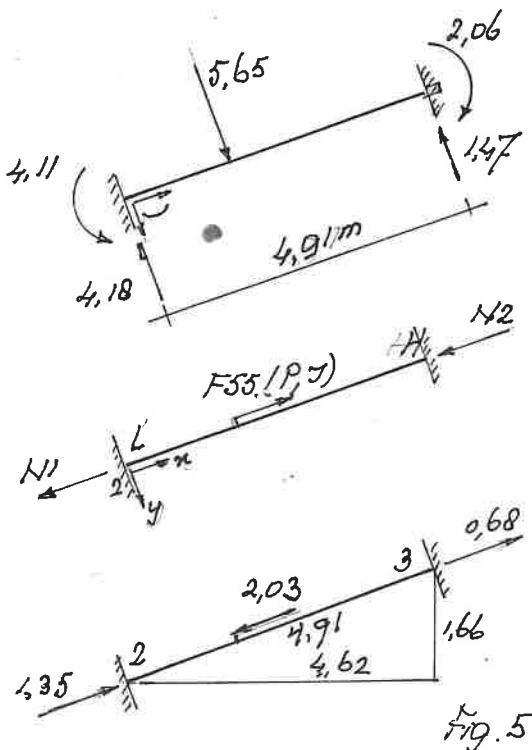


Fig. 5.

M1(2)=	4,11 kNm	M2(2)=	2,06 kNm
R1(2)=	4,18 kN	R2(2)=	1,47 kN
N1(2)=	-1,35 kN	N2(2)=	-0,68 kN

Primary forces and moments.

('The computer 'knows' best'...)

Beam 1.

Fig. 4.

Beam end forces and beam end moments are drawn with their real directions.

The relation between 'forces' and 'displacements' here below. The values of u are taken from page 11.

D5(2,3)	UH(2)	11,69	11,692
D6(2,3)	UV(2)	0,08	0,081
MF(2,3) = S5	UR(2)	7,99	7,990
D5(3,2)	UH(3)	20,35	20,354
D6(3,2)	UV(3)	24,24	24,237
MF(3,2)	UR(3)	1,32	1,316

f = S5 u u /EI

On the left the beam stiffness matrix S5 found on page *. For use the elements are divided by 100. Zero multiplications are omitted. For D5(2,3) is found 0,71 kN, to check.

D5(2,3) = 54,13(11,69) - 19,41(0,08) + 0,08(7,99) - 54,13(20,35) + 19,41(24,24) + 0,08(1,32) = 632,78 - 1,55 + 0,64 - 1105,55 + 470,50 + 0,11 = 0,93 kN to the right, due to displacements alone. The influence of the primary forces at beam end 2 must be added.

Fig. 5.

The horizontal components are (4,18/4,91)*1,66 = 1,41 kN to the left, and (1,35/4,91)*4,62 = 1,27 kN to the right. Then to the right minus to the left is 0,93 + 1,27 - 1,41 = 0,79 <> 0,71 kN

Another try with u with three decimals.

D5(2,3) = 54,13(11,692) - 19,41(0,081) + 0,08(7,990) - 54,13(20,354) + 19,41(24,237) + 0,08(1,316) = 632,89 - 1,57 + 0,64 - 1101,76 + 470,44 + 0,11 = 0,75 kN 0,75 + 1,27 - 1,41 = 0,61 <> 0,71 kN

A try with two more decimals for the elements of S5 which can be found.

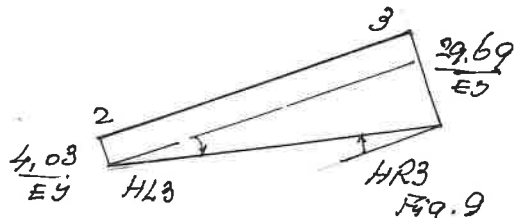
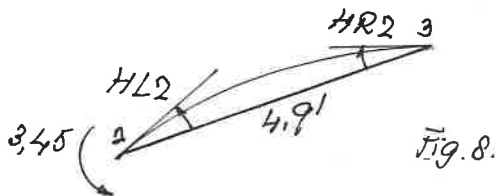
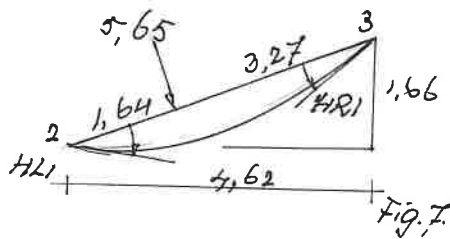
S55(2,1,1) = 54,1343206745647 is 54,1343
S55(2,1,2) = -19,4144161655519 is -19,4144
S55(2,1,3) = 0,08418481746388 is 0,0841
D5(2,3) = 54,1343(11,692) - 19,4144(0,081) + 0,0841(7,990) - 54,1343(20,354) + 19,4144(24,237) + 0,0841(1,316) = 632,94 - 1,57 + 0,67 - 1101,85 + 470,55 + 0,11 = 0,85 kN 0,85 + 1,27 - 1,41 = 0,71 = 0,71 kN!

And D6(2,3) with 'short' values, first try?

D6(2,3) = -19,41(11,69) + 7,08(0,08) + 0,23(7,99) - 19,41(20,35) - 7,08(24,24) + 0,23(1,32) - 226,90 + 0,57 + 1,84 + 394,99 - 171,62 + 0,30 = -0,82 kN, not downward but upward.

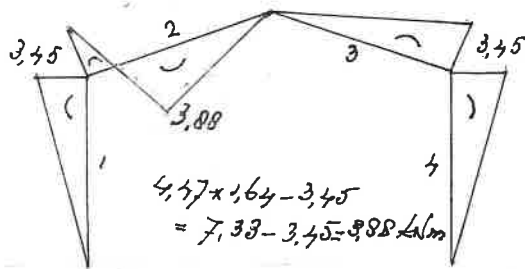
The vertical components of 4,18 and 1,35 are (4,18/4,91)*4,62 = 3,93 kN upward, and (1,35/4,91)*1,66 = 0,46 kN upward. 0,82 + 0,46 + 3,93 = 5,21 <> 5,00 kN

With the third way of calculation one will find 0,61 + 0,46 + 3,93 = 5,00 kN, wonderful.



$$\begin{aligned} U_x(2,3) &= 10,98 / EI & U_x(3,2) &= 10,96 / EI \\ U_y(2,3) &= 4,03 / EI & U_y(3,2) &= 29,69 / EI \end{aligned}$$

NB Excuse, SL1 is HL1, SR1 is HR1! Etc.



$$\begin{aligned} DA(1,2) &= 0,71 \text{ kN} & DA(2,1) &= -0,71 \text{ kN} \\ NA(1,2) &= 5,00 \text{ kN} & NA(2,1) &= -5,00 \text{ kN} \\ MF(1,2) &= 0,00 \text{ kNm} & MF(2,1) &= 3,45 \text{ kNm} \end{aligned}$$

$$\begin{aligned} DA(2,3) &= -4,47 \text{ kN} & DA(3,2) &= -1,18 \text{ kN} \\ NA(2,3) &= 2,36 \text{ kN} & NA(3,2) &= -0,33 \text{ kN} \\ MF(2,3) &= -3,45 \text{ kNm} & MF(3,2) &= 0,00 \text{ kNm} \end{aligned}$$

$$\begin{aligned} DA(3,4) &= 0,70 \text{ kN} & DA(4,3) &= -0,70 \text{ kN} \\ NA(3,4) &= 1,01 \text{ kN} & NA(4,3) &= -1,01 \text{ kN} \\ MF(3,4) &= 0,00 \text{ kNm} & MF(4,3) &= 3,45 \text{ kNm} \end{aligned}$$

$$\begin{aligned} DA(4,5) &= -0,71 \text{ kN} & DA(5,4) &= 0,71 \text{ kN} \\ NA(4,5) &= 1,00 \text{ kN} & NA(5,4) &= -1,00 \text{ kN} \\ MF(4,5) &= -3,45 \text{ kNm} & MF(5,4) &= 0,00 \text{ kNm} \end{aligned}$$

Checking joint rotations $UR(2)$ and $UR(3)$, or beam end slopes of beam 2.
Due to 5,65 kN.

Fig. 7.

$$\begin{aligned} SL1 &= (5,65) (1,64) (3,27) (4,91+3,27) / (6 \cdot 4,91 \cdot EI) \\ &= (30,30) (8,18) / (29,46 \cdot EI) \\ &= 8,41 / EI \text{ to the right.} \end{aligned}$$

$$\begin{aligned} SR1 &= (5,65) (1,64) (3,27) (4,91+1,64) / (6 \cdot 4,91 \cdot EI) \\ &= (30,30) (6,55) / (28,46 \cdot EI) \\ &= 6,74 / EI \text{ to the left.} \end{aligned}$$

Due to 3,45 kNm at beam end 2.

Fig. 8.

$$\begin{aligned} SL2 &= (3,45) (4,91) / (3 \cdot EI) = 5,65 / EI \text{ to the left.} \\ SR2 &= (3,45) (4,91) / (6 \cdot EI) = 2,82 / EI \text{ to the right.} \end{aligned}$$

Due to the beam end displacements perpendicular to the beam.

Fig. 9.

$$U_y(2,3) = 4,03 / EI \text{ and } U_y(3,2) = 29,69 / EI.$$

$$SL3 = (29,69 / EI - 4,03 / EI) / 4,91$$

$$= 5,23 / EI \text{ to the right.}$$

$$SR3 = SL3 = 5,23 / EI$$

Now to the right =

to the right minus to the left

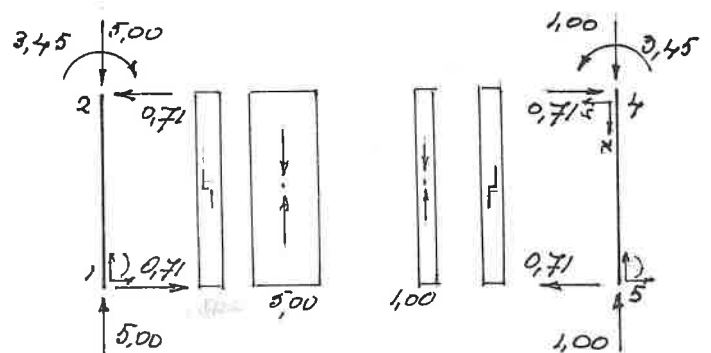
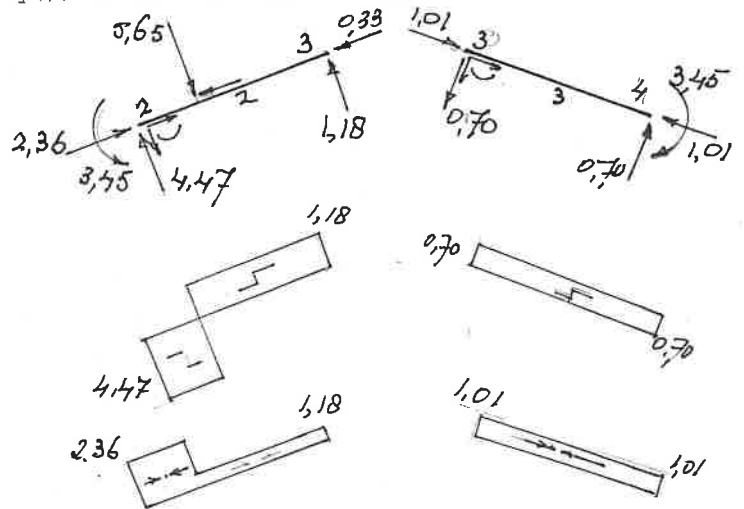
$$\text{At beam end 2 } (8,41 - 5,65 + 5,23) / EI = 7,99 / EI$$

is $UR(2) = 7,99 / EI$, correct.

$$\text{At beam end 3 } (2,82 + 5,23 - 6,74) / EI = 1,31 / EI$$

is $UR(3) = 1,32 / EI$, correct. Both positive answers, so as assumed to the right. Etc.

The diagrams are drawn with different scales. Transverse forces $DA(,)$ and normal forces $NA(,)$ have assumed directions according to the x and y axes of the member axis system x-y placed at the lowest member/beam end number. That applies to the displacements $U_x(,)$ and $U_y(,)$ of the beam ends as well.



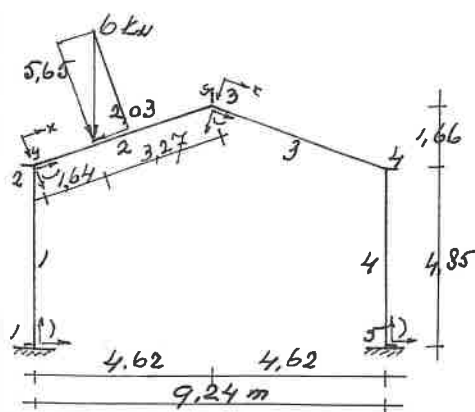


Fig. 1.

D5(1,2)= 0,91 kN D5(2,1)= -0,91 kN
D6(1,2)= -5,14 kN D6(2,1)= 5,14 kN
MF(1,2)= 1,09 kNm MF(2,1)= 3,30 kNm

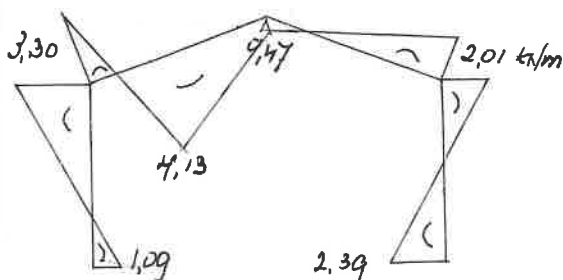
D5(2,3)= 0,91 kN D5(3,2)= -0,91 kN
D6(2,3)= -5,14 kN D6(3,2)= -0,86 kN
MF(2,3)= -3,30 kNm MF(3,2)= -0,47 kNm

D5(3,4)= 0,91 kN D5(4,3)= -0,91 kN
D6(3,4)= 0,86 kN D6(4,3)= -0,86 kN
MF(3,4)= 0,47 kNm MF(4,3)= 2,01 kNm

D5(4,5)= 0,91 kN D5(5,4)= -0,91 kN
D6(4,5)= 0,86 kN D6(5,4)= -0,86 kN
MF(4,5)= -2,01 kNm MF(5,4)= -2,39 kNm

UV(1)= 0,00 /EI UH(1)= 0,00 /EI
UR(1)= 0,00 /EI
UV(2)= 0,08 /EI UH(2)= 4,36 /EI
UR(2)= 5,35 /EI
UV(3)= 9,15 /EI UH(3)= 7,60 /EI
UR(3)= -2,85 /EI
UV(4)= 0,01 /EI UH(4)= 10,86 /EI
UR(4)= 0,92 /EI
UV(5)= 0,00 /EI UH(5)= 0,00 /EI
UR(5)= 0,00 /EI

RH(1)= 0,91 kN RV(1)= 5,14 kN
RM(1)= 1,09 kNm
RH(5)= -0,91 kN RV(5)= 0,86 kN
RM(5)= -2,39 kNm



Clicking M5MAX gives maximum, zero and member end moments.

Example. EX5b, EX5a plus PR(1)=1 and PR(5)=1.

Fig.1.

Like last example on page 11 but now without hinges, joint 1 and 5 are clamps.

Cursor in TN9, type 5, click Joint data to give joint data start values.

Next cursor in TSTRING to type

PV(1)=1 Enter, PH(1)=1 Enter, PR(1)=1 Enter, PV(5)=1 Enter, PH(5)=1 Enter, PR(5)=1 Enter.

Then click Coordinates, CSE=0, DS=1, NV=3, and type in TSTRING, like on page

I	X1(I)	Y1(I)	I	X1(I)	Y1(I)
1	0	6.51	4	9.24	1.66
2	0	1.66	5	9.24	6.51
3	4.62	0			

Cursor in TP9, type 4, Tab, cursor in TSTRING to type four rows of member data.

P9=4

P	LL	HH	NL	NH	BA	EI	EA
1	1	2	0	0	1	1	300
2	2	3	0	0	2	1	300
3	3	4	0	0	3	1	300
4	4	5	0	0	5	1	300

No hinges, all NL(P)=0 and NH(P)=0.

Click Beam load forces NFB(P), type in TSTRING, P NFB(P) I F22(P,I) L22(P,I)

2 1 1 5.65 1.64 Enter, and click Beam load forces NFA(P), type in TSTRING, P NFA(P) I F55(P,I) L55(P,I)

2 1 1 -2.03 1.64 Enter.

The force of 2.03 kN is directed towards the x axis of the member axis system, not as assumed thus a minus sign.

Click Calculate, Res(L,H) (H,L) and Reactions to get the results printed like shown on the left.

Next the case with the vertical force on the right on member 3, mirrored. Other data the same. When going on after the last results, first make the loads on member 2 zero with

Click Beam load forces NFB(P), type in TSTRING, P NFB(P) I F22(P,I) L22(P,I)

2 0 Enter, that's enough, and

click Beam load forces NFA(P), type in TSTRING, P NFA(P) I F55(P,I) L55(P,I)

2 0 Enter. Next the loads on member 3.

Click Beam load forces NFB(P), type in TSTRING, P NFB(P) I F22(P,I) L22(P,I)

3 1 1 5.65 3.27 Enter, and

click Beam load forces NFA(P), type in TSTRING, P NFA(P) I F55(P,I) L55(P,I)

3 1 1 2.03 3.27 Enter.

Click Again, Calculate and Reactions to get the results. Now one finds

UH(3)=-7,60/EI and earlier UH(3)= 7,60/EI,

UR(3)= 2,84/EI and earlier UR(3)=-2,85/EI,

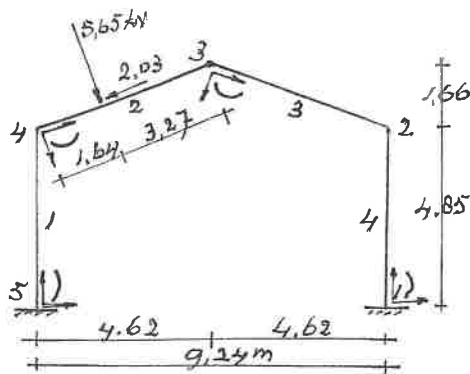
UR(2)=-0,92/EI and earlier UR(4)= 0,92/EI, etc.

RH(1)= 0,91 kN RV(1)= 0,86 kN

RM(1)= 2,39 kNm

RH(5)=-0,91 kN RV(5)= 5,14 kN

RM(5)=-1,09 kNm



N9= 5

I	FX	FY	UX	UY	MX	MY	SR
1	0	1	0	0	9.24	0	0
2	0	0	0	0	9.24	0	0
3	0	0	0	0	4.62	0	0
4	0	0	0	0	0.00	0	0
5	0	1	0	0	0.00	0	1

P9= 4

P	LL	HH	NL	NH	BA	EI	EA
1	4	5	0	0	5	1	300
2	3	4	0	0	4	1	300

NFB(2)=1 I F2 L2
1 5.65 1.64

NFA(2)=1 I F5 L5
1 2.03 3.27

P	LL	HH	NL	NH	BA	EI	EA
3	2	3	0	0	3	1	300
4	1	2	0	0	1	1	300

UV(1)=	0.00 /EI	UR(1)=	0.00 /EI
UV(2)=	0.00 /EI	UR(2)=	10.86 /EI
UV(3)=	0.01 /EI	UR(3)=	7.60 /EI
UV(4)=	0.02 /EI	UR(4)=	4.36 /EI
UV(5)=	0.00 /EI	UR(5)=	0.00 /EI

RE(1)=	-0.91 kN	RV(1)=	0.86 kN
RE(2)=	-2.39 kNm	RV(2)=	5.14 kN
RE(3)=	0.91 kN	RV(3)=	0.00 kN
RE(4)=	1.09 kNm	RV(4)=	0.00 kN

UX(4,5)=	0.00 /EI	UX(5,4)=	0.00 /EI
UY(4,5)=	-4.36 /EI	UY(5,4)=	0.00 /EI
UX(3,4)=	-4.06 /EI	UX(4,3)=	-4.08 /EI
UY(3,4)=	-11.18 /EI	UY(4,3)=	-1.55 /EI

Example. EX5c

Different joint numbering.

First type 5 in TP9, Tab, cursor in TSTRING.

Click Joint data and type in TSTRING

PH(1)=1 Enter, PV(1)=1 Enter, PR(1)=1 Enter,
PH(5)=1 Enter, PV(5)=1 Enter, PR(5)=1 Enter.

N9=5 joints.

I	X1(I)	Y1(I)	I	X1(I)	Y1(I)
1	9.24	6.51	4	0	1.66
2	9.24	1.66	5	0	6.51
3	4.62	0			

P9=4 members.

P	LL	HH	NL	NH	BA	EI	EA
1	4	5	0	0	5	1	300
2	3	4	0	0	4	1	300
3	2	3	0	0	3	1	300
4	1	2	0	0	1	1	300

P NFB(P) I F22(P,I) L22(P,I)
2 1 1 5.65 1.64

Click Beam load forces NFB(P), type in TSTRING
2,1,1,5.65,1.64 Enter.

P NFA(P) I F55(P,I) L55(P,I)
2 1 1 2.03 3.27

Click Beam load forces NFA(P), type in TSTRING
2,1,1,2.03,3.27 Enter.

Calculate etc.

DA(4,5)=	0.91 kN	DA(5,4)=	-0.91 kN
NA(4,5)=	5.14 kN	NA(5,4)=	-5.14 kN
MF(4,5)=	3.30 kNm	MF(5,4)=	1.09 kNm

DA(3,4)=	1.12 kN	DA(4,3)=	4.53 kN
NA(3,4)=	0.56 kN	NA(4,3)=	-2.59 kN
MF(3,4)=	-0.47 kNm	MF(4,3)=	-3.30 kNm

DA(2,3)=	0.51 kN	DA(3,2)=	-0.51 kN
NA(2,3)=	1.15 kN	NA(3,2)=	-1.15 kN
MF(2,3)=	2.01 kNm	MF(3,2)=	0.47 kNm

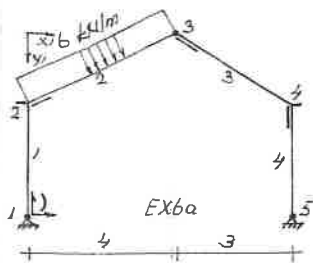
DA(1,2)=	-0.91 kN	DA(2,1)=	0.91 kN
NA(1,2)=	0.86 kN	NA(2,1)=	-0.86 kN
MF(1,2)=	-2.39 kNm	MF(2,1)=	-2.01 kNm

D5(4,5)=	-0.91 kN	D5(5,4)=	0.91 kN
D6(4,5)=	5.14 kN	D6(5,4)=	-5.14 kN
MF(4,5)=	3.30 kNm	MF(5,4)=	1.09 kNm

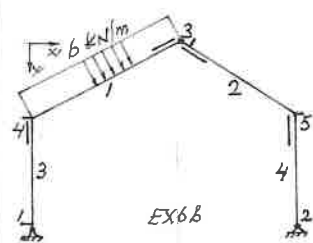
D5(3,4)=	-0.91 kN	D5(4,3)=	0.91 kN
D6(3,4)=	-0.86 kN	D6(4,3)=	-5.14 kN
MF(3,4)=	-0.47 kNm	MF(4,3)=	-3.30 kNm

D5(2,3)=	-0.91 kN	D5(3,2)=	0.91 kN
D6(2,3)=	-0.86 kN	D6(3,2)=	0.86 kN
MF(2,3)=	2.01 kNm	MF(3,2)=	0.47 kNm

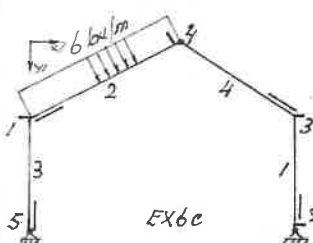
D5(1,2)=	-0.91 kN	D5(2,1)=	0.91 kN
D6(1,2)=	-0.86 kN	D6(2,1)=	0.86 kN
MF(1,2)=	-2.39 kNm	MF(2,1)=	-2.01 kNm



UV(1)=	0,00 /EI	UH(1)=	0,00 /EI
UV(2)=	0,26 /EI	UH(2)=	154,78 /EI
UR(2)=	40,28 /EI		
UV(3)=	4,46 /EI	UH(3)=	156,83 /EI
UV(4)=	0,34 /EI	UH(4)=	159,06 /EI
UR(4)=	28,35 /EI		
UV(5)=	0,00 /EI	UH(5)=	0,00 /EI
HE(1,2)=	57,25 /EI		
HE(3,2)=	-29,75 /EI		
HE(3,4)=	-16,12 /EI		
HE(5,4)=	65,35 /EI		



UV(1)=	0,00 /EI	UH(1)=	0,00 /EI
UR(1)=	57,22 /EI		
UV(2)=	0,00 /EI	UH(2)=	0,00 /EI
UV(3)=	4,53 /EI	UH(3)=	156,81 /EI
UR(3)=	-16,15 /EI		
UV(4)=	0,26 /EI	UH(4)=	154,72 /EI
UR(4)=	40,27 /EI		
UV(5)=	0,34 /EI	UH(5)=	159,08 /EI
UR(5)=	28,35 /EI		
HE(3,4)=	-29,72 /EI		
HE(2,5)=	65,37 /EI		



UV(1)=	0,26 /EI	UH(1)=	154,78 /EI
UR(1)=	40,28 /EI		
UV(2)=	0,00 /EI	UH(2)=	0,00 /EI
UR(2)=	65,35 /EI		
UV(3)=	0,34 /EI	UH(3)=	159,06 /EI
UR(3)=	28,35 /EI		
UV(4)=	4,46 /EI	UH(4)=	156,83 /EI
UR(4)=	-29,75 /EI		
UV(5)=	0,00 /EI	UH(5)=	0,00 /EI
HE(5,1)=	57,25 /EI		
HE(4,3)=	-16,12 /EI		

Example.

Three same 3-hinge frames with different joint and member numbering, and different places of the beam axis system $\vec{e}_1, \vec{e}_2$, and the same final results of the calculation.

EX6a

I	X1(I)	Y1(I)	I	X1(I)	Y1(I)
1	0	5	4	7	2
2	0	2	5	7	5
3	4	0			

P	LL	HH	NL	NH	BA	EI	EA
1	1	2	$\frac{1}{2}$	0	1	1	120
2	2	3	0	$\frac{1}{2}$	2	1	120
3	3	4	$\frac{1}{2}$	0	3	1	120
4	4	5	0	$\frac{1}{2}$	4	1	120

Four member ends with a hinge.

P	NQB(P)	I	Q3	Q4	L3	L4
2	1	1	6	6	0	4,47

HE(1,2)= 57,25/EI is UR(1)= 57,22/EI of EX6b,
HE(3,2)=-29,75/EI is UR(4)=-29,75/EI of EX6c,

HE(3,4)=-16,12/EI is UR(3)=-16,15/EI of EX6b,
HE(5,4)= 65,35/EI is UR(2)= 65,35/EI of EX6c.

EX6b

I	X1(I)	Y1(I)	I	X1(I)	Y1(I)
1	0	5	4	0	2
2	7	5	5	7	2
3	4	0			

P	LL	HH	NL	NH	BA	EI	EA
1	3	4	$\frac{1}{2}$	0	3	1	120
2	3	5	0	0	3	1	120
3	1	4	0	0	4	1	120
4	2	5	$\frac{1}{2}$	0	5	1	120

Two member ends with a hinge.

P	NQB(P)	I	Q3	Q4	L3	L4
1	1	1	-6	-6	0	4,47

HE(1,2)= 57,25/EI of EX6a now UR(1)= 57,22/EI,
HE(3,4)=-16,12/EI of EX6a now UR(3)=-16,15/EI.

EX6c

I	X1(I)	Y1(I)	I	X1(I)	Y1(I)
1	0	2	4	4	0
2	7	5	5	0	5
3	7	2			

P	LL	HH	NL	NH	BA	EI	EA
1	2	3	0	0	2	1	120
2	1	4	0	0	1	1	120
3	1	5	0	$\frac{1}{2}$	5	1	120
4	3	4	0	$\frac{1}{2}$	3	1	120

Two member ends with a hinge.

P	NQB(P)	I	Q3	Q4	L3	L4
2	1	1	6	6	0	4,47

UR(1)= 27,22/EI of EX6b now HE(5,1)= 57,25/EI,
UR(3)=-16,15/EI of EX6b now HE(4,3)=-16,12/EI.

To get an impression of the written code which can be done in other ways.

It's a try and it works.

```
Private Sub COK_Click()
If CSE=0 Then
T1=TSTRING.Text : NC=Len(T1)
NS=1
For J=1 To NV : E=""
For K=NS To NC
CR=Mid(T1,K,1)
If CR="," Then
NS=K+1 : Exit For
End If
E=E & CR
Next K
SELECTC
Next J
TSTRING.Text=""

ElseIf CSE=1 Then
T1=TSTRING.Text
T2=Mid(T1,1,2)

If T2="NL" Or T2="NH" Then
If Mid(T1,5,1)=" " Then
P=Val(Mid(T1,4,1))
VE=Val(Mid(T1,7,1))
ElseIf Mid(T1,6,1)=" " Then
P=Val(Mid(T1,4,2))
VE=Val(Mid(T1,8,1))
End If
If T2="NL" Then NL(P)=VE
If T2="NH" Then NH(P)=VE

ElseIf T2="PV" Or T2="PH" Or T2="PR" Then
If Mid(T1,5,1)=" " Then
I=Val(Mid(T1,4,1))
VE=Val(Mid(T1,7,1))
ElseIf Mid(T1,6,1)=" " Then
I=Val(Mid(T1,4,2))
VE=Val(Mid(T1,8,1))
End If
If T2="PV" Then PV(I)=VE
If T2="PH" Then PH(I)=VE
If T2="PR" Then PR(I)=VE

ElseIf T2="FX" Or T2="FY" Then
NC=Len(T1)
If Mid(T1,5,1)=" " Then
I=Val(Mid(T1,4,1))
VE=Val(Mid(T1,7,NC-6))
ElseIf Mid(T1,6,1)=" " Then
I=Val(Mid(T1,4,2))
VE=Val(Mid(T1,8,NC-7))
End If
If T2="FX" Then FX(I)=VE
If T2="FY" Then FY(I)=VE
End If
TSTRING=""
End If
End Sub
```

```
Private Sub SELECTC()
If DS=1 Then (NV=3)
Select Case J
Case 1 : I=Val(E)
Case 2 : X1(I)=Val(E)
Case 3 : Y1(I)=Val(E)
End Select

ElseIf DS=2 Then (NV=8)
Select Case J
Case 1 : P=Val(E)
Case 2 : LL(P)=Val(E)
Case 3 : HH(P)=Val(E)
Case 4 : NL(P)=Val(E)
Case 5 : NH(P)=Val(E)
Case 6 : BA(P)=Val(E)
Case 7 : EII(P)=Val(E)
Case 8 : EAA(P)=Val(E)
End Select

ElseIf DS=3 Then (NV=5)
Select Case J
Case 1 : P=Val(E)
Case 2 : NFB(P)=Val(E)
Case 3 : I=Val(E)
Case 4 : F22(P,I)=Val(E)
Case 5 : L22(P,I)=Val(E)
End Select

ElseIf DS=4 Then (NV=7)
Select Case J
Case 1 : P=Val(E)
Case 2 : NQB(P)=Val(E)
Case 3 : I=Val(E)
Case 4 : Q33(P,I)=Val(E)
Case 5 : Q44(P,I)=Val(E)
Case 6 : L33(P,I)=Val(E)
Case 7 : L44(P,I)=Val(E)
End Select

ElseIf DS=33 Then
Select Case J
Case 1 : P=Val(E)
Case 2 : NFA(P)=Val(E)
Case 3 : I=Val(E)
Case 4 : F55(P,I)=Val(E)
Case 5 : L55(P,I)=Val(E)
End Select

ElseIf DS=44 Then
Select Case J
Case 1 : P=Val(E)
Case 2 : NQA(P)=Val(E)
Case 3 : I=Val(E)
Case 4 : Q66(P,I)=Val(E)
Case 5 : Q77(P,I)=Val(E)
Case 6 : L66(P,I)=Val(E)
Case 7 : L77(P,I)=Val(E)
End Select

End If
End Sub
```

```

Private Sub LSHOW1_Click()
    CurrentY=0
    FontName="Courier New"
    FontSize=9
    For I=1 To N9
        If PV(I)=0 Then UV(I)=0
        If PH(I)=0 Then UH(I)=0
        If PR(I)=0 Then UR(I)=0
    Next I
    NLNHLERE
    CurrentX=210 : Print "N9= " & N9

    Print " I   FY   PV UV SV X1";
    Print " MZ   PR UR SR"
    For I=1 To N9
        CurrentX=105 : Print I;
        CurrentX=525 : Print FY(I);
        CurrentX=1050 : Print FV(I);
        CurrentX=1365 : Print UV(I);
        CurrentX=1680 : Print SV(I);
        CurrentX=2010
        Print Format(X1(I),"0.00");
        CurrentX=2520 : Print MZ(I);
        CurrentX=2940 : Print PR(I);
        CurrentX=3255 : Print UR(I);
        CurrentX=3570 : Print SR(I);
    Next I

    Print " I   FX   PH UH SH Y1"
    For I=1 To N9
        CurrentX=105 : Print I;
        CurrentX=525 : Print FX(I);
        CurrentX=1050 : Print PH(I);
        CurrentX=1365 : Print UH(I);
        CurrentX=1680 : Print SH(I);
        CurrentX=2010
        Print Format(Y1(I),"0.00")
    Next I
End Sub

```

```

Private Sub LSHOW2_Click()
    FontName="Courier New"
    FontSize=9
    NLNHLERE
    CurrentX=210 : Print "P9= " & P9
    Print " P   LL HH NL NH";
    Print " BA EI EA  L1"

```

```

For P=1 To P9 : L=LL(P) : H=HH(P)
D1=X1(H)-X1(L) : D2=Y1(H)-Y1(L)
L!!(P)=Sqr(D1^2+D2^2)
CurrentX=105 : Print P;
CurrentX=525 : Print LL(P);
CurrentX=840 : Print HH(P);
CurrentX=1260 : Print NL(P);
LE(P)=NL(P)
CurrentX=1575 : Print NH(P);
RE(P)=NH(P)
CurrentX=2100 : Print BA(P);
CurrentX=2505 : Print EII(P);
CurrentX=2820 : Print EAA(P);
CurrentX=3480
Print Format(L11(P),"0.00")

```

```

If NFB(P)>0 Then
    Print " NFB(" & P & ")=" & NFB(P);
    CurrentX=1365
    Print "I   F2   L2"

```

```

For I=1 To NFB(P)
    CurrentX=1260 : Print I;
    CurrentX=1680 : Print F22(P,I);
    CurrentX=2310
    Print Format(L22(P,I),"0.00")
Next I
End If

```

```

If NQB(P)>0 Then
    Print " NQB(" & P & ")=" & NQB(P);
    CurrentX=1260
    Print " I   Q3   Q4   L3   L4"
    For I=1 To NQB(P)
        CurrentX=1260 : Print I;
        CurrentX=1680 : Print Q33(P,I);
        CurrentX=2310 : Print Q44(P,I);
        CurrentX=2940
        Print Format(L33(P,I),"0.00");
        CurrentX=3570
        Print Format(L44(P,I),"0.00")
    Next I
End If

```

```

If NFA(P)>0 Then
    Print " NFA(" & P & ")=" & NFA(P);
    CurrentX=1365
    Print "I   F2   L2"
    For I=1 To NFA(P)
        CurrentX=1260 : Print I;
        CurrentX=1680 : Print F55(P,I);
        CurrentX=2310
        Print Format(L55(P,I),"0.00")
    Next I
End If

```

```

If NQA(P)>0 Then
    Print " NQA(" & P & ")=" & NQA(P);
    CurrentX=1260
    Print " I   Q3   Q4   L3   L4"
    For I=1 To NQA(P)
        CurrentX=1260 : Print I;
        CurrentX=1680 : Print Q66(P,I);
        CurrentX=2310 : Print Q77(P,I);
        CurrentX=2940
        Print Format(L66(P,I),"0.00");
        CurrentX=3570
        Print Format(L77(P,I),"0.00")
    Next I
End If

```

```

If NFB(P)>0 Or NQB(P)>0 Or _
NFA(P)>0 Or NQA(P)>0 Then
    If P<P9 Then
        Print " P   LL HH NL NH";
        Print " BA EI EA  L1"
    End If
End If
Next P : Print
End Sub

```

```

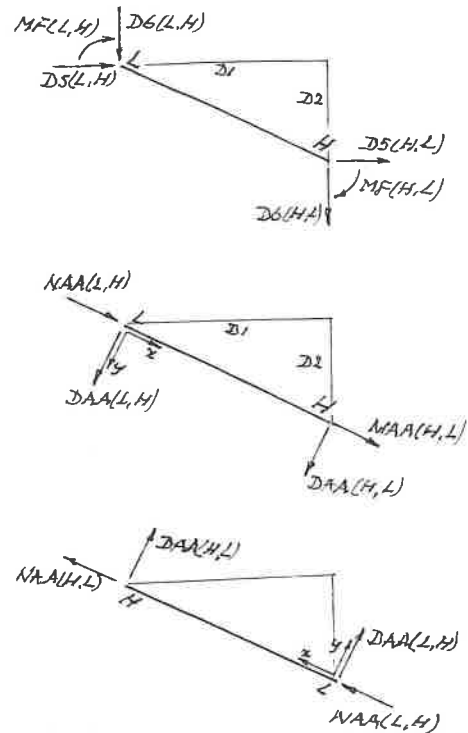
Private Sub NLNHLERE()
    For P=1 To P9 : L=LL(P) : H=HH(P)
        If BA(P)=L Then LE(P)=NL(P) _
        : RE(P)=NH(P)
        If BA(P)=H Then LE(P)=NH(P) _
        : RE(P)=NL(P)
    Next P
End Sub

```

```

Private Sub RESULTS()
For P=1 To P9 : L=LL(P) : H=HH(P)
FMT="0.00"
'Member end forces
If RST=1 Then
Print " D5(" & P & ",1=";
Else
Print " D5(" & L & ", " & H _
& ")=";
End If
CX=1995
T1=Format(D5(P,1),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
CurrentX=2205
If RST=1 Then
Print " D5(" & P & ",2)=";
Else
Print " D5(" & H & ", " & L _
& ")=";
End If
CX=4200
T1=Format(D5(P,2),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
CurrentX=2205
If RST=1 Then
Print " D6(" & P & ",1)=";
Else
Print " D6(" & L & ", " & H _
& ")=";
End If
CX=1995
T1=Format(D6(P,1),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
CurrentX=2205
If RST=1 Then
Print " D6(" & P & ",2)=";
Else
Print " D6(" & H & ", " & L _
& ")=";
End If
CX=4200
T1=Format(D6(P,2),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
CurrentX=2205
If RST=1 Then
Print " MF(" & P & ",1)=";
Else
Print " MF(" & L & ", " & H _
& ")=";
End If
CX=1995
T1=Format(MF(P,1),FMT & " kNm"
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
CurrentX=2205
If RST=1 Then
Print " MF(" & P & ",2)=";
Else
Print " MF(" & H & ", " & L _
& ")=";
End If
CX=4200
T1=Format(MF(P,2),FMT) & " kNm"
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1 : Print
Next P : Print
End Sub

```



```

Private Sub LDANA_Click()
FMT="0.00"
For P=1 To P: L=LL(P): H=HH(P)
Print " DA(" & L & ", " & H & ")=";
CX=2100
T1=Format(DAA(P,1),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
CurrentX=2205
Print " DA(" & H & ", " & L & ")=";
CX=4305
T1=Format(DAA(P,2),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
Print " NA(" & L & ", " & H & ")=";
CX=2100
T1=Format(NAA(P,1),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
CurrentX=2205
Print " NA(" & H & ", " & L & ")=";
CX=4305
T1=Format(NAA(P,2),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
Print " MF(" & L & ", " & H & ")=";
CX=2100
T1=Format(MF(P,1),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
CurrentX=2205
Print " MF(" & H & ", " & L & ")=";
CX=4305
T1=Format(MF(P,2),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1: Print
Next P
End Sub

```

```

Private Sub LREACTIONS_Click()
If UVHR=0 Then FMT="0.00" : UVHR=1 Else
FMT="0.000" : UVHR=0
For I=1 To N9 : JN(I)=0 : Next I
For P=1 To P9 : L=LL(P) : H=HH(P)
If NL(P)=0 Then JN(L)=1
If NH(P)=0 Then JN(H)=1
Next P

```

```

For I=1 To N9
Print " UV(" & I & ")="; : CX=1995
T1=Format(UV(I),FMT) & " /EI"
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
CurrentX=2205
Print " UH(" & I & ")="; : CX=4200
T1=Format(UH(I),FMT) & " /EI"
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1

```

```

If JN(I)=1 Then
Print "UR(" & I & ")="; : CX=1995
T1=Format(UR(I),FMT) & " /EI"
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1
End If
Next I : Print
'slopes separately calculated
For P=1 To P9 : L=LL(P) : H=HH(P)
If NL(P)=1 Then
Print HE(" % L & "," & H & ")=";
CX=2205
T1=Format(HE(P,1),"0.00") & " /EI"
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1
End If

```

```

If NH(P)=1 Then
Print HE(" % H & "," & L & ")=";
CX=2205
T1=Format(HE(P,2),"0.00") & " /EI"
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1
End If
Next P : Print
For I=1 To N9
If PH(I)=1 Then
Print " RH(" & I & ")=";
CX=1955
T1=Format((RH(I),"0.00") & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
End If
If PV(I)=1 Then
CurrentX=2205
Print " RV(" & I & ")=";
CX=4200
T1=Format((RV(I),"0.00") & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1
End If
If PR(I)=1 Then
Print " RM(" & I & ")=";
CX=1955
T1=Format((RM(I),"0.00") & " kNm"
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
End If
Next I : Print
End Sub

```

5d. Calculation of member end displacements w.r.t. x-y.

```

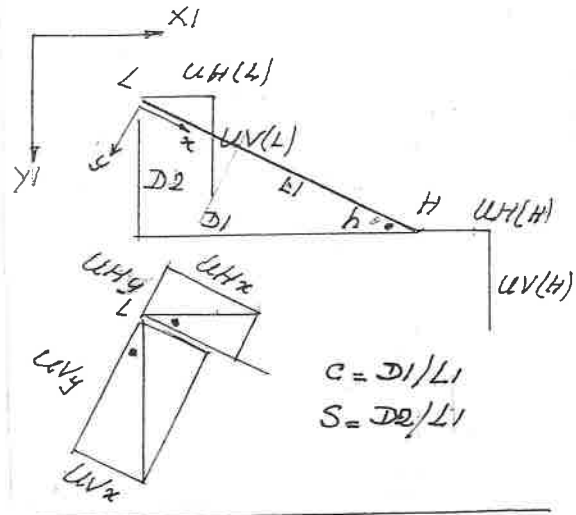
'member end L
UHx=UH(L)*C : UHy=UH(L)*S
UVy=UV(L)*C : UVx=UV(L)*S
Ux(P,1)=UHx+UVx : Uy(P,1)=UVy-UHy

```

```

'member end H
UHx=UH(H)*C : UHy=UH(H)*S
UVy=UV(H)*C : UVx=UV(H)*S
Ux(P,2)=UHx+UVx : Uy(P,2)=UVy-UHy

```



```

Private Sub LUxUy_Click()
For P=1 To P9 : L=LL(P) : H=HH(P)
FMT="0.00"
Print " Ux(" & L & "," & H &
& ")=";
CX=2205
T1=Format(Ux(P,1),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
CurrentX=2205
Print " Ux(" & H & "," & L &
& ")=";
CX=4410
T1=Format(Ux(P,2),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1

Print " Uy(" & L & "," & H &
& ")=";
CX=2205
T1=Format(Uy(P,1),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1;
CurrentX=2205
Print " Uy(" & H & "," & L &
& ")=";
CX=4410
T1=Format(Uy(P,2),FMT) & " kN "
TW=TextWidth(T1)
CurrentX=CX-TW : Print T1 : Print
Next P : Print
End Sub

```

```

Private Sub FRAMEMAINCALC()
'1. Composition of construction
'matrix CC with member matrices S5.
CONSTRMATCCFRAME
'2. Elements of force vector FF.
'2a. Joint load forces FX(I)
'and FY(I), and moment MZ(I).
N=3*N9
For I=1 To N9
A=3*I-2 : B=3*I-1 : C=3*I
FF(A)=FX(I) : FF(B)=FY(I)
FF(C)=MZ(I)
PP(A)=PH(I) : PP(B)=PV(I)
PP(C)=PR(I)
UU(A)=UH(I) : UU(B)=UV(I)
UU(C)=UR(I)
SS(A)=SH(I) : SS(B)=SV(I)
SS(C)=SR(I)
Next I

'2b. Primary forces and moments
'due to member loads perpendicular
'to the member axis.
For P=1 To P9 : L=LL(P) : H=HH(P)
EI=EII(P)
D1=X1(H)-X1(L) : D2=Y1(H)-Y1(L)
L1=Sqr(D1^2+D2^2)
If LE(P)=1 And RE(P)=1 Then
M1=0 : M2=0
BEAM1
ElseIf LE(P)=1 Or RE(P)=1 Then
M1=0 : M2=0 : M3=0 : M4=0
BEAM2
ElseIf LE(P)=0 And RE(P)=0 Then
BEAM3
End If
'primary moments and forces
M11(P)=M1 : R11(P)=R1
M22(P)=M2 : R22(P)=R2

S=D2/L1 : C=D1/L1
If BA(P)=L Then
D3(P,1)=-R1*S : D3(P,2)=-R2*S
D4(P,1)=R1*C : D4(P,2)=R2*C
MP(P,1)=M1 : MP(P,2)=-M2
HE(P,1)=H1 : HE(P,2)=H2

ElseIf BA(P)=H Then
D3(P,1)=R2*S : D3(P,2)=R1*S
D4(P,1)=-R2*C : D4(P,2)=-R1*C
MP(P,1)=-M2 : MP(P,2)=M1
HE(P,1)=H2 : HE(P,2)=H1
End If
Next P
'2c. Alteration of force vector FF.
For I=1 To N9
A=3*I-2 : B=3*I-1 : C=3*I
For P=1 To P9 : L=LL(P) : H=HH(P)
If I=L Then
FF(A)=FF(A)+D3(P,1)
FF(B)=FF(B)+D4(P,1)
FF(C)=FF(C)+MP(P,1)
ElseIf I=H Then
FF(A)=FF(A)+D3(P,2)
FF(B)=FF(B)+D4(P,2)
FF(C)=FF(C)+MP(P,2)
End If
Next P
Next I

```

```

'2d. Primary forces due to member
'loads along member axis x.
For P=1 To P9 : L=LL(P) : H=HH(P)
EA=EAA(P)
D1=X1(H)-X1(L) : D2=Y1(H)-Y1(L)
L1=Sqr(D1^2+D2^2)
S=D2/L1 : C=D1/L1

```

MEMBER

```

N11(P)=N1 : N22(P)=N2

```

```

D7(P,1)=N1*C : D7(P,2)=N2*C
D8(P,1)=N1*S : D8(P,2)=N2*S

```

```

Next P

```

```

'2e. Second alteration of force
'vector FF.

```

```

For I=1 To N9

```

```

A=3*I-2 : B=3*I-1

```

```

For P=1 To P9 : L=LL(P) : H=HH(P)

```

```

If I=L Then

```

```

FF(A)=FF(A)+D7(P,1)

```

```

FF(B)=FF(B)+D8(P,1)

```

```

ElseIf I=H Then

```

```

FF(A)=FF(A)+D7(P,2)

```

```

FF(B)=FF(B)+D8(P,2)

```

```

End If

```

```

Next P

```

```

Next I

```

```

'3. Alteration of force vector FF
'and construction matrix CC.

```

```

'3a. Of FF in case of prescribed
'displacements <>0.

```

```

For I=1 To N

```

```

If UU(I)<>0 Then

```

```

For K=1 To N

```

```

FF(K)=FF(K)-CC(K,I)*UU(I)

```

```

Next K

```

```

End If

```

```

Next I

```

```

'3b. Of FF and CC in case of prescribed
'displacements.

```

```

For I=1 To N

```

```

If PP(I)=1 Then

```

```

For J=1 To N

```

```

CC(I,J)=0 : CC(J,I)=0

```

```

Next J

```

```

CC(I,I)=1 : FF(I)=UU(I)

```

```

End If

```

```

Next I

```

```

'3c. Of FF and CC when the joint
'is a hinge.

```

```

For I=1 To N

```

```

If CC(I,I)=0 Then CC(I,I)=1 :

```

```

FF(I)=0

```

```

Next I

```

```

'3d. Of CC in case of springy
'supports.

```

```

For I=1 To N

```

```

If SS(I)>0 Then CC(I,I)=CC(I,I)+
SS(I)

```

```

Next I

```

```

'4a. Calculation of the unknown
'displacements UH(I), UV(I) and
'rotation UR(I).

```

```

For I=1 To N : BB(I)=FF(I)

```

```

For J=1 To N

```

```

AA(I,J)=CC(I,J)

```

```

Next J

```

```

Next I

```

GAUSS

```
For I=1 To N9
A=3*I-2 : B=3*I-1 : C=3*I
UH(I)=XX(A) : UU(A)=XX(A)
UV(I)=XX(B) : UU(B)=XX(B)
UR(I)=XX(C) : UU(C)=XX(B)
Next I
```

'4b. Slope deflections to be calculated seperately in case of hinged member ends..

```
For P=1 To P9 : L=LL(P) : H=HH(P)
D1=X1(H)-X1(L) : D2=Y1(H)-Y1(L)
L1=Sqr(D1^2+D2^2)
S=D2/L1 : C=D1/L1
ZL=UV(L)*C-UH(L)*S
ZH=UV(H)*C-UH(H)*S
H6=(ZH-ZL)/L1
H7=1.5*(ZH-ZL)/L1
If NL(P)=1 And NH(P)=1 Then
HE(P,1)=HE(P,1)+H6
HE(P,2)=HE(P,2)+H6
ElseIf NL(P)=1 Then
HE(P,1)=HE(P,1)+H7-0.5*UR(H)
ElseIf NH(P)=1 Then
HE(P,2)=HE(P,2)+H7-0.5*UR(L)
End If
Next P
```

'5. Calculation of the member end forces and moments w.r.t. construction axis system X1-Y1.
'5a. Due to the displacements alone.

```
For P=1 To P9 : L=LL(P) : H=HH(P)
EA=EAA(P) : EI=EII(P)
```

MEMBERMATS5FRAME

```
TT(1)=3*L-2 : TT(2)=3*L-1
TT(3)=3*L
TT(4)=3*H-2 : TT(5)=3*H-1
TT(6)=3*H
For I=1 To 6 : FK(P,I)=0
For J=1 To 6 : A=TT(J)
FK(P,I)=FK(P,I)+S5(I,J)*UU(A)
Next J
Next I
```

'5b. Due to displacements and member loads.

```
D5(P,1)=FK(P,1)-D3(P,1)-D7(P,1)
D6(P,1)=FK(P,2)-D4(P,1)-D8(P,1)
MF(P,1)=FK(P,3)-MP(P,1)
D5(P,2)=FK(P,4)-D3(P,2)-D7(P,2)
D6(P,2)=FK(P,5)-D4(P,2)-D8(P,2)
MF(P,2)=FK(P,6)-MP(P,2)
```

'5c. Calculation of the member end forces w.r.t. the member axes system x-y.

```
D1=X1(H)-X1(L) : D2=Y1(H)-Y1(L)
L1=Sqr(D1^2+D2^2)
S=D2/L1 : C=D1/L1
```

```
NAA(P,1)=D5(P,1)*C+D6(P,1)*S
NAA(P,2)=D5(P,2)*C+D6(P,2)*S
DAA(P,1)=D6(P,1)*C-D5(P,1)*S
DAA(P,2)=D6(P,2)*C-D5(P,2)*S
```

5d. Calculation of member end displacements w.r.t. x-y.

```
'member end L
UHx=UH(L)*C : UHy=UH(L)*S
UVy=UV(L)*C : UVx=UV(L)*S
Ux(P,1)=UHx+UVx : Uy(P,1)=UVy-UHy
'member end H
UHx=UH(H)*C : UHy=UH(H)*S
UVy=UV(H)*C : UVx=UV(H)*S
Ux(P,2)=UHx+UVy : Uy(P,2)=UVy-UHy
Next P
```

'6. Calculation of the joint forces KH(I), KV(I) and KM(I).

'6c. Due to the displacements alone.

CONSTRMATCCFRAME

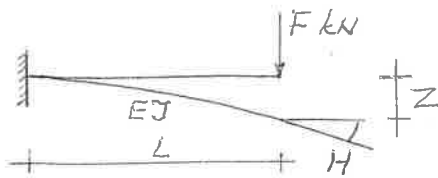
```
For I=1 To N9
A=3*I-2 : B=3*I-1 : C=3*I
KH(I)=0 : KV(I)=0 : KM(I)=0
For J=1 To N
KH(I)=KH(I)+CC(A,J)*UU(J)
KV(I)=KV(I)+CC(B,J)*UU(J)
KM(I)=KM(I)+CC(C,J)*UU(J)
Next J
```

'6b. Due to the displacements, and member loads perpendicular to and along the member axis.

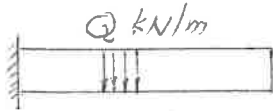
```
For P=1 To P9 : L=LL(P) : H=HH(P)
If I=L Then
KH(I)=KH(I)-D3(P,1)-D7(P,1)
KV(I)=KV(I)-D4(P,1)-D8(P,1)
KM(I)=KM(I)-MP(P,1)
ElseIf I=H Then
KH(I)=KH(I)-D3(P,2)-D7(P,2)
KV(I)=KV(I)-D4(P,2)-D8(P,2)
KM(I)=KM(I)-MP(P,2)
End If
Next P
Next I
```

'7. Calculation of the reactions.

```
For I=1 To N9
If SH(I)>0 Then
RH(I)=-SH(I)*UH(I)
Else
RH(I)=KH(I)-FX(I)
End If
If SV(I)>0 Then
RV(I)=-SV(I)*UV(I)
Else
RV(I)=-KV(I)+FY(I)
End If
If SR(I)>0 Then
RM(I)=-SR(I)*UR(I)
Else
RM(I)=KM(I)-MZ(I)
End If
Next I
End Sub
```



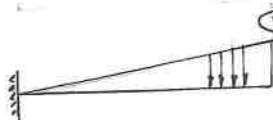
$$H = \frac{F \cdot L^2}{2 \cdot EI} \quad Z = \frac{F \cdot L^3}{3 \cdot EI}$$



$$H = \frac{Q \cdot L^3}{6 \cdot EI} \quad Z = \frac{Q \cdot L^4}{8 \cdot EI}$$



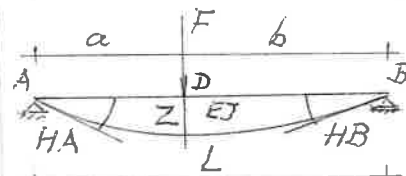
$$H = \frac{Q \cdot L^3}{24 \cdot EI} \quad Z = \frac{Q \cdot L^4}{30 \cdot EI}$$



$$H = \frac{Q \cdot L^3}{8 \cdot EI} \quad Z = \frac{11Q \cdot L^3}{120 \cdot EI}$$



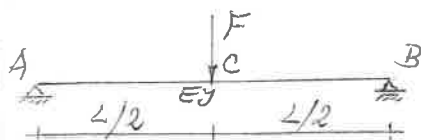
$$H = \frac{M \cdot L}{EI} \quad Z = \frac{M \cdot L^2}{2 \cdot EI}$$



$$H_A = \frac{F \cdot a \cdot b \cdot (L + b)}{6 \cdot L \cdot EI}$$

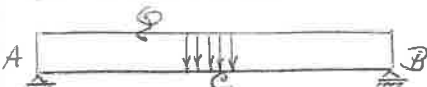
$$H_B = \frac{F \cdot a \cdot b \cdot (L + a)}{6 \cdot L \cdot EI}$$

$$Z_D = \frac{F \cdot a^2 \cdot b^2}{3 \cdot L \cdot EI}$$



$$H_A = H_B = \frac{F \cdot L^2}{16 \cdot EI}$$

$$Z_C = \frac{F \cdot L^3}{48 \cdot EI}$$



$$H_A = H_B = \frac{Q \cdot L^3}{24 \cdot EI}$$

$$Z_C = \frac{5 \cdot Q \cdot L^4}{384 \cdot EI}$$



$$H_A = \frac{Q \cdot L^3}{45 \cdot EI}$$

$$H_B = \frac{7 \cdot Q \cdot L^3}{360 \cdot EI}$$

$$Z_C = \frac{(5 \cdot Q \cdot L^4 / (384 \cdot EI))}{2}$$

Standard formulas for simple beams.

E is modulus of elasticity in kN/m²
EI is bending stiffness, EI is E*I with
I is moment of inertia in m⁴

EI is (kN/m²)*m⁴ is kNm²

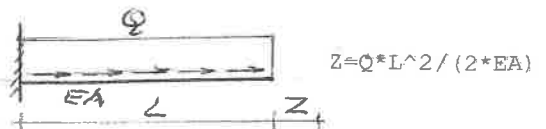
EA is strain stiffness, EA is E*A with
A is cross sectional area in m²

EA is (kN/m²)*m² is kN

Displacement Z in m, angle H in radians



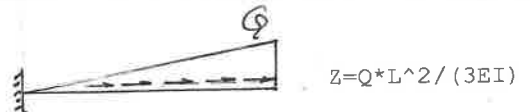
$$Z = \frac{F \cdot L}{EA}$$



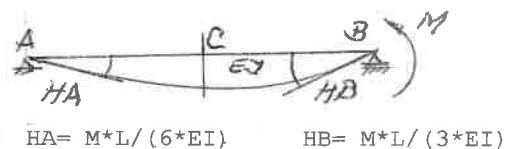
$$Z = \frac{Q \cdot L^2}{2 \cdot EA}$$



$$Z = \frac{Q \cdot L^2}{6 \cdot EA}$$



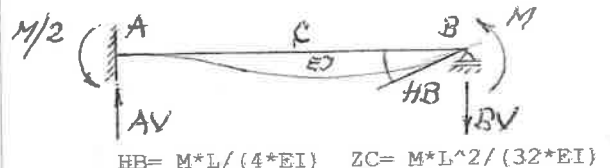
$$Z = \frac{Q \cdot L^2}{3 \cdot EA}$$



$$H_A = \frac{M \cdot L}{6 \cdot EI}$$

$$H_B = \frac{M \cdot L}{3 \cdot EI}$$

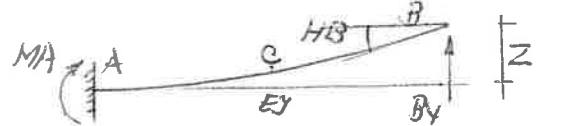
$$Z_C = \frac{M \cdot L^2}{16 \cdot EI}$$



$$H_B = \frac{M \cdot L}{4 \cdot EI}$$

$$Z_C = \frac{M \cdot L^2}{32 \cdot EI}$$

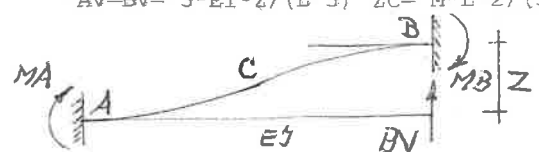
$$AV = BV = \frac{3 \cdot M}{2 \cdot EI}$$



$$MA = \frac{3 \cdot EI \cdot Z}{L^2}$$

$$H_B = \frac{3 \cdot Z}{2 \cdot L}$$

$$AV = BV = \frac{3 \cdot EI \cdot Z}{L^3} \quad Z_C = \frac{M \cdot L^2}{32 \cdot EI}$$



$$MA = MB = \frac{6 \cdot EI \cdot Z}{L^2}$$

$$Z_C = \frac{Z}{2}$$

$$AV = BV = \frac{12 \cdot EI \cdot Z}{L^3}$$