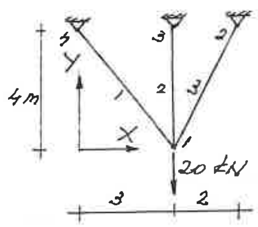
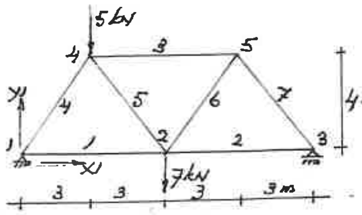
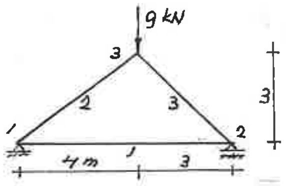
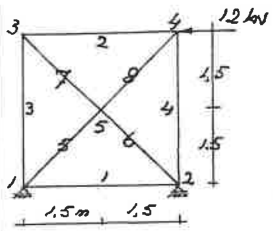
Part 5

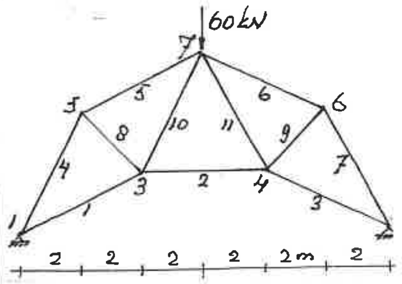
Plane constructions of which the member ends are hinges. Trusses.











2.1. The relation between member end forces and member end displacements.

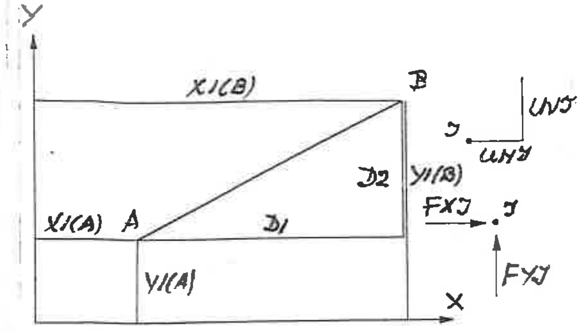


Fig.1a.

Fig.1a.
The X-Y axis system is the accepted construction axis system. Starting point is a member as drawn on the left.
The horizontal displacement UHI of a joint I (one or more member ends) is assumed as the X axis to the right, the vertical displacement UVI as the Y axis upwards.

On the joints act horizontal joint load forces FXI, assumption as the X axis to the right, and vertical joint load forces FYI, assumption as the Y axis upwards.

(The vertical joint load forces are mostly directed downwards, one may assume that direction as well, and just go on working with it.)

It is assumed that the coordinates of member end B of member AB are larger than those of member end A. The sides of the triangle then become
D1=X1(B)-X1(A) and D2=Y1(B)-Y1(L).

With D1 and D2 the member length becomes

L1=Sqr(D1^2+D2^2) and further are

Sin(h)=D2/L1 or S=D2/L1 and
Cos(h)=D1/L1 or C=D1/L1.

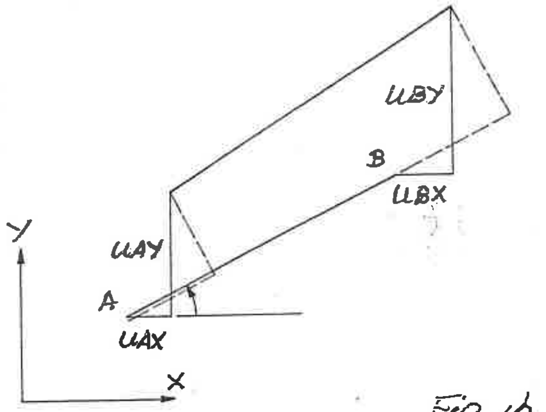


Fig. 1b.

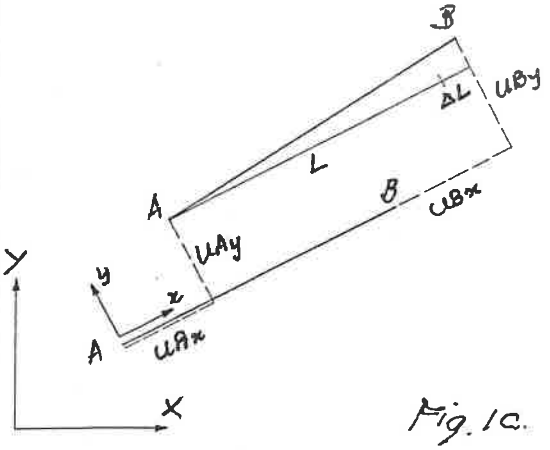


Fig. 1c.

Fig.1b en 1c.
The member ends displace as assumed for the joints to the right and upwards.
The member itself has an own member axis system of which the origin is assumed to be at A and the x axis is directed from A to B.
Later, the joints, and thus the member ends, are to be numbered. Then A will be given the lowest member end number L, and B the highest member end number H. The member axis system then will always be placed at the lowest member end number.

If one assumes that displacement UBx of member end B along the member is larger than displacement UAx of member end A, then the member will become ΔL longer.
Since the displacements UAy and UBy perpendicular to the member axis are small w.r.t. member length L, one can write
ΔL=UBx-UAx.

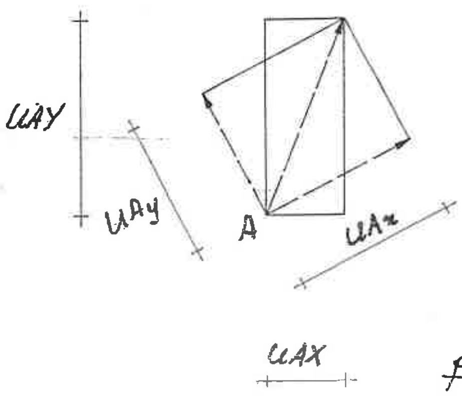


Fig.2a.

Fig.2a.
The displacements UAx and UAy w.r.t. the member axes sytem x-y, shall be expressed in the displacements UAX and UAY w.r.t the construction axes system X-Y. Because, see later, the member end forces according to the X and Y direction shall be expressed in the displacements w.r.t. construction axes sytem X-Y by means of the member stiffness matrix S5.

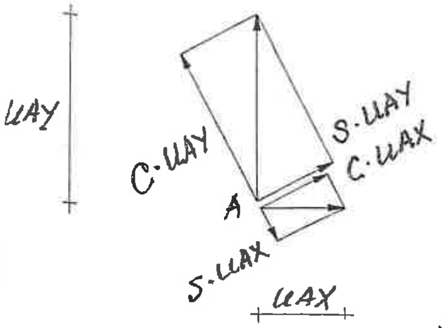


Fig.2b.

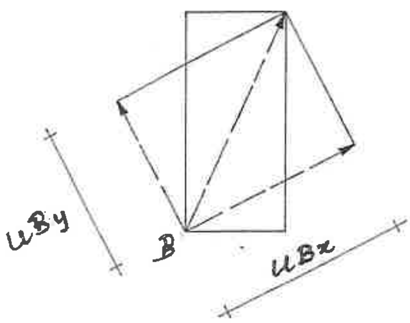


Fig.3a.

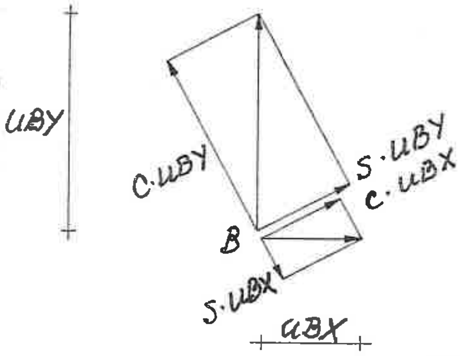


Fig.3b.

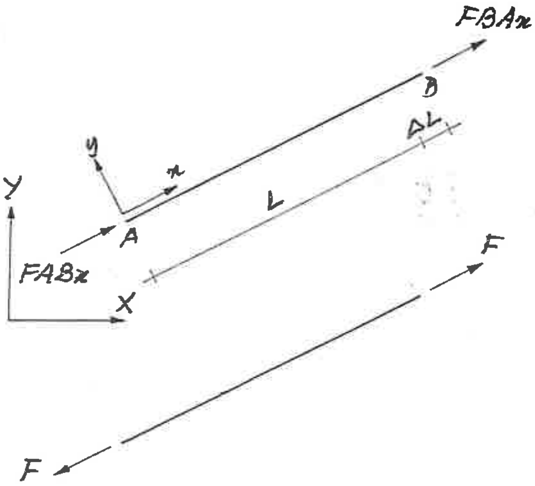


Fig.4.

The displacements of member end A.

Fig.2a and 2b.
The as vectors drawn displacements UAX and UAY are resolved in a displacement along, and a displacement perpendicular to the member axis.

The component of UAX along the member axis is Cos(h)*UAX, written shorter C*UAX, and the component of UAY along the member axis is Sin(h)*UAY, and shorter S*UAY.
Then the displacement UAx, fig.2a, along member axis x is
UAx= C*UAX+S*UAY.

Perpendicular to the member axis the components are
Sin(h)*UAX or S*UAX, and
Cos(h)*UAY or C*UAY.
Paying attention to the directions then for the displacement UAy according to member axis y follows
UAy= C*UBX-S*UAX or, in other order,

UAy=-S*UAX+C*UAY.

The displacements of member end B.

Fig.3a en 3b.
Similar as for member end A follows

UBx= C*UBX+S*UBY and

UBy=-S*UBX+C*UBY.

Now the relation between member end forces w.r.t. to the member axis system x-y, and the displacements w.r.t. the construction axis system.

Fig.4.
The member becomes ΔL=UBx-UAx longer. The member is a tensile member. On the member ends act tensile forces of F kN.

With Hooke is ΔL=FL/EA or F=(EA/L)ΔL.
The member stiffness factor is R=EA/L. Then is

F=R(UBx-UAx) or F=R(-UAx+UBx).

The assumed direction for the member end forces FABx and FBAx is the same as for the displacements UAx and UBx, the same direction as member axis x.
Both member end forces FABx and F at A, and both member end forces FBAx and F at B, must be equal, thus follows

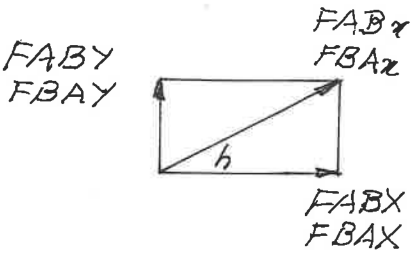
FABx=-F so that FABx=R(UAx-UBx), and

FBAx= F so that FBAx=R(-UAx+UBx).

When the earlier determined UAx and UBx are substituted in both equations, then follow

FABx=R((C*UAX+S*UAY)-(C*UBX+S*UBY)) and

FBAx=R(-(C*UAX+S*UAY)+(C*UBX+S*UBY)).



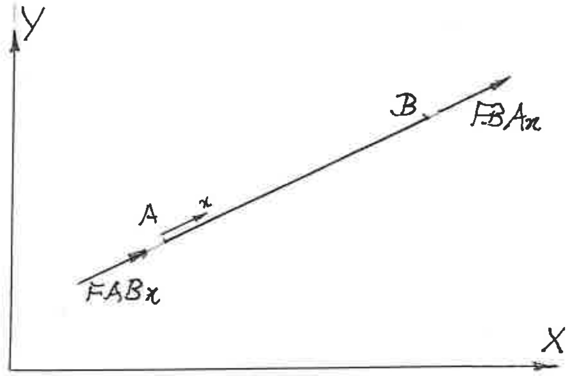


Fig.5.

R*C^2	R*S*C	-R*C^2	-R*S*C
R*S*C	R*S^2	-R*S*C	-R*S^2
-R*C^2	-R*S*C	R*C^2	R*S*C
-R*S*C	-R*S^2	R*S*C	R*S^2

S5

There appear in matrix S5 three different elements with plus or minus sign, R*C^2, R*S*C and R*S^2, with

R=EA/L, C=Cos(h) and S=Sin(h).

The matrix is symmetric w.r.t. the main diagonal.

Fig.6.

Fig.5.
The assumption for the direction of the horizontal member end forces FABX and FBAX is as for the horizontal displacements to the right, and the assumption for the direction of the vertical member end forces FABY and FBAY is as for the vertical displacements upward.
These member end forces are the components of FABx and FBAx.

Cos(h)=FABX/FABx so that FABX=FABx*Cos(h)

Sin(h)=FABY/FABx FABY=FABx*Sin(h)

Cos(h)=FBAX/FBAx FBAX=FBAx*Cos(h)

Sin(h)=FBAY/FBAx FBAY=FBAx*Sin(h)

With C=Cos(h) and S is Sin(h) then follow

FABX=FABx*C and FBAX=FBAx*C
FABY=FABx*S and FBAY=FBAx*S.

The equations for the according to the member axis acting member end forces FABx and FBAx, expressed in the displacements w.r.t. the X-Y-axes system were found on the preceding page.

FABx=R((C*UAX+S*UAY)-(C*UBX+S*UBY)) and

FBAx=R(-(C*UAX+S*UAY)+(C*UBX+S*UBY)).

When these are substituted in the equations of the horizontal en vertical member end forces at A and B, then follow

FABX= R(C*C*UAX+S*C*UAY-C*C*UBX-S*C*UBY)

FABY= R(S*C*UAX+S*S*UAY-S*C*UBX-S*S*UBY)

FBAX= R(-C*C*UAX-S*C*UBY+C*C*UBX+S*C*UBY)

FBAY= R(-S*C*UAX-S*S*UAY+S*C*UBX+S*S*UBY)

and written in matrix form

$$\begin{bmatrix} FABX \\ FABY \\ FBAX \\ FBAY \end{bmatrix} = \begin{bmatrix} R*C*C & R*S*C & -R*C*C & -R*S*C \\ R*S*C & R*S*S & -R*S*C & -R*S*S \\ -R*C*C & -R*S*C & R*C*C & R*S*C \\ -R*S*C & -R*S*S & R*S*C & R*S*S \end{bmatrix} \cdot \begin{bmatrix} UAX \\ UAY \\ UBX \\ UBY \end{bmatrix}$$

f S5 u

f is the force vector,

S5 is the member stiffness matrix, and

u is the displacement vector.

Fig.6.
These are the elements of matrix S5 as they will be calculated in the examples and the programme to be written.
worden berekend. (Sometimes stress stiffnes R put before matrix S5.)

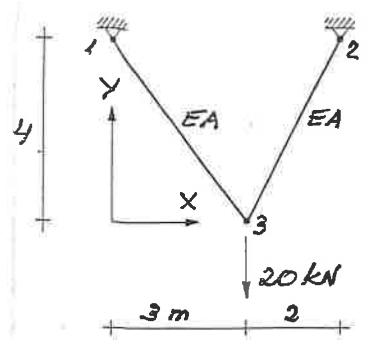


Fig.1.

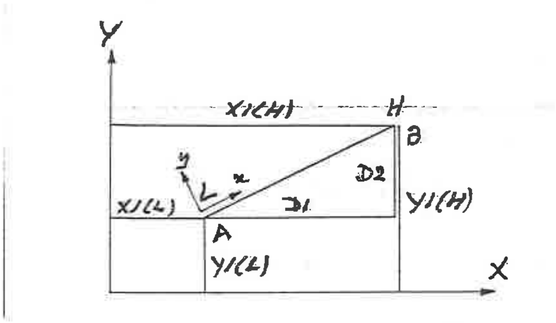


Fig.2.

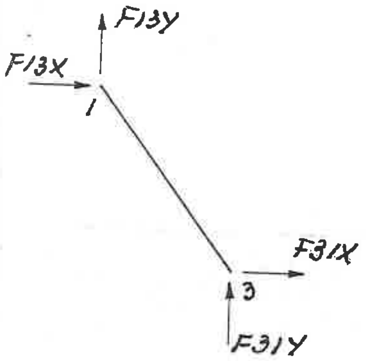


Fig.3a.

$$\begin{bmatrix} F13X \\ F13Y \\ F31X \\ F31Y \end{bmatrix} = \frac{EA}{1000} \begin{bmatrix} 72 & -96 & -72 & 96 \\ -96 & 128 & 96 & -128 \\ -72 & 96 & 72 & -96 \\ 96 & -128 & -96 & 128 \end{bmatrix} \begin{bmatrix} UH1 \\ UV1 \\ UH3 \\ UV3 \end{bmatrix}$$

S51

Fig.3b.

Example.

Fig.1.
The construction consists of two members and three joints. Members, and joints, are numbered in arbitrary order, but starting with 1 and increasing with 1.
The construction axes system X-Y is assumed as drawn in the figure.
Then the joint coordinates become in meters m, X1() for X and Y1() for Y, as written below.

X1(1)=0	X1(2)=5	X1(3)=3
Y1(1)=4	Y1(2)=4	Y1(3)=0

Fig.2.
This is the situation for a member A-B as assumed on page 2 , for which applies
D1=X1(B)-X1(A) and D2=Y1(B)-Y1(A).

For each member is assumed that the lowest member end number L is A, and the highest member end number H is B.

Member 1.
D1=X1(H)-X1(L)=X1(3)-X1(1)=3-0= 3
D2=Y1(H)-Y1(L)=Y1(3)-Y1(1)=0-4=-4
The member length becomes
L1=SQR(D1^2+D2^2)
=SQR(3^2+(-4)^2)=SQR(9+16)=SQR(25)=5,00 m.

With member cross-section A in m2, and modulus of elasticity E in kN/m2, follows stress stiffness EA (E times A) in kN.
With length L1 in meters follows member stiffness factor R=EA/L1 in kN/m.
For member 1 is R=EA/L1=EA/5,00= 0,2EA kN/m.

Further is S=D2/L1=-4/5=-0,8 and
C=D1/L1= 3/5= 0,6.

There are three different combinations of R, C and S which form, with + or -, the elements of matrix S5. See page .

R*C^2= 0,2EA*(0,6)^2 = 0,072EA
R*S*C= 0,2EA*(-0,8)(0,6)=-0,096EA
R*S^2= 0,2EA*(-0,8)^2 = 0,128EA

Fig.3a.
Member 1 is drawn with at member end 1 the member end forces F13X and F13Y with directions as assumed.
After F follows the number of the member end on which the force acts, after the number of the other member end. And finally X for horizontal and Y for vertical.
At member end 3 are drawn the member end forces F13X and F31Y with ndirections as assumed, to the right and upward.

Fig.3b.
Here is given the relation in matrix form between member end forces and displacements of member 1 by means of stiffness matrix S51.
The three terms calculated above have been multiplied by 1000 and divided by EA. Therefore before the matrix EA/1000.

Member 2.

Fig.4a en 4b.
D1=X1(H)-X1(L)=X1(3)-X1(2)=3-5=-2
D2=Y1(H)-Y1(L)=Y1(3)-Y1(2)=0-4=-4

L1=SQR(D1^2+D2^2)
=SQR((-2)^2+(-4)^2)=SQR(4+16)=SQR(20)=4,47 m

The member stiffness factor is
R=EA/L1=EA/4,47=0,224EA

Sin(h) is S and Cos(h) is C, see page 3 ,

then become S=D2/L1=-4/4,47=-0,895
C=D1/L1=-2/4,47=-0,447

R*C^2= 0,224EA*(-0,447)^2 =0,045EA
R*S*C= 0,224EA*(-0,895)(-0,447)=0,090EA
R*S^2= 0,224EA*(-0,895)^2 =0,179EA

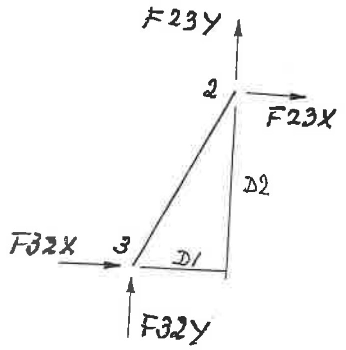


Fig.4a.

Multiplied by 1000 and divided by EA, and and given a + or a -, one finds as on page the the elements of member stiffness matrix S52 as shown on the left.

$$\begin{bmatrix} F23X \\ F23Y \\ F32X \\ F32Y \end{bmatrix} = \frac{EA}{1000} \underbrace{\begin{bmatrix} 45 & 90 & -45 & -90 \\ 90 & 179 & -90 & -179 \\ -45 & -90 & 45 & 90 \\ -90 & -179 & 90 & 179 \end{bmatrix}}_{S52} \cdot \begin{bmatrix} UH2 \\ UV2 \\ UH3 \\ UV3 \end{bmatrix}$$

Fig.4b.

This way the member end forces w.r.t. the construction axes system X-Y are expressed in the member end (joint) displacements w.r.t. the same axes system by means of member stiffnes matrix S52.

Both sets of equations, of member 1 and member 2, can be composed to one set of equations as shown below. (EA/1000 is omitted.)

$$\begin{bmatrix} F13X \\ F13Y \\ . \\ . \\ F31X \\ F31Y \end{bmatrix} = \underbrace{\begin{bmatrix} 72 & -96 & . & . & -72 & 96 \\ -96 & 128 & . & . & 96 & -128 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ -72 & 96 & . & . & \underline{72} & \underline{-96} \\ 96 & -128 & . & . & \underline{-96} & \underline{128} \end{bmatrix}}_{C1} \cdot \begin{bmatrix} UH1 \\ UV1 \\ UH2 \\ UV2 \\ UH3 \\ UV3 \end{bmatrix}$$

$$\begin{bmatrix} . \\ . \\ F23X \\ F23Y \\ F32X \\ F32Y \end{bmatrix} = \underbrace{\begin{bmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & 45 & 90 & -45 & -90 \\ . & . & 90 & 179 & -89 & -179 \\ . & . & -45 & -90 & \underline{45} & \underline{90} \\ . & . & -90 & -170 & \underline{90} & \underline{179} \end{bmatrix}}_{C2} \cdot \begin{bmatrix} UH1 \\ UV1 \\ UH2 \\ UV2 \\ UH3 \\ UV3 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} F13X \\ F13Y \\ F23X \\ F23Y \\ F31X+F32X \\ F31Y+F32Y \end{bmatrix}}_{\underline{f}} = \underbrace{\begin{bmatrix} 72 & -96 & . & . & -72 & 96 \\ -96 & 128 & . & . & 96 & -128 \\ . & . & 45 & 90 & -45 & -90 \\ . & . & 90 & 179 & -90 & -179 \\ -72 & 96 & 45 & -90 & \underline{117} & \underline{-6} \\ 96 & -128 & -90 & -179 & \underline{-6} & \underline{307} \end{bmatrix}}_{CC} \cdot \underbrace{\begin{bmatrix} UH1 \\ UV1 \\ UH2 \\ UV2 \\ UH3 \\ UV3 \end{bmatrix}}_{\underline{u}}$$

The dots indicate zeros.
For joint 3 there are 4 elements elements of both matrices S5 which coincide in construction matrix CC.
For joint 3 there are 4 elements of both matrices S5 which coincide in construction matrix CC.

CC(5,5)= C1(5,5)+ C2(5,5)
=S51(3,3)+S52(3,3)
=72+45=117
CC(5,6)= C1(5,6)+ C2(5,6)
=S51(3,4)+S52(3,4)
=-96+90=-6
CC(6,5)=S51(4,3)+S52(4,3)
=-96+90=-6
CC(6,6)=S51(4,4)+S52(4,4)
=128+179=307

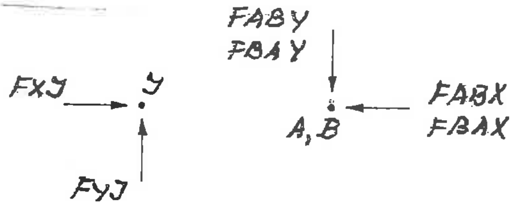


Fig.5.

Fig.5.
The joint load forces of joint I are
FXI, assumption to the right, and
FYI, assumption upward.

(The vertical load is mostly directed downward. One may assume that FYI is also directed downward and just working further with it, also when writing the program.)

joint 1	FX1=0 kN	FY1=0 kN
joint 2	FX2=0 kN	FY2=0 kN
joint 3	FX3=0 kN	FY3=-20 kN

FY3 is not directed as assumed upward, but directed downward, therefore the minus sign.

On the member ends A and B act horizontal member end forces FABX and FBAX, assumption directed to the right, and vertical member end forces FABY and FBAY, assumption directed upward.

On the joints act forces as large but opposite directed forces. From what is stated before then follows that the on the joint acting horizontal member end forces FABX and FBAX as assumption are directed to the left, and the on the joint acting vertical forces FABY and FBAY as assumption are directed downward.

The elements of force vector f is found from the horizontal and vertical equilibrium of the joints.

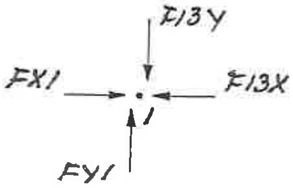


Fig.6a.

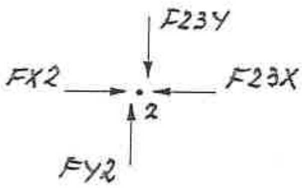


Fig.6b.

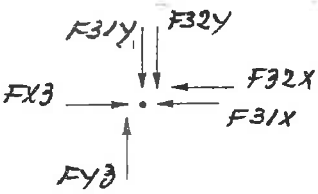


Fig.6c.

Fig.6a.

Σ hor. joint 1 =0	FX1-F13X=0	
	0-F13X=0	F13X=0
Σ vert. joint 1 =0	FY1-F13Y=0	
	0-F13Y=0	F13Y=0

Fig.6b.

Σ hor. joint 2 =0	FX2-F23X=0	
	0-F23X=0	F23X=0
Σ vert. joint 2 =0	FY2-F23Y=0	F23Y=0

Fig.6c.
Σ hor. joint 3 =0
FX3-F31X-F32X=0 0-F31X-F32X=0 F31X+F32X=0

Σ vert. joint 3 =0
FY3-F31Y-F32Y=0 -20-F31Y-F32Y=0 F31Y+F32Y=-20

All forces, ofcourse, in kN.

F13X	0	UH1=0
F13Y	0	UV1=0
F23X	0	UH2=0
F23Y	0	UV2=0
F31X+F32X	0	
F31Y+F32Y	-20	

Fig.7.

Fig.7.
This way the elements of force vector f due to the joint load forces are found.

The horizontal and vertical displacements of joint 1 and 2 are known, prescribed, UH1=0, UV1=0, UH2=0 and UV2=0.

The concerning first four elements of f are also zero and don't need therefore to be altered.

$$\frac{EA}{1000}\begin{bmatrix}1&.&.&.&.&.\\ .&1&.&.&.&.\\ .&.&1&.&.&.\\ .&.&.&1&.&.\\ .&.&.&.&117&-6\\ .&.&.&.&-6&307\end{bmatrix}\cdot\begin{bmatrix}UH1\\UV1\\UH2\\UV2\\UH3\\UV3\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\\-20\end{bmatrix}$$

Fig.8.

Fig.8.
Since the first four displacements are prescribed, the first four equation could be dropped. But to keep the number of equations the same the first four rows and columns of construction matrix CC are filled with zeros and the concerning elements on the main diagonal are made 1. (The programme to be written works with all the six equations.)
Here the two unknowns UH3 and UV3 will be solved from the last two equations.
For convenience EA/1000 is omitted for a while. After solution of the unknowns they will be multiplied by 1000/EA then.

117UH3 -6UV3= 0 1) times 307/6 gives 1')

-6UH3+307UV3=-20 2)

5987UH3-307UV3= 0 1')

2) plus 1') gives

5981UH3=-20 from which UH3=-3,34*10^3,

and subst. in 1) or 2) follows UV3=-65,2*10^3.

And multiplied by 1000/EA become

<u>UH3=-3,34/EA</u> and <u>UV3=-65,2/EA.</u>

Fig.9.
Now all displacements are known for each member the member end forces can be calculated by means of their stiffness matrices S5.

$$\begin{bmatrix}F13X\\F13Y\\F31X\\F31Y\end{bmatrix}=\frac{EA}{1000}\begin{bmatrix}72&-96&-72&96\\-96&128&96&-128\\-72&96&72&-96\\96&-128&-96&128\end{bmatrix}\cdot\begin{bmatrix}UH1\\UV1\\UH3\\UV3\end{bmatrix}$$

S51

Calculation of the member end forces of member 1. UH1=0 and UV1=0.

F13X=(EA/1000)(72UH1-96UV1-72UH3+96UV3)

=(EA/1000)(-72(-3,34/EA)+96(-65,2/EA))

=(EA/1000)(241/EA-6259/EA)

F13X=(EA/1000)(-6019/EA) so that <u>F13X=-6 kN.</u>

A negative answer, thus the force is not as assumed directed to the right but to the left.

F13Y=(EA/1000)(-96UH1+128UV1+96UH3-128UV3)

=(EA/1000)(96(-3,34/EA)-128(-65,2/EA))

=(EA/1000)(-321/EA+8346/EA)

F13Y=(EA/1000)(8025/EA) zodat <u>F13Y= 8 kN.</u>

A positive answer, thus the force is as assumed directed upward.
Further one finds F31X= 6 kN en F31Y= 8 kN.
And for member 2, using fig.4b of page 5 one finds F23X= 6 kN en F23Y= 12 kN
and F32X=-6 kN en F32Y=-12 kN.
The figure shows the forces drawn with their real directions.

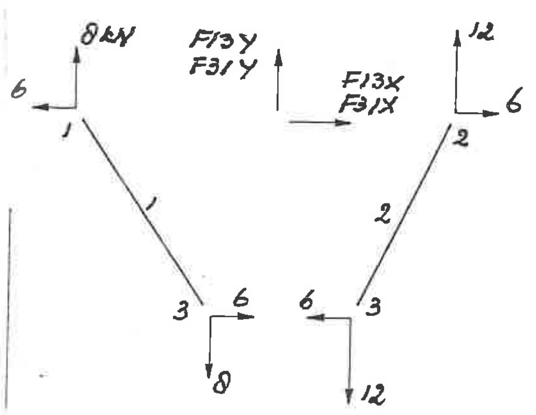


Fig. 9.

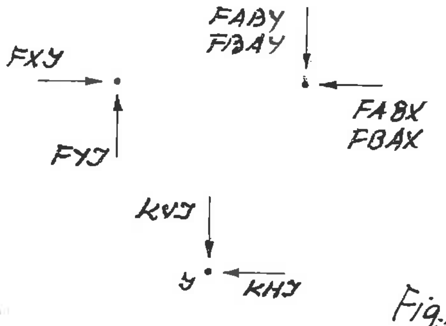


Fig.10.

$$\underline{k} = \begin{bmatrix} KH1 \\ KV1 \\ KH2 \\ KV2 \\ KH3 \\ KV3 \end{bmatrix} \quad \begin{bmatrix} F13X \\ F13Y \\ F23X \\ F23Y \\ F31X+F32X \\ F31Y+F32Y \end{bmatrix} \quad \underline{f} = \begin{bmatrix} -6 \\ 8 \\ 6 \\ 12 \\ 0 \\ -20 \end{bmatrix} =$$

$$C = \begin{bmatrix} 72 & -96 & . & . & -72 & 96 \\ -96 & 128 & . & . & 96 & -128 \\ . & . & 45 & 90 & -45 & -90 \\ . & . & 90 & 179 & -90 & -179 \\ -72 & 96 & 45 & -90 & \underline{117} & \underline{-6} \\ 96 & -128 & -90 & -179 & \underline{-6} & \underline{307} \end{bmatrix} \quad \underline{u} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -3,34/EA \\ -65,2/EA \end{bmatrix}$$

maal EA/1000

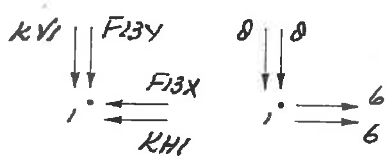


Fig.11a.

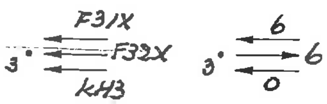


Fig.11b.

Fig.10.
The elements of joint force vector $\underline{k}$, is the total force vector $\underline{f}$, are calculated by means of the original, thus not altered construction stiffness matrix CC.

As assumed on the member ends act horizontal member end forces to the right, and vertical member end forces upward.

On the joints act
horizontal joint forces KHI, assumption to the left, and
vertical joint forces KVI, assumption downward.
A joint force is equal to a row of matrix CC times the displacement vector $\underline{u}$.

UH1=0   UV1=0   UH2=0   UV2=0
UH3=-3,34/EA   UV3=-65,2/EA

Fig.11a.
KH1 is the first row of CC times column $\underline{u}$.

KH1=(EA/1000)(72UH1-96UV1+0UH2+0UV2
-72UH3+96UV3)

=(EA/1000)(-72(-3,34/EA)+96(-65,2/EA))

=(EA/1000)(241/EA-6259/EA)

KH1=(EA/1000)(-6019/EA)   so that   KH1= -6 kN.
There is only one horizontal member end force acting on joint 1, then the joint force equals that member end force, KH1=F13X. Earlier was found F13X= -6 kN, so KH1= -6 kN.

Now without UH1, UV1, UH2 and UV2, they are zero.
KV1 is the second row of CC times column $\underline{u}$.

KV1=(EA/1000)(96UH3-128UV3)

=(EA/1000)(96(-3,34/EA)-128(-65,2/EA)

=(EA/1000)(-321/EA+8346/EA)

KV1=(EA/1000)(8025/EA)   so that   KV1= 8 kN.
KV1=F13Y= 8 kN one will find also this way
KH2=F23X= 6 kN and KV2=F23Y= 12 kN.

Fig.11b.
KH3 is the fifth row of CC times $\underline{u}$.

KH3=(EA/1000)(-72UH1+96UV1-45UH2-90UV2
+117UH3-6UV3)

=(EA/1000)(117(-3,34/EA)-6(-65,2/EA)

=(EA/1000)(-391/EA+391/EA) gives KH3= 0 kN.

On joint 3 act two horizontal member end forces. Joint force KH3 is equal to the sum of these member end forces, KH3=F31X+F32X.
Already found F31X= 6 kN and F32X=-6 kN, then KH3=6-6=0 kN.

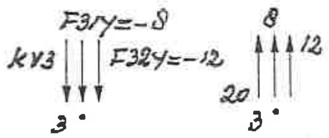


Fig.11c.

Fig.11c.
KV3 is the sixth row of C times column u.

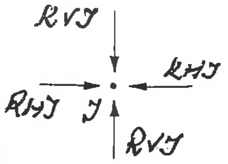
KV3=(EA/1000)(96UH1-128UV1-90UH2-179UV2

-6UH3+307UV3)

=(EA/1000)(-6(-3,34/EA)+307(-65,2/EA))

=(EA/1000)(20/EA-20016/EA)

KV3=(EA/1000)(-19996/EA) so that KV3=-20 kN.



Joint force KV3 is equal to the sum of the member end forces acting on the joint,
KV3=F31Y+F32.
Found was F31Y=-8 kN and F32Y=-12 kN then is KV3=-8+(-12)=-20 kN. A negative answer, thus the force is not directed downward as assumed but upward.

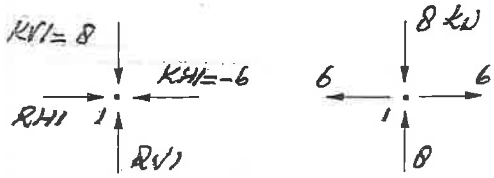


Fig.12a.

Calculation of the reactions.
Fig.12a.
The horizontal reachtion RHI at a joint I is assumed to be directed to the right,
the vertical reaction RVI at a joiunt I is assumed to be directed upward.
By means of the joint forces (which in fact replace/represent the on the joint acting member end forces) the reactions are calculated.

Found was KH1=-6 kN and KV1= 8 kN.

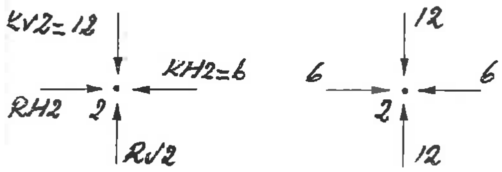


Fig.12b.

Σ hor. joint 1 =0
RH1-KH1=0 or RH1-(-6)=0 so that RH1=-6 kN.
A negative answer, the reaction force is not directed to the right as assumed, but to the left.
Σ vert. joint 1 =0
RV1-KV1=0 of RV1-8=0 so that RV1= 8 kN.
A positive answer, the vertical reaction is directed to the as assumed.

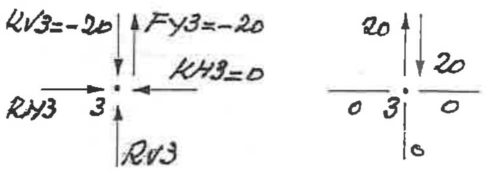


Fig.12c.

Fig.12b.
Found was KH2= 6 kN and KV2=12 kN.

Σ hor. joint 2 =0
RH2-KH2=0 or RH2-6=0 so that RH2= 6 kN.
A positive answer, the reaction is directed to the right.
Σ vert. joint 2 =0
RV2-KV2=0 or RV2-12=0 so that RV2=12 kN.
This reaction is directed upward as assumed.

Fig.12c.
Found was KH3= 0 kN and KV3=-20 kN.
Σ hor. joint 3 =0
RH3-KH3=0 or RH3-0=0 so that RH3= 0 kN.
Zero, correct, this reaction cannot exist.
Σ vert. joint 3 =0
There's also the joint load force FY3=-20 kN.
RV3-KV3+FY3=0 or
RV3-(-20)+(-20)=0 so that RV3= 0 kN.
There's indeed no vertical reaction.

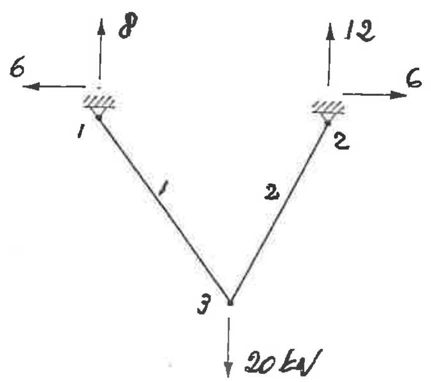


Fig.13.

Fig.13
The figure shows the reactions drawn with their real directions.

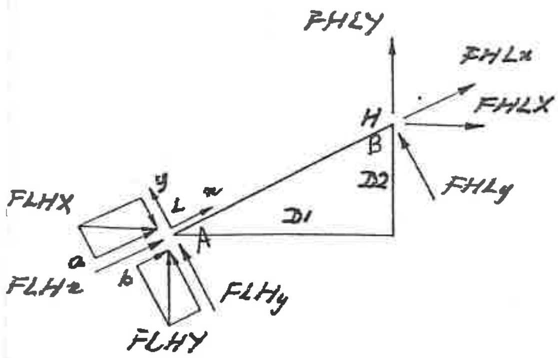


Fig.14.

2.2. Calculation of the member end forces w.r.t. the member axes system x-y.

Fig.14.
As on page 4 the lowest member end number L is assumed at A, the highest member end number H at B.

The member end forces FLHx and FHLx are directed according the x-axis from L to H.
The member end forces FLHy and FHLy perpendicular to the member are directed according the y-axis of the member axes system x-y.

The components along the member of the member end forces FLHX and FLHY , at member end L, follow using similarity of triangles.

a/FLHX=D1/L1=C ⇒ a=FLHX*C
b/FLHY=D2/L1=S ⇒ b=FLHY*S so that

FLHx = FLHX*C + FLHY*S.

Think of Cos(h)=D1/L1 and Sin(h)=D2/L1.

Next the components of FLHX and FLHY at member end L.

c/FLHX=D2/L1=S ⇒ c=FLHX*S
d/FLHY=D1/L1=C ⇒ d=FLHY*C so that

FLHy =-FLHX*S + FLHY*C.

In the same way one finds at member end H
FHLx = FHLX*C + FHLY*S and

FHLy =-FHLX*S + FHLY*C.

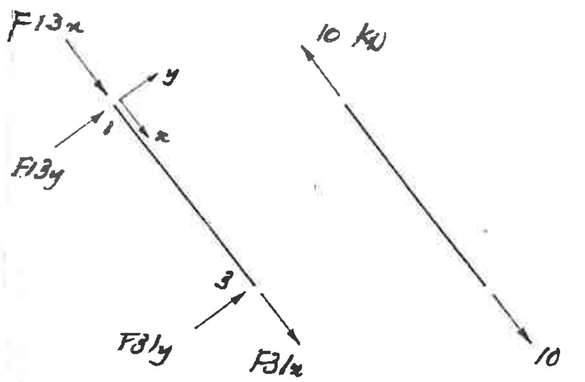


Fig.15a.

Member 1. S=-0,8 and C=0,6.
Fig.15a.
F13X=-6 F13Y= 8 F31X= 6 F31Y=-8 kN

F13x=F13X*C+F13Y*S
=(-6)(0,6)+8(-0,8)=-3,6-6,4=-10 kN

A negative answer, the force does not push on member end 1 but pulls at it.

F13y=-F13X*S+F13Y*C
=-(-6)(-0,8)+8(0,6)=-4,8+4,8= 0 kN

F31x=F31X*C+F31Y*S
=6(0,6)+(-8)(-0,8)=3,6+4,0= 10 kN

A positive answer, the force pulls at member end 3 as assumed.

F31y=-F31X*S+F31Y*C
=-6(-0,8)+(-8)(0,6)=4,8-4,8=0 kN

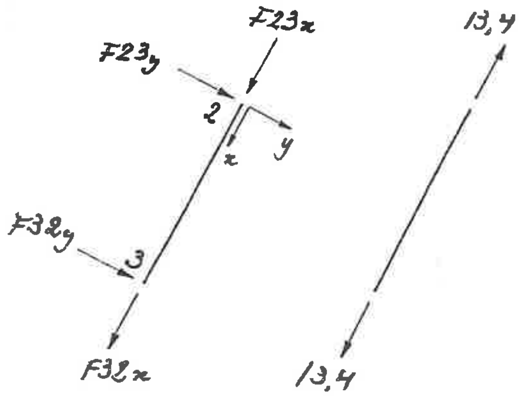


Fig.15b.

Member 2. S=-0895 and C=-0,447
Fig.15b.
F23X= 6 F23Y=12 F32X=-6 F32Y=-12 kN

F23x=6(-0,447)+12(-0,895)=-2,7-10,7=-13,4 kN

F32x=-6(-0,447)+(-12)(-0,895)=2,7+10,7=13,4 kN

For the member end forces perpendicular to the member one finds F23y=0 and F32y=0.

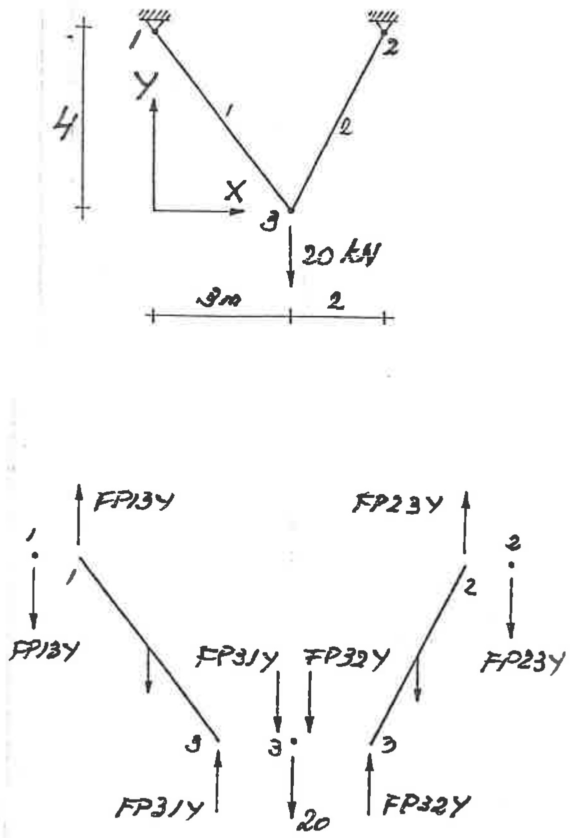


Fig. 16.

$$\begin{bmatrix} F13X \\ F13Y \\ F23X \\ F23Y \\ F31X+F32X \\ F31Y+F32Y \end{bmatrix} \quad \begin{bmatrix} FX1 \\ FY1-FP13Y \\ FX2 \\ FY2-FP23Y \\ FX3 \\ FY3-FP31Y-FP32Y \end{bmatrix}$$

f

$$\begin{bmatrix} 0 \\ -0,75 \\ 0 \\ -0,67 \\ 0 \\ -21,42 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -21,42 \end{bmatrix}$$

f

Fig.17.

2.3. Primary forces due to own weight.

Fig.16.
The same construction. For the members is now assumed an own weight of 0,3 kN/m. Is is a vertical downward directed uniformly distributed load. By this arise on the member ends of the loosened member vertical upward direced member-end forces.

For member 1 with length 5,00 m
forces of (5,00*0,3)/2=0,75 kN, and

for member 2 with length 4,47 m
forces of (4,47*0,3)/2=0,67 kN.

On the joints these forces are directed downward, opposite to the assumed direction of the on the joints acting joint load forces and opposite to the assumed direction of the on the joints acting primary forces. Then follow

for joint 1 FP13Y=-0,75 kN,

for joint 2 FP23Y=-0,67 kN, and

for joint 3 FP31Y=-0,75 kN and FP32Y=-0,67 kN.

The joint load forces are

FX1=0   FY1=0   FX2=0   FY2=0   FX3=0   FY3=-20 kN.

Fig.17.
Force vector f is filled with joint load forces which follow with Σ hor. =0 and Σ vert. =0.

Now the primary forces are added.

F13X=FX1 = 0 kN

F13Y=FY1+FP13Y=0-0,75 = -0,75 kN

F23X=FX2 = 0 kN

F23Y=FY2+FP23Y=0-0,67 = -0,67 kN

F31X+F32X=FX3 = 0 kN

F31Y+F32Y=FY3+FP31Y+FP32Y
=-20-0,75 -0,67=-21,42 kN

The first four displacements are prescribed and equal zero. To keep the number of equations the same row 1 to 4, and column 1 to 4, of matrix CC are filled with zeros. The four concerning elements on the main diagonal are made 1.
The elements 1 to 4 of force vector f are given the values of the prescribed displacements.
(Done for later programming.)

117UH3 -6UV3= 0 *1000
-6UH3+307UV3=-21,42 *1000 after solving

UH3=-3,58/EA and UV3=-69,8/EA.

(With GAUSS part **12** page 10,
UH3=-3,58/EA and UV3=69,84/EA.)

$$\begin{bmatrix} F13X \\ F13Y \\ F31X \\ F31Y \end{bmatrix} = \begin{bmatrix} 72 & -96 & -72 & 96 \\ -96 & 128 & 96 & -128 \\ -72 & 96 & 72 & -96 \\ 96 & -128 & -96 & 128 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ -3,58/EA \\ -69,8/EA \end{bmatrix}$$

times EA/1000

$\underline{f}$ CC $\underline{u}$

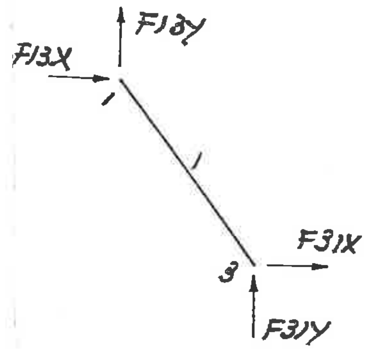


Fig.18a.

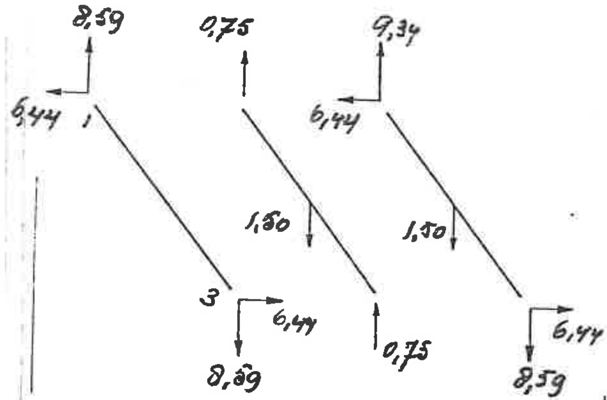


Fig.18b.

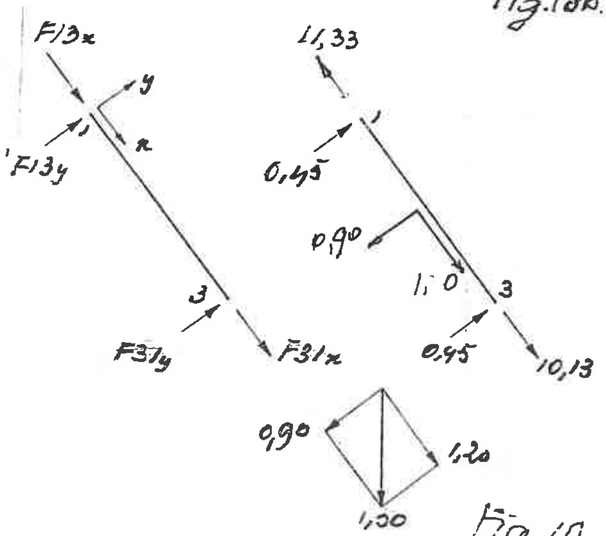


Fig. 19.

Now when all displacements are known for each member the member end forces can be calculated with $\underline{f}$ = S5 $\underline{u}$

Fig.18a.
Member 1.
UH1 and UV1 are zero and are omitted in the equations.

F13X=(EA/1000)(-72UH3+96UV3)

=(EA/1000)(-72(-3,58/EA)+96(-69,8/EA))

=(EA/1000)(258/EA-6701/EA)

F13X=(EA/1000)(-6443/EA) so that F13X=-6,44 kN.

F13Y=(EA/1000)(96UH3-128UV3) F13Y= 8,59 kN

F31X=(EA/1000)(72UH3-96UV3) F31X= 6,44 kN

F31Y=(EA/1000)(-96UH3+128UV3) F31Y=-8,59 kN

These are the member end forces due to the displacements alone. The resultant of both forces at a member end falls along the member axis. (8,59 divided by 4 times 3 is 6,44.)

Fig.18a en 18b.
One obtaines the final member end forces by adding up the primary forces to the just calculated member end forces.
As said before the assumed direction of a member end force is upward.

F13Y becomes F13Y-FP13Y= 8,59-(-0,75)= 9,34 kN.
F31Y becomes F31Y-FP31Y=-8,59-(-0,75)=-7,84 kN.

Σ hor. =0 6,44-6,44=0
Σ vert. =0 9,34-1,50-7,84=0
Σ mom. member end 1 =0
1,50*1,5+7,84*3-6,44*4=2,25=23,52-25,76=0,01≈0

Calculation of the member end forces w.r.t. the member axis system x-y, including own weight.

Fig.19.
S=-0,8 C=0,6
FLHx= FLHX*C+FLHY*S
F13x= F13X*C+F13Y*S
= (-6,44)(0,6)+(9,34)(-0,8)=-11,33 kN

FLHy=-FLHX*S+FLHY*C
F13y=-F13X*S+F13Y*C
=-(-6,44)(-0,8)+(9,34)(0,6)= 0,45 kN

FHLx= FHLX*C+FHLY*S
F31x= F31X*C+F31Y*S
= (6,44)(0,6)+(-7,84)(-0,8)= 10,13 kN

FHLy=-FHLX*S+FHLY*C
F31Y=-F31X*S+F31Y*C
=-(6,44)(-0,8)+(-7,84)(0,6)= 0,45 kN
At the member on the right the member end forces are drawn with their real direction.
The resultant of the distributed load is 5,00 times 0,3 is 1,50 kN.

Fig.20a.
Member 2.
UH2 and UV2 are zero and are omitted in the equations.

$$\begin{bmatrix} F23X \\ F23Y \\ F32X \\ F32Y \end{bmatrix} = \begin{bmatrix} 45 & 90 & -45 & -90 \\ 90 & 179 & -90 & -179 \\ -45 & -90 & 45 & 90 \\ -90 & -179 & 90 & 179 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ -3,58/EA \\ -69,8/EA \end{bmatrix}$$

F23X=(EA/1000)(-45UH3-90UH4)

=(EA/1000)(-45(-3,58/EA)-90(-69,8/EA))

=(EA/1000)(161/EA+6282/EA)

F23X=(EA/1000)(6443/EA) so that F23X= 6,44 kN.

F23Y=(EA/1000)(-90UH3-179UV3) F23Y= 12,82 kN

F32X=(EA/1000)(45UH3+90UV3) F32X= -6,44 kN

F32Y=(EA/1000)(90UH#+179UV3) F32Y=-12,82 kN

These are the member end froces due to the displacements alone. The resultants at the member ends fall along the member axis.
(12,82 gedeeld door 4 maal 2 is 6,41≈6,44.)

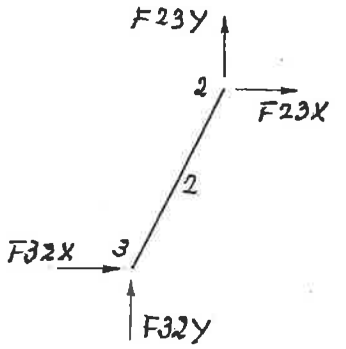


Fig.20a.

Fig.20a en 20b.
The final member end forces are obtained by adding up to them the primary forces.

FP23Y=-0,67 kN and FP32Y=-0,67 kN.

F23Y becomes F23Y-FP23Y= 12,82-(-0,67)=13,49kN.

F32Y wordt F32Y-FP32Y=-12,82-(-0,67)=-12,15 kN.

Σ hor. =0 6,44-6,44=0
Σ vert. =0 13,49-1,34-12,15=0
Σ mom. member end 3 =0
1,34*1+6,44*4-13,49*2=27,10-26,98=0,12≈0

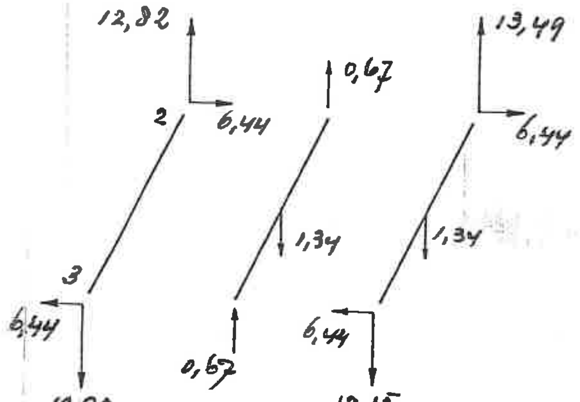


Fig.20b.

Calculation of the member end forces w.r.t. the member axis sytem x-y, including own weight.

Fig.21.
S=-0,895 C=-0,447
FLHx= FLHX*C+FLHY*S
F23x= F23X*C+F23Y*S
= (6,44)(-0,447)+(13,49)(-0,895) =-14,95 kN

FLHy=-FLHX*S+FLHY*C
F23y=-F23X*S+F23Y*C
=-(6,44)(-0,859)+(13,49)(-0,447) = -0,27 kN

FHLx= FHLX*C+FHLY*S
F32x= F32X*C+F32Y*S
=(-6,44)(-0,447)+(-12,15)(-0,895)= 13,75 kN

FHLy=-FHLX*S+FHLY*C
F32Y=-F32X*S+F32Y*C
=-(-6,44)(-0,895)+(-12,15)(-0,447)=-0,33 kN

(F23y is not as large as F32y, they are small compaired with F23x and F32x.)

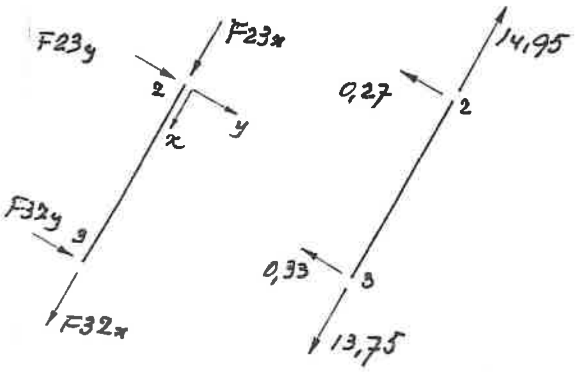


Fig.21.

72	-96	.	.	-72	96		0
-96	128	.	.	96	-128		0
.	.	45	90	-45	-90		0
.	.	90	179	-90	-179	.	0
-72	96	45	-90	117	-6		-3,58/EA
96	-128	-90	-179	-6	307		-69,8/EA

times EA/1000

C u

Calculation of the joint forces KHI and KVI by means of the original construction matrix CC.
See page 8 .
Zero multiplications are omitted.
KH1 is the first row of CC times u.

KH1=(EA/1000)(-72UH3-96UV3)
=(EA/1000)(-72(-3,58/EA)-96(-69,8)/EA))
=(EA/1000)(258/EA-6701/EA) = -6,44 kN
KV1=(EA/1000)(96UH3-128UV3)
=(EA/1000)(96(-3,58/EA)-128(-69,8/EA))
=(EA/1000)(-344/EA+8934/EA)= 8,59 kN

These are the joint forces KHI and KVI due to the displacements alone.
KHI does not change because there is no primary force FP13X.
The vertical primary force is FP13Y=-0,75 kN.
KV1 becomes KV1-FP13Y= 8,59-(-0,75)= 9,34 kN, and is eqaual to the earlier found final force F13Y= 9,34 kN.

KH2=(EA/1000)(-45(-3,58/EA)-90(-69,8)/EA))
=(EA/1000)(161/EA+6282/EA) = 6,44 kN
KV2=(EA/1000)(-90(-3,58/EA)-179(-69,8/EA))
=(EA/1000)(322/EA+12494/EA)= 12,82 kN
KH2 does not change..

KV2 becomes KV2-FP23Y= 12,82-(-0,67)= 13,49 kN, and is equal to the earlier found final force F23Y= 13,49 kN.

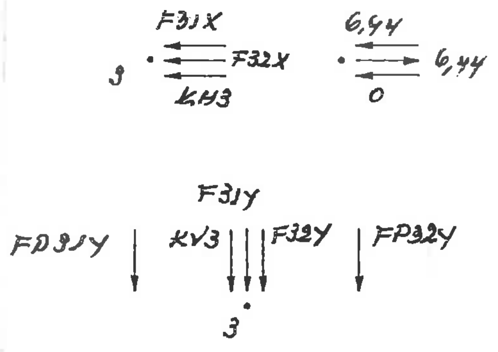


Fig. 22.

Fig.22.
KH3=(EA/1000)(117(-3,58/EA)-6(-69,8/EA))
=(EA/1000)(-419/EA+419/EA) = 0 kN and is equal to F31X+F32X=6,44-6,44=0.

KV3=(EA/1000)(-6(-3,58/EA)+307(-69,8)/EA))
=(EA/1000)(21/EA-21429/EA) =-21,41 kN

KV3 becomes KV3-FP31Y-FP32Y
-21,41-(-0,75)-(-0,67)=-19,99 kN and is equal to F31Y+F32Y=-7,84-12,15=-19,99 kN.

Calculation of the reactions.

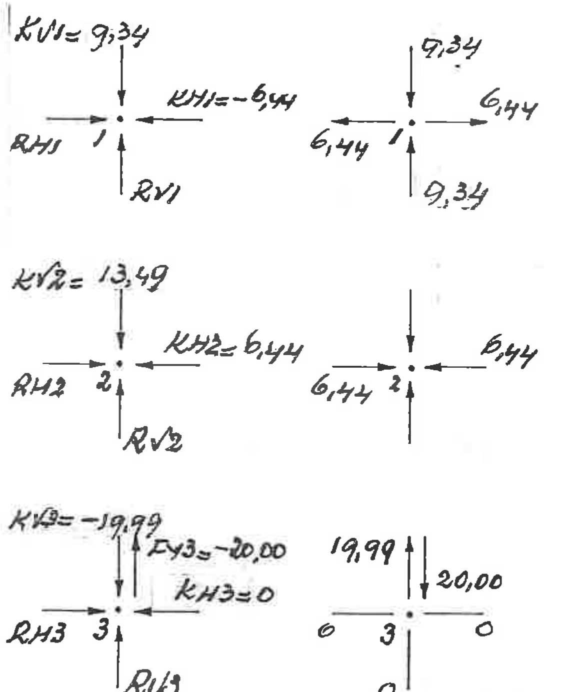


Fig.23.

Fig.23.
On the left the joint forces KHI and KVI are drawn with their assumed directions and their calculated values are added. Further the reactions RHI and RVI are drawn with their assumed directions. Notice FY3.

Σ hor. joint 1 =0
RH1-KH1=0 RH1=KH1=-6,44 kN
Σ vert. joint 2 =0
RV1-KV1=0 RV1=KV1= 9,34 kN
Σ hor. joint 2 =0
RH2-KH2=0 RH2=KH2= 6,44 kN
Σ vert. joint 2 =0
RV2-KV2=0 RV2=KV2=13,49 kN
Σ hor. joint 3 =0
RH3-KH3=0 RH3=KH3 0 kN
Σ vert. joint 3 =0
RV3-KV3+FY3=0
RV3=KV3-FY3=-19,99-(-20,00)= 0,01≈0 kN

On the right the forces on the joints are drawn with their real directions.

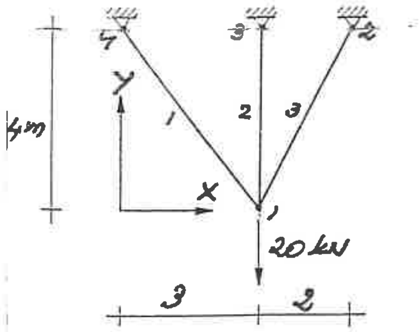


Fig.1.

$$\begin{bmatrix} F14X \\ F14Y \\ F41X \\ F41Y \end{bmatrix} = \frac{EA}{1000} \begin{bmatrix} 72 & -96 & -72 & 96 \\ -96 & 128 & 96 & -128 \\ -72 & 96 & 72 & -96 \\ 96 & -128 & -96 & 128 \end{bmatrix} \cdot \begin{bmatrix} UH1 \\ UV1 \\ UH4 \\ UV4 \end{bmatrix}$$

member 1

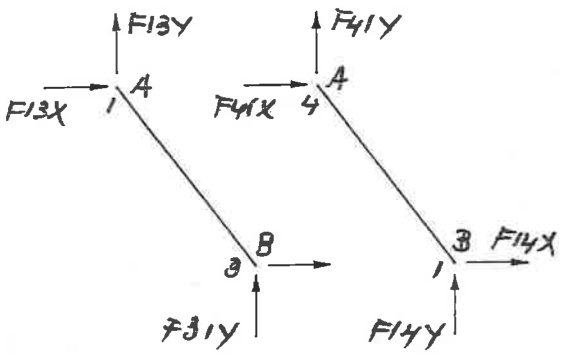


Fig.2.

$$\begin{bmatrix} F12X \\ F12Y \\ F21X \\ F21Y \end{bmatrix} = \frac{EA}{1000} \begin{bmatrix} 45 & 90 & -45 & -90 \\ 90 & 179 & -90 & -179 \\ -45 & -90 & 45 & 90 \\ -90 & -179 & 90 & 179 \end{bmatrix} \cdot \begin{bmatrix} UH1 \\ UV1 \\ UH2 \\ UV2 \end{bmatrix}$$

member 3

Fig.3.

$$\begin{bmatrix} F13X \\ F13Y \\ F31X \\ F31Y \end{bmatrix} = \frac{EA}{1000} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 250 & 0 & -250 \\ 0 & 0 & 0 & 0 \\ 0 & -250 & 0 & 250 \end{bmatrix} \cdot \begin{bmatrix} UH1 \\ UV1 \\ UH3 \\ UV3 \end{bmatrix}$$

member 2

Fig.4.

Example.

Fig.1.
A statically indeterminate member construction. A member is added to the previous example. Own weight is not considered.
The joint coordinates are

X1(1)=3 X1(2)=5 X1(3)=3 X1(4)=0 m
Y1(1)=0 Y1(2)=4 Y1(3)=4 Y1(4)=4 m.

Member 1.
Fig.2.

```
D1=X1(H)-X1(L)=X1(4)-X1(1)=0-3=-3 m
D2=Y1(H)-Y1(L)=Y1(4)-Y1(1)=4-0= 4 m
L1=Sqr(D1^2+D2^2)
  =Sqr((-3)^2+4^2)=Sqr(9+16)=Sqr(25)=5,00 m
```

The member stiffness factor is

```
R=EA/5,00= 0,2EA kN/m.
S=D2/L1= 4/5= 0,8
C=D1/L1=-3/5=-0,6

R*C^2= 0,2EA*(-0,6)^2    = 0,072EA
R*S*C= 0,2EA*(0,8)(-0,6)=-0,096EA
R*S^2= 0,2EA*(0,8)^2     = 0,128EA
```

One finds the same member stiffness matrix as on page . Plus and minus sign of S and C are exchanged but the values of the elements are the same because of the squares of S and C, and S times C.
The relation f = S5 u between member end forces and displacements is the same, ofcourse.
Because of the way of numbering the member end forces have other names, and UH3 and UV3 have become now UH4 and UV4.
For member 1 of the previous example was

F13X=(EA/1000)(72UH1-96UV1-72UH3+96UV3)

and with A and B

FABX=(EA/1000)(72UHA-96UVA -72UHB+96UVB). 1)

For member 1 now follows for the 'same' force

F41X=(EA/1000)(-72UH1+96UV1+72UH4-96UV4) or

FABX=(EA/1000)(-72UHB+96UVB +72UHA-96UVA). 2)

1) and 2) are equal, F13X=F41X, and so on.

Fig.3.
For member 3 one finds the same relation as for member 2 of page 13 .

The vertical member 2.
Fig.4.

```
D1=X1(H)-X1(L)=X1(3)-X1(1)=3-3=0 m
D2=Y1(H)-Y1(L)=Y1(3)-Y1(1)=4-0=4 m
L1=Sqr(D1^2+D2^2)=Sqr(0^2+4^2)=Sqr(16)=4 m
R=EA/4= 0,250EA kN/m
S=D2/L1=4/4=1
C=D1/L1=0/4=0

R*C^2= 0,250EA*(0)^2 = 0
R*S*C= 0,250EA*(1)(0)= 0
R*S^2= 0,250EA*(1)^2 = 0,250EA
```

$$\begin{bmatrix} F12X \\ F12Y \end{bmatrix} = \begin{bmatrix} 45 & 90 \\ 90 & 179 \end{bmatrix} \cdot \begin{bmatrix} UH1 \\ UV1 \end{bmatrix}$$

$$\begin{bmatrix} F13X \\ F13Y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 250 \end{bmatrix} \cdot \begin{bmatrix} UH1 \\ UV1 \end{bmatrix}$$

$$\begin{bmatrix} F14X \\ F14Y \end{bmatrix} = \begin{bmatrix} 72 & -96 \\ -96 & 128 \end{bmatrix} \cdot \begin{bmatrix} UH1 \\ UV1 \end{bmatrix}$$

Fig.5.
The three sets of four equations can be composed to one single set of eight equations.

At joint 1 three members meet. Therefore four elements of the three member matrices S5 coincide in construction matrix CC.
They are the four elements of the left upper corners of the S5's which with the displacements UH1 and UV1 of joint 1 deliver their contribution to the member end forces of the members with member end 1.

```
CC(1,1)=  45+   0 +  72= 117
CC(1,2)=  90+   0 -  96=  -6
CC(2,1)=  90+   0 -  96=  -6
CC(2,2)= 179+ 250 + 128= 557
```

$$\underset{\underline{f}}{\begin{bmatrix} F12X+F13X+F14X \\ F12Y+F13Y+F14Y \\ F21X \\ F21Y \\ F31X \\ F31Y \\ F41X \\ F41Y \end{bmatrix}} = \frac{EA}{1000} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix} \underset{C}{\overset{\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix}}{\begin{bmatrix} \underline{117} & \underline{-6} & -45 & -90 & 0 & 0 & -72 & 96 \\ \underline{-6} & \underline{557} & -90 & -179 & 0 & -250 & 96 & -128 \\ -45 & -90 & 45 & 90 & . & . & . & . \\ -90 & -179 & 90 & 179 & . & . & . & . \\ 0 & 0 & . & . & 0 & 0 & . & . \\ 0 & -250 & . & . & 0 & 250 & . & . \\ -72 & 96 & . & . & . & . & 72 & -96 \\ 96 & -128 & . & . & . & . & -96 & 128 \end{bmatrix}}} \cdot \underset{\underline{u}}{\begin{bmatrix} UH1 \\ UV1 \\ UH2 \\ UV2 \\ UH3 \\ UV3 \\ UH4 \\ UV4 \end{bmatrix}} \quad \underset{\underline{f}}{\begin{bmatrix} 0 \\ -20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}$$

Fig.5.

The force vector is filled with joint load forces which are all zero exept the vertical one, FY1=-20 kN, the second element of force vector $\underline{f}$.
The displacements of the joints 2, 3 and 4 are prescribed and equal zero. The rows and columns 3 to 8 are filled with zeros and the diagonal elements CC(3,3) to CC(8,8) are made zero. (Also the zero of CC(5,5) becomes a 1.)
Force vector $\underline{f}$ does not change because the prescribed displacements are zero, the elements 3 to 8 are zero. Left 2 eq.
(EA/1000)(117UH1- 6UV1)= 0 *1000
(EA/1000)(-6UH1+557UV1)=-20 *1000
Solving 'by hand' gives

<u>UH1=-1,84/EA</u> and <u>UV1=-35,9/EA</u>.

(With GAUSS part **12** page 10,
UH1=-1,84/EA and UV1=-35,93/EA.)

```
F12X=-3,31 kN     F12Y=-6,59 kN
F21X= 3,31 kN     F21Y= 6,59 kN

F13X= 0    kN     F13Y=-8,98 kN
F31X= 0    kN     F31Y= 8,98 kN

F14X= 3,31 kN     F14Y=-4,42 kN
F41X=-3,31 kN     F41Y= 4,42 kN
```

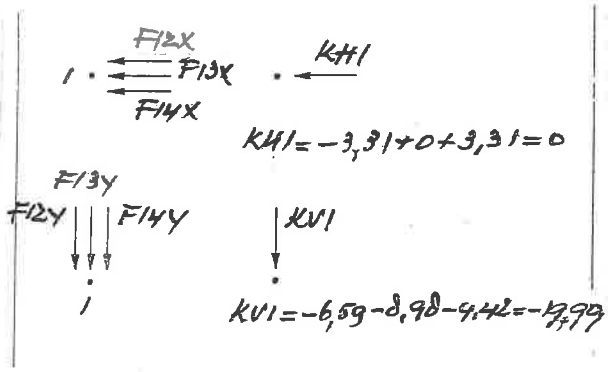


Fig.6.

Fig.6.
The joint forces at joint 1.
KH1=F12X+F13X+F14X=-3,31+0 +3,31= 0 kN
KV1=F12Y+F13Y+F14Y=-6,59-8,98-4,42=-19,99 kN,
a negative answer, so not directed downward as assumed but upward, and thus in equilibrium with the downward directed joint load force of 20,00 kN.

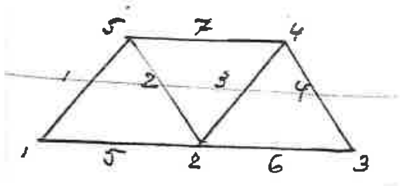


Fig.1.

```
Private Sub CONSTRMATCCTRUSS()
N=2*N9
For I=1 To N : For J=1 To N
CC(I,J)=0 : Next J : Next I
For P=1 To P9 : L=LL(P) : H=HH(P)
EA=EAA(P)
MEMBERMATS5TRUSS
TT(1)=2*L-1 : TT(2)=2*L
TT(3)=2*H-1 : TT(4)=2*H
For I=1 To 4 : I1=TT(I)
For J=1 To 4 : J1=TT(J)
CC(I1,J1)=CC(I1,J1)+S5(I,J)
Next J
Next I
Next P

End Sub
```

	1								5	
	1	2	3	4	5	6	7	8	9	10
1	.	.							.	.
2	.	.							.	.
3										
4										
5										
6										
7										
8										
9	.	.							.	.
10	.	.							.	.

C

	J= 1	2	3	4
I=1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

S5 of member 1

	1		5	
	J1= 1	2	9	10
I1=1	1,1	1,2	1,9	1,10
2	2,1	2,2	2,9	2,10
9	9,1	9,2	9,9	9,10
10	10,1	10,2	10,9	10,10

	1	5
1	[1,1]	[1,5]
5	[5,1]	[5,5]

Fig.2.

2.4. Private Sub CONSTRMATCCTRUSS()

Fig.1.
This truss has N9=5 joints so that the number of equations is N=2*N9=2*5=10.
The number of members is P9=5
Joints and members are numbered in arbitrary order.
First the elements of construction matrix CC are set zero.
For I=1 To N : For J=1 To N
CC(I,J)=0 : Next J : Next I
For each member P with
lowest member end number L=LL(P) and
highest member end number H=HH(P)
the strain stiffness EA=EAA(P).

With the subroutine
MEMBERMATS5TRUSS (page 20) the elements of member stiffness matrix S5 are determined which will be taken in matrix CC after that.
De rijnummers van C zijn I1=T(I) en
de kolomnummers van C zijn J1=T(J) die bepaald worden met
TT(1)=2*L-1 : TT(2)=2*L and
TT(3)=2*H-1 : TT(4)=2*H. With
CC(I1,J1)=CC(I1,J1)+S5(I,J) the elements of CC are determined. The new CC(I1,J1) is the old CC(I1,J1) plus S5(I,J).

Fig.2.
For member P=1 with
L=LL(P)=LL(1)=1 and H=HH(P)=HH(1)=5 become
TT(1)=2*1-1= 2-1=1 and TT(2)=2*1=2,
TT(3)=2*5-1=10-1=9 and TT(4)=2*5=10.

The first row of S5 to the first row of CC.

```
I=1   I1=TT(I)=TT(1)=1
J=1   J1=TT(J)=TT(1)=1    CC(1,1)  =  0  +S5(1,1)
J=2   J1=TT(J)=TT(2)=2    CC(1,2)  =  0  +S5(1,2)
J=3   J1=TT(J)=TT(3)=9    CC(1,9)  =  0  +S5(1,3)
J=4   J1=TT(J)=TT(4)=10   CC(1,10)=   0  +S5(1,4)
```

The second row of S5 to the second row of CC.

```
I=2   I1=TT(I)=TT(2)=2
J=1   J1=TT(1)=1          CC(2,1)  =  0  +S5(2,1)
J=2   J1=TT(2)=2          CC(2,2)  =  0  +S5(2,2)
J=3   J1=TT(3)=9          CC(2,9)  =  0  +S5(2,3)
J=4   J1=TT(4)=10         CC(2,10)=   0  +S5(2,4)
```

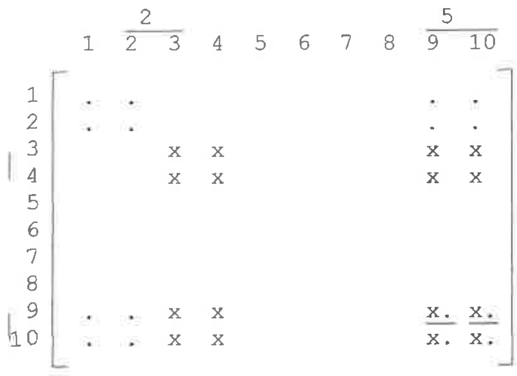
The third row of S5 to the nineth row of CC.

```
I=3   I1=TT(I)=TT(3)=9
J=1   J1=TT(1)=1          CC(9,1)  =  0  +S5(3,1)
J=2   J1=TT(2)=2          CC(9,2)  =  0  +S5(3,2)
J=3   J1=TT(3)=9          CC(9,9)  =  0  +S5(3,3)
J=4   J1=TT(4)=10         CC(9,10)=   0  +S5(3,4)
```

The fourth row of S5 to the tenth row of CC.

```
I=4   I1=TT(I)=TT(4)=10
J=1   J1=1               CC(10,1)  =  0  +S5(4,1)
J=2   J1=2               CC(10,2)  =  0  +S5(4,2)
J=3   J1=9               CC(10,9)  =  0  +S5(4,3)
J=4   J1=10              CC(10,10)=   0  +S5(4,4)
```

Matrix S5 can be divided in four submatrices and indicate them as 'elements' with the member end numbers [1,1], [1,5], [5,1] and [5,5]. One sees at once where the arrive in matrix CC.

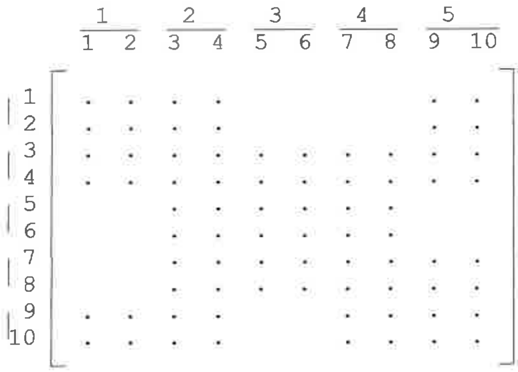


CC

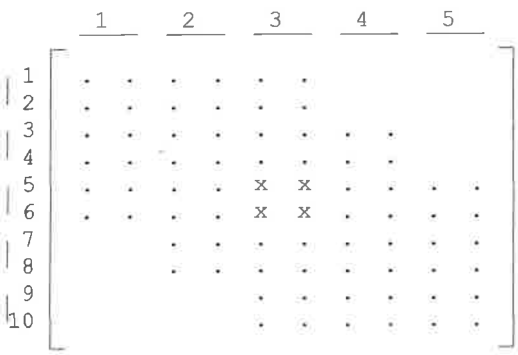
	3 (2)	4 (2)	9 (5)	10 (5)
3	3,3	3,4	3,9	3,10
4	4,3	4,4	4,9	4,10
9	9,3	9,4	9,9	9,10
10	10,3	10,4	10,9	10,10

member 2

	2	5
2	[2,2]	[2,5]
5	[5,2]	[5,5]



CC



CC Fig.5.

Fig.3.
For member P=2 is L=LL(2)=2 and H=HH(2)=5.
Then become

TT(1)=2*L-1=2*2-1=3, TT(2)=2*L=2*2=4,
TT(3)=2*H-1=2*5-1=9 and TT(4)=2*H=2*5=10.

The first row of S5 to matrix CC.
CC(I1,J1)=CC(I1,J1)+S5(I,J)
I=1 I1=TT(I)=TT(1)=3
J=1 J1=TT(J)=TT(1)=3 CC(3,3) = 0 +S5(1,1)
J=2 J1=TT(J)=TT(2)=4 CC(3,4) = 0 +S5(1,2)
J=3 J1=TT(J)=TT(3)=9 CC(3,9)= 0 +S5(1,3)
J=4 J1=TT(J)=TT(4)=10 CC(3,10)= 0 +S5(1,4)
The first row of S5 comes in the third row of CC.

The second row of S5 to matrix CC.
I=2 I1=TT(I)=TT(2)=4 and so on.

The third row of S5 to matrix CC.
I=3 I1=TT(I)=TT(3)=9
J=1 J1=TT(J)=TT(1)=3 CC(9,3) = 0 +S5(3,1)
J=2 J1=TT(J)=TT(2)=4 CC(9,4) = 0 +S5(3,2)
J=3 J1=TT(J)=TT(3)=9 CC(9,9) =C(9,9) +S5(3,3)
J=4 J1=TT(J)=TT(4)=10 CC(9,10)=C(9,10)+S5(3,4)
The last two elements had already got a value of matrix S5 of member 1, the two underlined elements.

The fourth row of S5 to matrix CC.
I=4 I1=TT(I)=TT(4)=10
The fourth row of S5 comes in the tenth row of CC.
One sees that the submatrices [2,2] and [5,5] of the main diagonal of the S5 of member P=2 come on the main diagonal of matrix CC. nd that applies for each member.
Since member 1 and member 2 have the same joint number their submatrices [5,5] coincide on the main diagonal in matrix CC.
Member 7 as well has a member end number 5 so that also that submatrix [5,5] coincides with that of member 1 and 2 in matrix CC.
Submatrices which are not lying on the main diagonals of the S5's, are lying outside the main diagonal of C and never coincide.
The way of member numbering determines the order in which the member matrices S5 come in matrix CC.

Fig.4.
Construction matrix CC looks like this when all matrices S5 of the 7 members have give place in matrix CC.

Fig.5.
Another joint numbering gives another filling of matr'x CC like the figure shows.

At joint 3 meet the members 2, 3, 5 and 6 so that four submatrices [3,3] coincide. I

The members 5, 6, and 7 are horizontal. Then their S=D2/L1=0/L1=0. Then there will appear a number of zeros in their matrices S5. Question is which elements become zero, and, will it be possible that there arise zeros on the main diagonal of construction matrix CC.

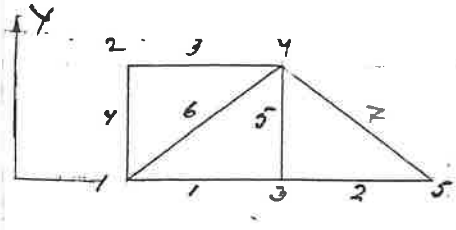


Fig. 6.

Fig. 6.
An example with also horizontal and vertical members.

R*C^2	R*S*C	-R*C^2	-R*S*C	R=EA/L1
R*S*C	R*S^2	-R*S*C	-R*S^2	C=D1/L1
-R*C^2	-R*S*C	R*C^2	R*S*C	S=D2/L1
-R*S*C	-R*S^2	R*S*C	R*S^2	(p. 3,15)

S5

For the horizontal members is D2=0, so S=0, and C=D1/L1=D1/D1=1. Then are
R*C^2=R, R*S*C=0 and R*S^2=0.
For the vertical members is D1=0, so C=0, and S=D2/L1=D2/D2=1. Then are
R*C^2=0, R*S*C=0 and R*S^2=R.

	1		3	
	1	2	5	6
1	R1	0	-R1	0
2	0	0	0	0
5	-R1	0	R1	0
6	0	0	0	0

member 1 horizontal

	3		5	
	5	6	9	10
5	R2	0	-R2	0
6	0	0	0	0
9	-R2	0	R2	0
10	0	0	0	0

member 2 horizontal

	2		4	
	3	4	7	8
3	R3	0	-R3	0
4	0	0	0	0
7	-R3	0	R3	0
8	0	0	0	0

member 3 horizontal

	1		2	
	1	2	3	4
1	0	0	0	0
2	0	R4	0	-R4
3	0	0	0	0
4	0	-R4	0	R4

member 4 vertical

	3		4	
	5	6	7	8
5	0	0	0	0
6	0	R5	0	-R5
7	0	0	0	0
8	0	-R5	0	R5

member 5 vertical

[1,1]	[1,4]
[4,1]	[4,4]

member 6

[4,4]	[4,5]
[5,4]	[5,5]

member 7

By means of these data the member stiffness matrices of the five members are composed with the stiffness factors R1 to R5 of the three horizontal and the two vertical members.
And with help of the tabel here below these S5's are provided with row and column numbers of matrix CC, and with joint numbers, thus member end numbers.

P	L	TT(1)	TT(2)	H	TT(3)	TT(4)
1	1	1	2	3	5	6
2	3	5	6	5	9	10
3	2	3	4	4	7	8
4	1	1	2	2	3	4
5	3	5	6	4	7	8
6	1	1	2	4	7	8
7	4	7	8	5	9	10

What is happening on the main diagonal?
After member 1 have become CC(1,1)=R1 and CC(5,5)=R1. CC(2,2) and CC(6,6) have remained zero.
After member 2 have become CC(5,5)=R1+R2 and CC(9,9)=R2.
After member 3 is CC(3,3)=R3 and CC(7,7)=R3. CC(4,4) and CC(8,8) zijn nu gebleven.
After member 4 is CC(2,2)=R4 and CC(4,4)=R4. CC(1,1) was already R1 and CC(3,3) already R3.
After member 5 is CC(6,6)=R5 and CC(8,8)=R5. CC(5,5) was already R1+R2 and CC(7,7) already R3.
The diagonal elementCC(10,10) is still zero.
There are yet the sloping members 6 and 7 with which the underlined elements of CC are altered. CC(10,10) has become unequal zero. All diagonal elements are now unequal zero.

	1		2		3		4		5	
	1	2	3	4	5	6	7	8	9	10
1	R1	0	0	0	-R1	0	-	-		
2	0	R4	0	-R4	0	0	-	-		
3	0	0	R3	0			-R3	0		
4	0	-R4	0	R4			0	0		
5	-R1	0			R1+R2	0	0	0	-R2	0
6	0	0			0	R5	0	-R5	0	0
7	-	-	-R3	0	0	0	R3	0_	-	-
8	-	-	0	0	0	-R5	0_	R5	-	-
9					-R2	0	-	-	R2	0_
10					0	0	-	-	0	0_

C

For a truss there do not arise zeros on the main diagonal. This can happen for continuous beams and frames.

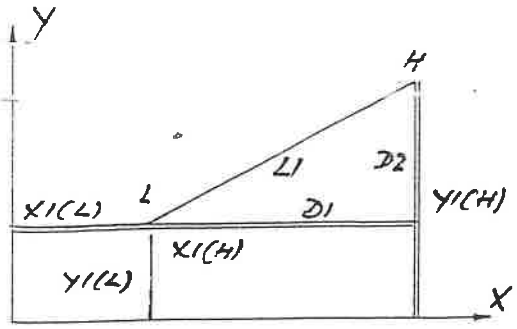


Fig.7.

R*C^2	R*S*C	-R*C^2	-R*S*C
R*S*C	R*S^2	-R*S*C	-R*S^2
-R*C^2	-R*S*C	R*C^2	R*S*C
-R*S*C	-R*S^2	R*S*C	R*S^2

S5

	1	2	3	4
1	A1	A2	-A1	-A2
2	.	A3	-A2	-A3
3	.	.	A1	A2
4	.	.	.	A3

```
Private Sub MEMBERMATS5TRUSS()
D1=X1(H)-X1(L) : D2=Y1(H)-Y1(L)
L1=Sqr(D1^2+D2^2)
S=D2/L1 : C=D1/L1 : R=EA/L1
A1=R*C^2 : A2=R*S*C : A3=R*S^2

S5(1,1)=A1 : S5(1,2)=A2
S5(1,3)=-A1 : S5(1,4)=-A2
S5(2,2)=A3 : S5(2,3)=-A2
S5(2,4)=-A3
S5(3,3)=A1 : S5(3,4)=A2
S5(4,4)=A3
A=2 : For I=1 To 3
For J=A To 4
S5(J,I)=S5(I,J)
Next J
Next I

End Sub
```

A1	A2	-A1	-A2
A2	A3	-A2	-A3
-A1	-A2	A1	A2
-A2	-A3	A2	A3

S5[1,1]	S5[1,2]
S5[2,1]	S5[2,2]

Fig.8.

2.5. Private Sub MEMBERMATS5TRUSS()

Fig.7.
Before the calculation of the elements of member stiffness matrix S5 of a member P become first
D1=X1(H)-X1(L) : D2=Y1(H)-Y1(L),
L1=Sqr(D1^2+D2^2) and
S=D2/L1 : C=D1/L1 : R=EA/L1.

Before calling this subroutine in the subroutine
CONSTRMATCCTRUSS (page 17) there's written
For P=1 To P9 : L=LL(P) : H=HH(P) en
EA=EAA(P).
In member stiffness matrix S5 occur three different combinations of of R, C and S which have a plus or minus sign in matrix S5.
These three are
A1=R*C^2 : A2=R*S*C : A3=R*S^2.

Next follow for the elements of the upper triangle, including the diagonal,
in the first row
S5(1,1)=A1 : S5(1,2)=A2 : S5(1,3)=-A1
S5(1,4)=-A2, and so on.
The matrix is symmetric w.r.t. the main diagonal. The lower triangle can then be filled as follows.

```
A=2    I=1    J=A To 4=2 To 4
                    S5(J,I)=S5(I,J)
              J=2   S5(2,1)=S5(1,2)
              J=3   S5(3,1)=S5(1,3)
              J=4   S5(4,1)=S5(1,4)   A=A+1=2+1=3
       I=2    J=A To 4=3 To 4
              J=3   S5(3,2)=S5(2,3)
              J=4   S5(4,2)=S5(2,4)   A=A+1=3+1=4
       I=3    J=4   S5(4,3)=S5(3,4)
```

This way the empty columns below the main diagonal are filled. And finally
End Sub.

Well, it's simplier to provide all 16 elements with a value, For I=1 to 4 and For J=1 To 4.

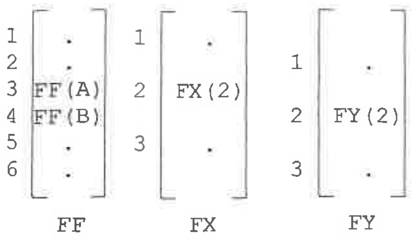
Or one fills the first two rows.
The third row is the first row times -1, and the fourth row is the second row times -1.

```
For I=3 To 4 : For J=1 To 4
S5(I,J)=S5(I-2,J)
Next J : Next I
```

Fig.8.
Or one divides S5 in four submatrices. These submatrices have on the same places elements as large as, with a + or a -.
Submatrix S5[1,1] is then filled with
S5(1,1)=A1 : S5(1,2)=A2 and
S5(2,1)=A2 : S5(2,2)=A3.
After that three times a submatrix is filled by means of

```
For K=1 To 3
If K=1 Then A=0 : B=2 : C=-1
If K=2 Then A=2 : B=0    (C was alraedy -1)
If K=3 Then B=2 : C=1    (A was already 2)
For I=1 To 2 : For J=1 To 2
S5(I+A,J+B)=C*S5(I,J) : Next J : Next I
Next K
```

Exaggerating a little bit?...



	A=2*I-1	B=2*I
I=2	A=2*2-1=3	B=2*2=4
I=3	A=2*3-1=5	B=2*3=6

Fig.1.

```
Private Sub TRUSSMAINCALC()

'1. Composition of construction
'matrix CC with member matrices S5.

CONSTRMATCCTRUSS      page 17

'2.Elements of force vector FF.
'2a. Joint load forces FX(I)
'and FY(I).
N=2*N9
For I=1 To N9
A=2*I-1 : B=2*I
FF(A)=FX(I)  : FF(B)=FY(I)
PP(A)=PH(I)  : PP(B)=PV(I)
UU(A)=UH(I)  : UU(B)=UV(I)
SS(A)=SH(I)  : SS(B)=SV(I)
Next I
```

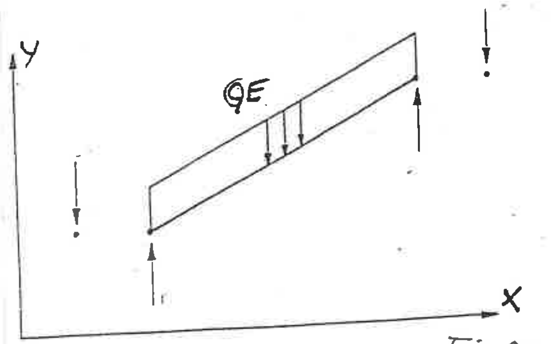


Fig.2a.

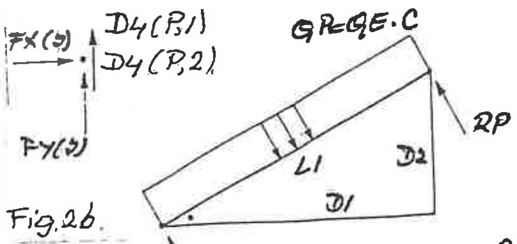


Fig.2b.

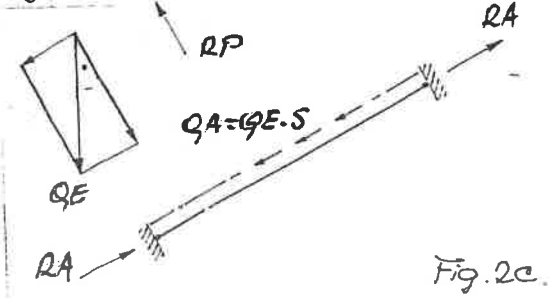


Fig.2c.

2.6. Private Sub TRUSSMAINCALC()

With this subroutine the main calculation is carried out for plane trusses.

First with subroutine

1. CONSTRMATCCTRUSS
construction matrix CC is filled with the member stiffness matrices S5.

2. The elements of force vector FF.
2a. The joint load forces.

There are N9 joints. For each joint there are two displacements, UH(I) and UV(I), thus the number of equations becomes N=2*N9.

First have been put in
the joint load forces FX(I) and FY(I),
PH(I) and PV(I) for either or not being prescribed of the displacements UH(I) and UV(I), which are set zero except those which are prescribed, which can be zero as well, and the spring constants SH(I) and SV(I) of the springy supports, for the other joints they are set zero.

Fig.1.
These data being put in are placed in the total vector FF, VV, UU and SS by means of A=2*I-1 and B=2*I.

2b. Primary forces due to own weight. See next page for the code.

Fig.2a.
For each member own weight is QE=QEE(P). It's a vertical downward directed uniformly distributed load. Since the member ends are seen as hinges both member end reactions are RT=QE*L1/2. On the joints act forces as large as but opposite directed.

Fig.2b.
For the on the joints acting primary forces D4(P,1) at member end L, and D4(P,2) at member end H the same direction is assumed as for the joint load forces FY(I), so upward. Noticing the directions of the on the joints acting forces of fig. 2a and 2b, become D4(P,1)=-RT and D4(P,2)=-RT. And then the next member with Next P.

Fig.2c.
QE seen as two load cases.
QP perpendicular to the member axis follows with
QP/QE=D1/L1=C so that QP=QE*C. Both reactions are as large as because both member ends are hinges, RP=QP*L1/2.

QA along the member axis follows with
QA/QE=D2/L1=S so that QA=QE*S. Both reactions are as large as, RA=QA*L1/2, because the member cross-section is constant.
Then the horizontal components of RP and RA are equally large and opposite directed and the vertical components deliver together RT.

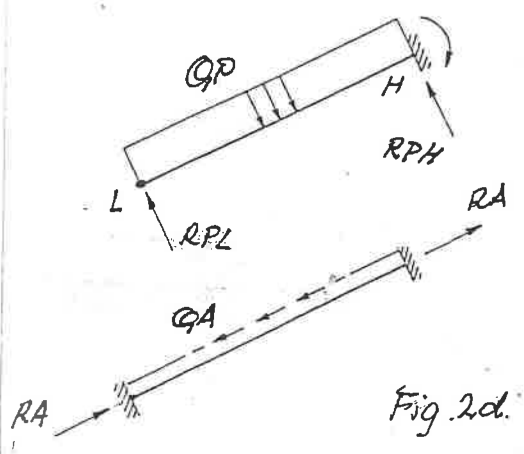


Fig.2d.

```
'2b. Primary forces due to own
'weight.
For P=1 To P9 : L=LL(P) : H=HH(P)
QE=QEE(P)
D1=X1(H)-X1(L)  : D2=Y1(H)-Y1(L)
L1=Sqr(D1^2+D2^2)  : L11(P)=L1
RT=QE*L1/2
D4(P,1)=-RT  : D4(P,2)=-RT
Next P

'2c. Alteration of force vector FF.
For I=1 To N9
B=2*I
For P=1 TO P9 : L=LL(P) : H=HH(P)
If I=L Then
FF(B)=FF(B)+D4(P,1)
Else If I=H Then
FF(B)=FF(B)+D4(P,2)
End If
Next P
Next I
```

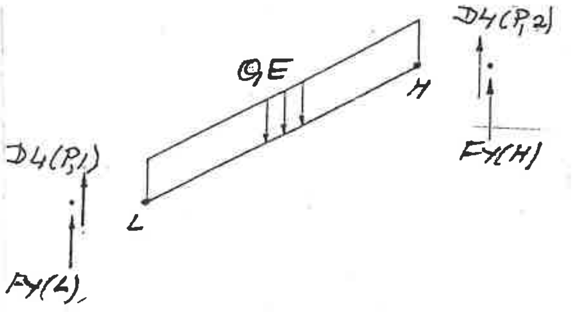


Fig.3.

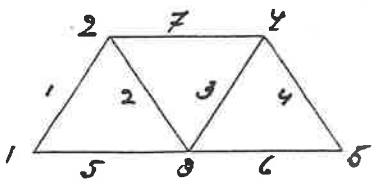


Fig.4.

(Fig.2d.
With this member, whioch is not a member of a truss, is given a few info of a subroutine called QEWEIGHT with which primary forces and moments due to own weight are calculated for frames.
Is member end H rigidly connected with joint H and member end L hinglike with joint L, then it is different from the member of the preceding page.
QP perpendicular to the member axis delivers the reactions RPL and RPH which are not! equally large, moreover there is also a reaction moment at H.
QA along the member delivers also now two equally large reactions RA. Then subroutine MEMBER for axially loaded members, and subroutines BEAM1, BEAM2 and BEAM3 for perpendicular loaded beams, see part 8, are applied.)

But now further with trussen and other plane member constructions of which the member ends are hinges.

2c. Alteration of force vector FF.

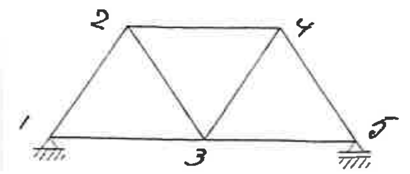
Fig.3.
Since the on the joints L and H acting primary forces due to own weight are known, and their assumed directions are the same as those of the joint load forces FY(I), they can be added to the previous elements of force vector FF.
For I=1 To N9
B=2*I
For a joint I will be checked for each member P=1 To P9 if that member delivers a primary force on that joint. The concerning element in total vector FF is indicated with FF(B).
The elements B=2*I are the elements FF(2), FF(4) and so on. These are the places of the vertical joint load forces FY(I).
The member has two member ends with member end are joint numbers as well, L and H.
If joint number I=L then D4(P,1)is added to F(B). With (P,1) meaning L-H, then becomes F(B)=F(B)+D4(P,1), or
if joint number I=H then D4(P,2) is added to F(B). With (P,2) meaning H-L, then becomes F(B)=F(B)+D4(P,2).
On the way as follows.
Fig.4.

```
I=1   B=2*I=2*1=2         FF(B)=FF(2)
P=1   I=L   1=1           FF(2)=FF(2)+D4(1,1)
P=2   I<>L=2 and I<>H=3
P=3   I<>L=3 and I<>H=4  and so on.
P=5   I=L   1=1       FF(2)=FF(2)+D4(5,1) a.s.o.
```

The members 1 and 5 deliver a primary force on joint 1.

```
I=2   B=2*I=2*2=4         FF(B)=FF(4)
P=1   I=H   2=2           FF(4)=FF(4)+D4(1,2)
P=2   I=L   2=2           FF(4)=FF(4)+D4(2,1)
P=7   I=L   2=2           FF(4)=FF(4)+D4(7,1)
```

The members 1, 2 and 7 deliver a primary force on joint 2. For joint I=3 with B=2*I=6 are added to FF(B)=FF(6)
D4(2,2) of member 2, D4(3,1) of member 3, D4(5,2) of member 5, D4(6,1) of member 6.
And so on.



3. Alteration of force vector FF and constructionmatrix CC.
3a. Of F in case of prescribed displacements <>0.

Fig.5.
Suppose that for joint 5 the vertical displacement UV(5) is prescribed. With B=2*I it would be UU(B)=UU(10) in total vector UU.

```
    1  2  3  4  5  6  7  8  9    10
 1  .  .  .  .  .  .          CC(1,10)     .       FF(1)
 2  .  .  .  .  .  .          CC(2,10)     .       FF(2)
 3  .  .  .  .  .  .  .  .                 .         .
 4  .  .  .  .  .  .  .  .                 .         .
 5  .  .  .  .  .  .  .  .  .     .        .         .
 6  .  .  .  .  .  .  .  .  .  CC(K,10)    .       FF(K)
 7        .  .  .  .  .  .  .     .        .         .
 8        .  .  .  .  .  .  .     .        .         .
 9              .  .  .  .  .     .        .         .
10              .  .  .  .  .  CC(N,10)  UU(10)    FF(N)
              CC                           UU        FF
```

Is UU(I)=UU(10)<>0 then all elements of FF For K=1 To N, are lessened by CC(K,10)*UU(10).

```
FF(1)=FF(1)-CC(1,10)*UU(10)
FF(2)=FF(2)-CC(2,10)*UU(10)
FF(3)=FF(3)-CC(3,10)*UU(10)
FF(4)=FF(4)-CC(4,10)*UU(10)
FF(5)=FF(5)-CC(5,10)*UU(10)
```
And so on.

```
'3. Alteration of force vector FF and
'construction matrix CC.
'3a. Of FF in case of prescribed
'displacements <>0.
For I=1 To N
If UU(I)<>0 Then
For K=1 To N
FF(K)=FF(K)-CC(K,I)*UU(I)
Next K
End If
Next I
'3b. Of FF and CC in case of pres-
'cribed displacements.
For I=1 To N
If PP(I)=1 Then
For J=1 To N
CC(I,J)=0 : CC(J,I)=0
Next J
CC(I,I)=1 : FF(I)=UU(I)
End If
Next I
'3c. Of CC in case of springy
'supports.
For I=1 To N
If SS(I)>0 Then CC(I,I)=CC(I,I)+SS(I)
Next I
```

3. Alteration of force vector FF and construction matrix C.
3a. Of FF in case of prescribed displavements <>0.
Fig.6.
Notice that For I=1 To N is 1 To 2*N9
Is PP(I)=0 then displacement UU(I) is not prescribed..
Is PP(I)=1 then displacement UU(I) is prescribed, a horizontal displacement UH() or a vertical displacement UV().
In this case here UU(1), UU(2) and UU(5).
They are given in UU as
horizontal displacement UH(1),
vertical displacement UV(1), and
vertical displacement UV(5).
For example PP(2)=1, then UU(2) is prescribed and the second row and second column of CC are filled with zeros with CC(I,J)=0 : CC(J,I)=0.
And then, after Next J, the element on the main diagonal is made C(I,I)=C(2,2)=1 and FF(I)=FF(2) gets the value of UU(I)=UU(2) with FF(I)=UU(I).

3c. Of CC in case of springy supports.
If SS(I)>0 then a spring constant has been put in which is added to the concerning element on the main diagonal, CC(I,I)=CC(I,I)+SS(I).

```
    1  2  3  4  5  6  7  8  9  10
 1  1  0  0  0  0  0                 UH(1)    UU(1)
 2  0  1  0  0  0  0                 UV(1)    UU(2)
 3  0  0  .  .  .  .  .  .             .        .
 4  0  0  .  .  .  .  .  .             .        .
 5  0  0  .  .  .  .  .  .  .  0       .        .
 6  0  0  .  .  .  .  .  .  .  0       .        .
 7        .  .  .  .  .  .  .  0       .        .
 8        .  .  .  .  .  .  .  0       .        .
 9              .  .  .  .  .  0       .        .
10              0  0  0  0  0  1     UV(5)    UU(10)
 X
                CC                              UU
```

4. Computation of the unknown displacements UH(I) and UV(I).
(See the code on the next page.)
For this all elements of column FF are placed in column BB and is matrix AA filled with the elements of construction matrix CC.

With subroutine <u>GAUSS</u>, the unknowns of XX() are solved. Next the elements of XX() are placed in the sub-vectors UH() and UV(), and in total vector UU().

Fig.6. (Not shown elements of CC are also made zero in the code, but they were already zero.)

```
'4. Calculation of the unknown
'displacements UH(I) and UV(I).
For I=1 To N : BB(I)=FF(I)
For J=1 To N
AA(I,J)=CC(I,J)
Next J
Next I
'The solution of N=2*N9 equations.
GAUSS
For I=1 To N9
A=2*I-1 : B=2*I
UH(I)=XX(A) : UV(I)=XX(B)
UU(A)=XX(A) : UU(B)=XX(B)
Next I
```

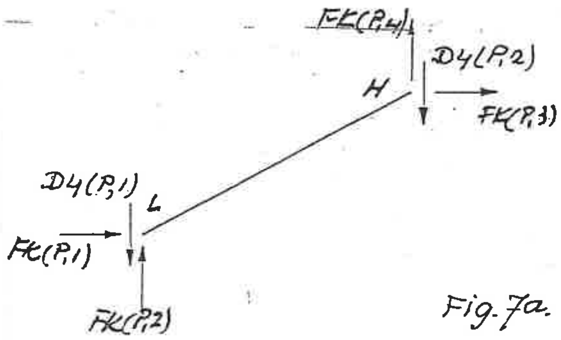


Fig.7a.

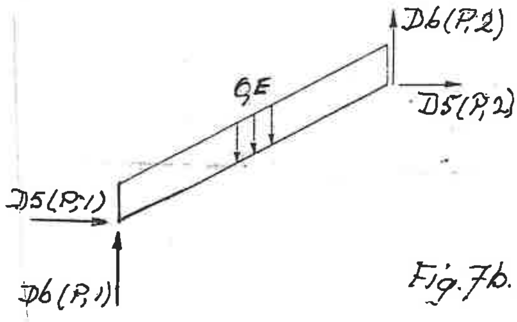


Fig.7b.

```
'5. Calculation of the member end
'forces w.r.t. the construction
'axes system X-Y.
'5a. Due to the displacements
'alone.
For P=1 To P9 : L=LL(P) : H=HH(P)
EA=EAA(P)
MEMBERMATS5TRUSS       page 20
TT(1)=2*L-1 : TT(2)=2*L
TT(3)=2*H-1 : TT(4)=2*H
For I=1 To 4 : FK(P,I)=0
For J=1 To 4 : A=TT(J)
FK(P,I)=FK(P,I)+S5(I,J)*UU(A)
Next J
Next I

'5b. Due to displacements and
'own weight.
D5(P,1)=FK(P,1)
D6(P,1)=FK(P,2)-D4(P,1)
D5(P,2)=FK(P,3)
D6(P,2)=FK(P,4)-D4(P,2)
```

	UU	UH	UV		XX
1	.			1	.
2	.	.		2	.
3	UU(A)	UH(2)	.	3	XX(A)
4	UU(B)		UV(2)	4	XX(B)
5	.	.		5	.
6	.		.	6	.

For example I=2.   A=2*I-1=3   B=2*I=4
UH(I)=XX(A) UH(2)=XX(3) UV(I)=XX(B) UV(2)=XX(4)
UU(A)=XX(A) UU(3)=XX(3) UU(B)=XX(B) UU(4)=XX(4)

5. Calculation of the member end forces w.r.t. the construction axes system X-Y.
5a. Due to the displacements alone..

Fig.7a.
The relation between member end forces and displacements is f = S5 u.

	f	S5				UU	u
FLHX	FK(P,1)	11	12	13	14	UU(A)	UH(L)
FLHY	FK(P,2)	21	22	23	24	UU(A)	UV(L)
FHLX	FK(P,3)	31	32	33	34	UU(A)	UH(H)
FHLY	FK(P,4)	41	42	43	44	UU(A)	UV(H)

For member P the strain stiffness EA=EAA(P).
With subroutine
MEMBERMATS5TRUSS
member stiffness matrix S5 is formed.
Each of the four member end forces FK(P,I) is equal to a row of S5 times column u.
FK(P,1) and FK(P,2) at member end L, and FK(P,3) and FK(P,4)at member end H.

There are For I=1 To 4 member end forces. First always FK(P,I)=0. After that follow for J=1 To 4 the four element multiplications.

The elements of u are abtained from totalvector UU by means of TT(1) to TT(4).
Is the lowest member end number L=1, and the highest member end number H=4, then follow
TT(1)=2*L-1=2*1-1=1,   TT(2)=2*L=2*1=2 en
TT(3)=2*H-1=2*4-1=7,   TT(4)=2*H=2*4=8.

For example the calculation of FK(P,3).

```
I=3   A=TT(J)
J=1   A=TT(1)=1   FK(P,3)=    0    +S5(3,1)*UU(1)
J=2   A=TT(2)=2   FK(P,3)=FK(P,3)+S5(3,2)*UU(2)
J=3   A=TT(3)=7   FK(P,3)=FK(P,3)+S5(3,3)*UU(7)
J=4   A=TT(4)=8   FK(P,3)=FK(P,3)+S5(3,4)*UU(8)
```

5b. Due to the displacements and own weight.

Fig.7b. The final member end forces are D5(P,1) and D6(P,1) at member end L, and D5(P,2) and D6(P,2) at member end H. Their directions are assumed as of FK(P,I).
The on the joints acting primary forces D4(P,1) at L and D4(P,2) at H are assumed upward. On the member ends act forces as large as but opposite directed, so downward, see fig.7a.
Then follow
D5(P,1)=FK(P,1) and D6(P,1)=FK(P,2)-D2(P,1),
D5(P,2)=FK(P,3) and D6(P,2)=FK(P,4)-D2(P,2).

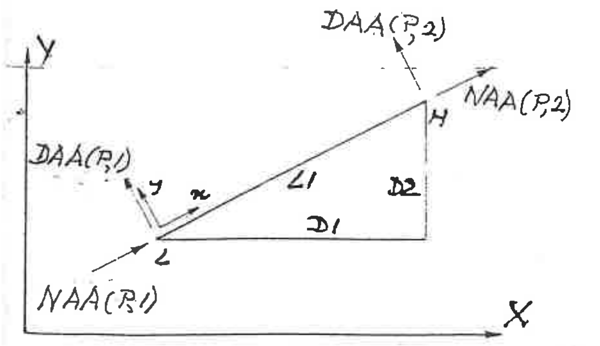


Fig. 8a.

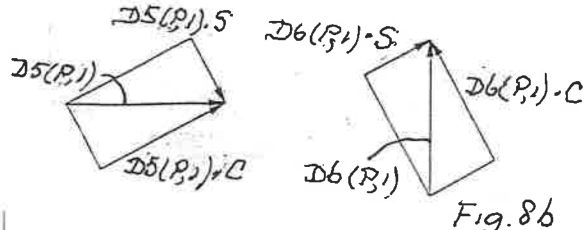


Fig. 8b

```
'5c.Computation of the member end
'forces w.r.t. the member axes
'system x-y.
D1=X1(H)-X1(L) : D2=Y1(H)-Y1(L)
L1=Sqr(D1^2+D2^2)
S=D2/L1 : C=D1/L1
NAA(P,1)=D5(P,1)*C+D6(P,1)*S
NAA(P,2)=D5(P,2)*C+D6(P,2)*S
DAA(P,1)=D6(P,1)*C-D5(P,1)*S
DAA(P,2)=D6(P,2)*C-D5(P,2)*S
Next P
'6. Computation of the joint for-
'ces KH(I) and KV(I).
'6a. Due to the displacements
'alone.
CONSTRMATCCTRUSS      page 17
For I=1 To N9
A=2*I-1 : B=2*I
KH(I)=0 : KV(I)=0
For J=1 To N
KH(I)=KH(I)+CC(A,J)*UU(J)
KV(I)=KV(I)+CC(B,J)*UU(J)
Next J
```

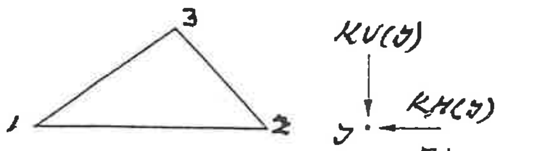
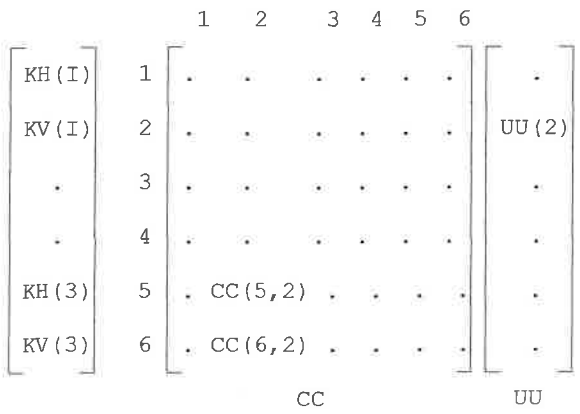


Fig. 9a.



5c. Calculation of member end forces w.r.t. the member axes system x-y.

Fig.8a.
For the member end forces NAA(P,1) at L and NAA(P,2) at H is assumed that they are directed according to member axis x, so from L to H.
For the member end forces DAA(P,1) at L and DAA(P,2) at H is assumed that they are directed according to member axis y.
First D1 and D2 are calculated, after that L1, and S and C.

Fig.8b.
The component Dx of D5(P,1) at L of fig.7b alon the member axis is found with similarity of triangles.
Dx/D5(P,1)=D1/L1=C ⇒ Dx=D5(P,1)*C
Likewise for the component Dy of D4(P,1) perpendicular to the member axis.
Dy/D5(P,1)=D2/L1=S ⇒ Dy=D5(P,1)*S
Next there is the component Dx of the vertical force D6(P,1) at member end L.
Dx/D6(P,1)=D2/L1=S ⇒ Dx=D6(P,1)*S
And also a component Dy of D5(P,1).
Dy/D6(P,1)=D1/L1=C ⇒ Dy=D6(P,1)*C
In same way one finds the components of D5(P,2) and D6(P,2) at member end H.
Dx=D5(P,2)*C Dy=D5(P,2)*S
Dx=D6(P,2)*S Dy=D6(P,2)*C

At member end L.
Both components Dx along the member have the same direction as NAA(P,1), then follows
NAA(P,1)=D5(P,1)*C+D6(P,1)*S.
The component Dy of D5(P,1) is opposite directed to DAA(P,1), and the component Dy of D5(P,1) has the same direction as DAA(P,1), so that
DAA(P,1)=D6(P,1)*C-D5(P,1)*S.
At member end H likewise.
NAA(P,2)=D5(P,2)*C+D6(P,2)*S
DA(P,2)=D6(P,2)*C-D5(P,2)*S
And this for each member.
Next P.

6. Calculation of the joint forces KH(I) and KV(I).
6a. Due to the displacements alone.
Fig.9a.
The joint forces are calculated by means of the original, not altered, construction matrix CC which is formed with subroutine CONSTRMATCCTRUSS.
A=2*I-1 is the row number of CC for KH(I), and B=2*I is the row number CC for KV(I).
A joint force is equal to a row of CC times column UU.
Suppose joint I=3 with A=2*3-1=5 and B=2*3=6.

```
J=1   KH(3)=  0  +CC(5,1)*UU(1)
      KV(3)=  0  +CC(6,1)*UU(1)
J=2   KH(3)=KH(3)+CC(5,2)*UU(2)
      KV(3)=KV(3)+CC(6,2)*UU(2)
J=3   KH(3)=KH(3)+CC(5,3)*UU(3)
      KV(3)=KV(3)+CC(6,3)*UU(3)
```

and so on up to and including J=N.

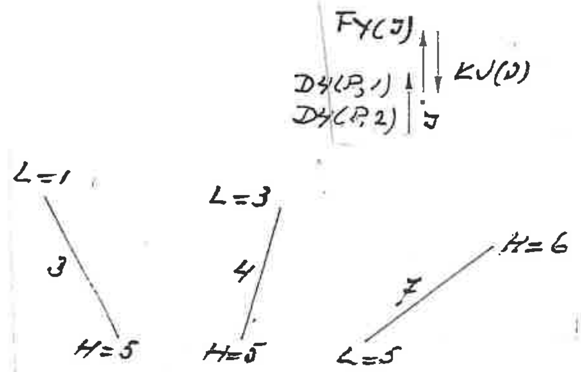
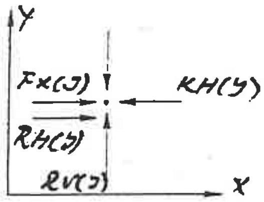


Fig.9b.

```
'6b. Due to the displacements
'and own weight of the members.
For P=1 To P9 : L=LL(P) : H=HH(P)
If I=L Then
KV(I)=KV(I)-D4(P,1)
ElseIf I=H Then
KV(I)=KV(I)-D4(P,2)
End If
Next P
Next I
```



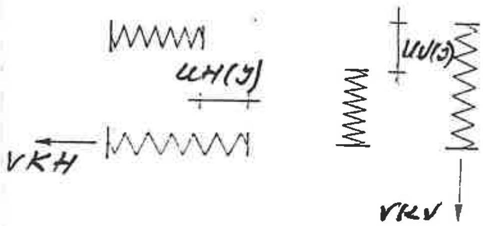


Fig.10.

```
'7. Calculation of the reactions.
For I=1 To N9
If SH(I)>0 Then
RH(I)=-SH(I)*UH(I)
Else
RH(I)=KH(I)-FX(I)
End If
If SV(I)>0 Then
RV(I)=-SV(I)*UV(I)
Else
RV(I)=KV(I)-FY(I)
End If
Next I

End Sub
```

6b. Due to the displacements and own weight of the members.

Fig.9b.
Due to the own weight arise on joint L a primary force D4(P,1), and on joint H a primary force D4(P,2).
Like the joint load forces their direction was assumed upward.

For a joint I=1 To N9 (see preceding page) all members are checked to see if that member delivers a primary force on joint I=L or I=H.

Suppose that at joint 5 the members P=3, 4 and 7 meet, then follow for P=1 To P9,
P=1 and P=2 deliver no contribution,
P=3 I=H=5 KV(5)=KV(5)-D4(3,2),
P=4 I=H=5 KV(5)=KV(5)-D4(4,2),
P=5 en P=6 leveren geen bijdrage,
P=7 I=L=5 KV(5)=KV(5)-D4(7,1),
P=8 to P9 no contribution.
(Ofcourse, one may program in such way that only the members meeting at a joint I are considered.)

7. Computation of the reactions.

Fig.10.
The vertical reaction is mostly directed upward therefore this reaction RV(I) is assumed to be directed upward.
The direction of the horizontal reaction RH(I) is assumed to be directed to the right.

Is joint I horizontally springy supported then the spring constant is SH(I)>0. With a displacement UH(I) as assumed to the right the spring reaction is
VKH=SH(I)*UH(I) directed to the left, that is opposite to the assumption for RH(I), so that
RH(I)=-SH(I)*UH(I).

If the joint is not horizontally springy supported then follows RH(I) with Σ hor. =0. There is also the joint load force FX(I) as assumed directed to the right. Then follows
RH(I)+FX(I)-KH(I)=0 from which
RH(I)=KH(I)-FX(I).

Is joint I vertically springy supported then is SV(I)>0. Displaces the joint over UV(I) as assumed upward then arises the downward directed spring reaction VKV=SV(I)*UV(I).
VKV and RV(I) are opposite directed so that
RV(I)=-SV(I)*UV(I).

Is the joint not springy supported, then the reaction follows with Σ vert. =0.
RV(I)+FY(I)-KVI=0 from which
RV(I)=KV(I)-FY(I).

(There is not worked with a total reaction vector when calculating the reactions, one may do so, ofcourse.)

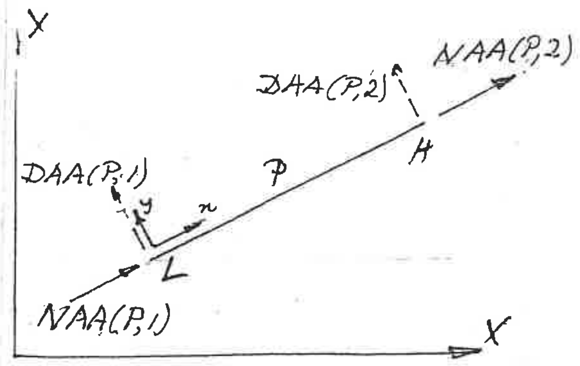


Fig.1.

The normal forces on the member ends with assumed directions for
NAA(P,1) at member end L, and
NAA(P,2) at member end H.

ans. NAA(P,1) positive, compression

ans. NAA(P,1) negative, tension

ans. NAA(P,2) positive, tension

ans. NAA(P,2) negative, compression

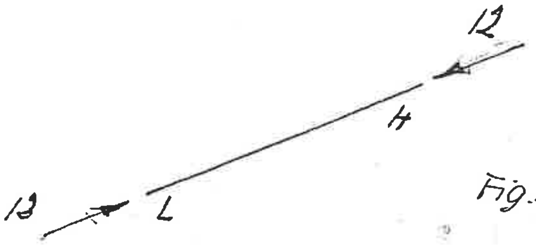


Fig.2a.

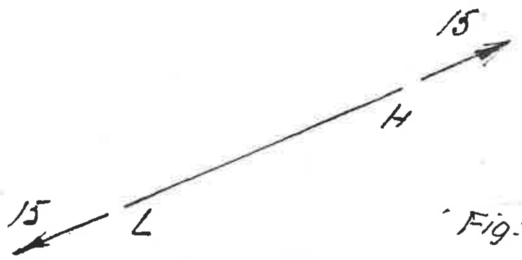


Fig.2b.

For the normal force NFF(P) in a member, that is for the normal cross-sections between member end L and member end H one can write

For P=1 To P9		For P=1 To P9
NFF(P)=-NAA(P,1)	or	NFF(P)=NAA(P,2)
Next P		Next P

2.7. 'Member forces' which are not the member end forces.

Fig.1.
After the main calculation with subroutine TRUSSMAINCALC (page 21) the member end forces NAA(P,1) and NAA(P,2) have become known. When own weight is omitted then DAA(P,1)=0 and DAA(P,2)=0.

The normal force in a member.

Member end force NAA(P,1) at member end L.
Is the answer for NAA(P,1) positive, then the force is directed according the assumption, so as the x-axis from L to H.
The force pushes on member end L, then the member is a compression member.
Is the answer for NAA(P,1) negative, then the force is opposite directed and pulls at member end L, the member is a tension member.

Member end force NAA(P,2) at member end H.
Is the answer for NAA(P,2) positive, then the force is directed as assumed from L To H. The force pulls at member end H, then the member is a tension member.
Is the answer for NAA(P,2) negative, then the force is opposite directed and pushes on member end H, the member is a compression member.

Determining the member forces NFF(P).

Fig.2a en 1.
Is NAA(P,1)=12 kN, a positive answer, then the force is directed as assumed. The force pushes on member end L. If NAA(P,1)=12 kN then is NAA(P,2)=-12 kN, a negative answer, then the force is opposite directed to the assumed direction, so does not pull at but pushes on member end H.
Conclusion: member P is a compression member.
If one agrees upon to indicate this with a minus sign then one can write
NFF(P)=-NAA(P,1) or NFF(P)=NAA(P,2).

Fig.2b en 1.
Is NAA(P,1)=-15 kN, a negative answer, then the force is opposite directed to the assumed direction. The force does not push on but pulls at member end L. Is NAA(P,1)=-15 kN then is NAA(P,2)=15 kN, a positive answer and pulls at member end H.
Conclusion: member P is a tension member.
If one agrees upon to indicate this with a plus sign then one can write
NFF(P)=-NAA(P,1) or NFF(P)=NAA(P,2).

Thus:
Is the answer for NFF(P) positive, then the member is a tension member.
Is the answer for NAF(P) negative then the member is a compression member.
So, saying + means tension member and - means compression member applies only because of the agreement for NFF(P) made in the end of the programme. And saying that at the very beginning... would cause problems, confusion...

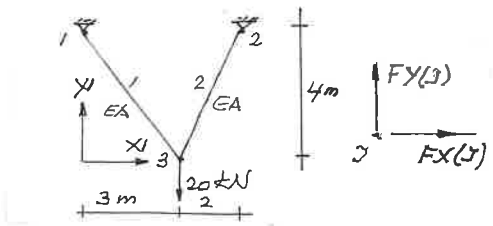


Fig.1.

Input of data.

Type 3 in TN9, Tab, and in TSTRING

1,0,1,0,0,0,1,0,4 Enter,

2,0,1,0,5,0,1,0,4 Enter and

3,0,0,0,3,-20,0,0,0 Enter.

Type 2 in TP9, Tab, and in TSTRING

1,1,3,1,0 Enter and 2,2,3,1,0 Enter.

Next click Show 2 times to see the data put in.
Or click EX1 to EX1a for these data.

Click Calculate to calculate the displacements UH(I) and UV(I), reactions RH(I) assumed to the right, and RV(I) assumed upward.

P	L	H	A1	QE
1	1	3	1	0,3
2	2	3	1	0,3

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	4
2	0	1	0	5	0	1	0	4
3	0	0	0	3	-20	0	0	0

UH1=	0,00/EA	RH1=	-6,43 kN
UV1=	0,00/EA	RV1=	9,32 kN
UH2=	0,00/EA	RH2=	6,43 kN
UV2=	0,00/EA	RV2=	13,52 kN
UH3=	-3,93/EA		
UV3=	-69,88/EA		

With a click on member end forces the member end forces
NAA(L,H),NAA(H,L), D5(L,H), D5(H.L), D6(L,H), D6(L,H) are printed.

member 1			
D5(1,3)=	-6,43 kN	D5(3,1)=	6,43 kN
D6(1,3)=	9,32 kN	D6(3,1)=	-7,82 kN
NAA(1,3)=	-11,31 kN	NAA(3,1)=	10,11 kN
DAA(1,3)=	0,45 kN	DAA(3,1)=	0,45 kN
member 2			
D5(2,3)=	6,43 kN	D5(3,2)=	-6,43 kN
D6(2,3)=	13,52 kN	D6(3,2)=	-12,18 kN
NAA(2,3)=	-14,97 kN	NAA(3,2)=	13,77 kN
DAA(2,3)=	-0,30 kN	DAA(3,2)=	-0,30 kN

Assumed direction RH(I) to the right and RV(I) upward.
Or click EX1 to EX1a, Show etc.

2.8. TRUSSPROGRAM111 Assumptions

Joint assumptions.

I FX PH UH SH X1 FY PV UV SV Y1

I joint number
FX(I) horizontal joint load force to the right

PH(I)=0
joint displacement UH(I) not prescribed
PH(I)=1
joint displacement UH(I) is prescribed
UH(I)=0 or <>0

UH(I) in EA (EA is strain stiffness)
a horizontal displacement
SH(I) horizontal spring constant in EA
not in examples

X1(I) joint distance determined by the assumed drawn place of the the X1-Y1 axis system.

FY(I) vertical joint load force, upward.

PV(I)=0
joint displacement UV(I) not prescribed
PV(I)=1
joint displacement UV(I) is prescribed
UH(I)=0 or <>0

UV(I) in EA (EA is strain stiffness)
a vertical displacement
SV(I) vertical spring constant s in kN/m
not applied in examples

Y1(I) joint distance determined by the assumed drawn place of the axis system.

Member assumptions.

P LL(P) HH(P) A11(P) QEE)P)

P member number
LL(P) lowest member end number
HH(P) highest member end number

A11(P) strain stiffness is EAA(P), in EA

QEE(P) own weight kN/m

Example.

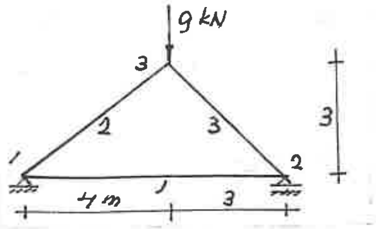
Fig.1.

N9=3 joints

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	4
2	0	1	0	5	0	1	0	4
3	0	0	0	3	-20	0	0	0

P9=2 members

P	L	H	A1	QE
1	1	3	1	.3
2	2	3	1	.3



I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	0
2	0	0	0	7	0	1	0	0
3	0	0	0	4	-9	0	0	3

P	L	H	A1	QE
1	1	2	1	0
2	1	3	1	0
3	2	3	1	0

```
UH1=    0,00/EA     RH1=   0,00 kN
UV1=    0,00/EA     RV1=   3,86 kN
UH2=   36,00/EA
UV2=    0,00/EA     RV2=   5,14 kN
UH3=   11,17/EA
UV3=  -68,47/EA

member 1
 D5(1,2)=  -5,14 kN    D5(2,1)=   5,14 kN
 D6(1,2)=   0,00 kN    D6(2,1)=   0,00 kN
NAA(1,2)=  -5,14 kN   NAA(2,1)=   5,14 kN
DAA(1,2)=   0,00 kN   DAA(2,1)=   0,00 kN
member 2
 D5(1,3)=   5,14 kN    D5(3,1)=  -5,14 kN
 D6(1,3)=   3,86 kN    D6(3,1)=  -3,86 kN
NAA(1,3)=   6,43 kN   NAA(3,1)=  -6,43 kN
DAA(1,3)=   0,00 kN   DAA(3,1)=   0,00 kN
member 3
 D5(2,3)=  -5,14 kN    D5(3,2)=   5,14 kN
 D6(2,3)=   5,14 kN    D6(3,2)=  -5,14 kN
NAA(2,3)=   7,27 kN   NAA(3,2)=  -7,27 kN
DAA(2,3)=   0,00 kN   DAA(3,2)=   0,00 kN
```

CSE=0 NAADAA D5D6 Again
N9= OK Cls
P9= EX1 EX2 EX3 Calculate End
EX displacements and member end forces PrF
Show STORE NR= ? GET AOAgain

```
NAA( 1,1)=  -5,14   NAA( 1,2)=   5,14  kN
DAA( 1,1)=   0,00   DAA( 1,2)=   0,00  kN

NAA( 2,1)=   6,43   NAA( 2,2)=  -6,43  kN
DAA( 2,1)=   0,00   DAA( 2,2)=   0,00  kN

NAA( 3,1)=   7,27   NAA( 3,2)=  -7,27  kN
DAA( 3,1)=   0,00   DAA( 3,2)=   0,00  kN
```

```
 D5( 1,1)=  -5,14    D5( 1,2)=   5,14  kN
 D6( 1,1)=   0,00    D6( 1,2)=   0,00  kN

 D5( 2,1)=   5,14    D5( 2,2)=  -5,14  kN
 D6( 2,1)=   3,86    D6( 2,2)=  -3,86  kN

 D5( 3,1)=  -5,14    D5( 3,2)=   5,14  kN
 D6( 3,1)=   5,14    D6( 3,2)=  -5,14  kN
```

TRUSSPROGRAM111, the form controls.

Number of joints N9=, text box TN9.

Number of members, text box TP9.

TSTRING is the large text box for input of joint and member data, after input press Enter or click OK.
Click Show 2 times after all input to show them on the form.
Or click EX to get the same data.

Click Calculate to carry out the calculations.
Click Cls to clear the form, and when wanted if necessary.

Click displacements and member end forces, displacements UH(I) and UV(I), and reactions RH(I) and RV(I) are printed on the form.

Next click shows successively for each meber P

member end forces D5(L,H) and D5(H,L), D6(L,H) and D6(H,L), and the

member end forces NAA(L,H) and NAA(H,L), DAA(L,H) and DAA(H,L).

When clicking NAADAA member end forces NAA(P,1) and NAA(P,2) and DAA(P,1) and DAA(P,2) appear on the form, and

when clicking D5D6 member end forces D5(P,1) and D5(P,2) and D6(P,1) and D6(P,2) appear on the form,

see the assumed directions next page.
NAADAA and D5D6, left mouse button (P,1) and (P,2), right mouse button (L,H) and (H,L).
A click on Again makes all displacements UH(I)=0 and UV(I)=0, sets to start position to make the text boxes empty and puts the cursor in text box TN9.

EX1, EX2 and EX3 are examples with given data, to click on, to appear e.g. EX1a, EX1b etc.

A click on AOagain to start all over again.

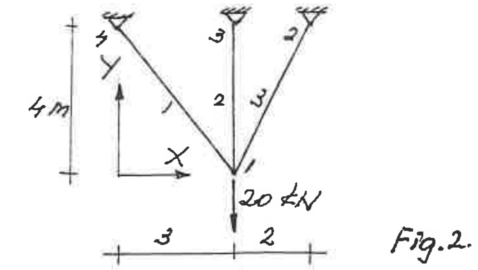
One may change data.
Then click CSE=0 to CSE=1 getting red and type in TSTRING the wanted change.

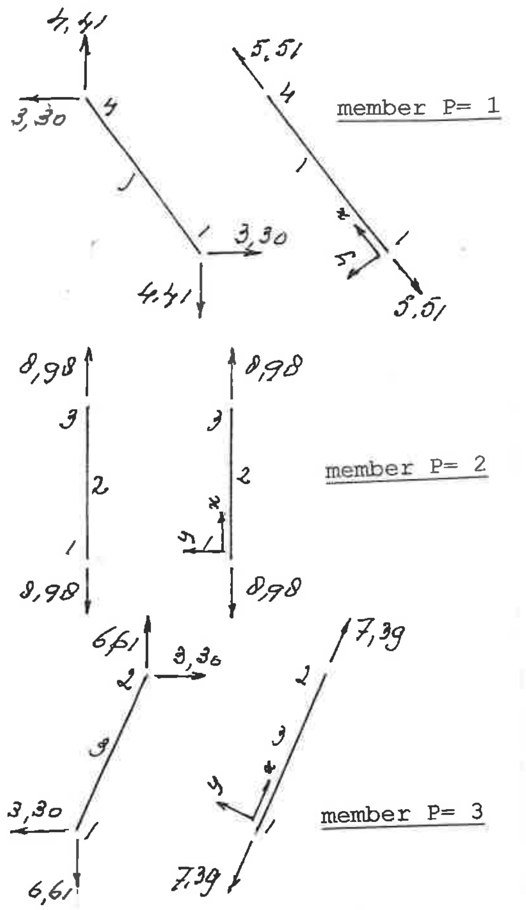
Chnaging the support data e.g., type in TSTRING

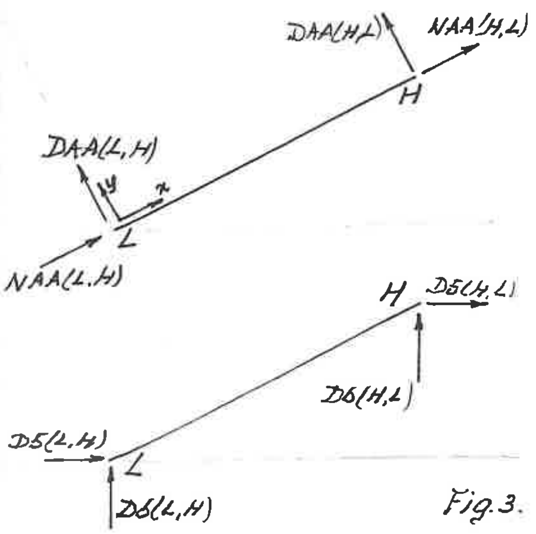
PH(1)=0 Enter and PH(2)=1 Enter.
Click Cls and Show to see the change, correct?

FX(I) and FY(I) can be changed.

Changing UH(I) and UV(I), if wanting to do so the first click Again to make all displace-*ments zero for the following main calculations.*







Example.   Click EX1a to EX1b.

Fig.2.

N9=4 joints

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	0	0	3	-20	0	0	4
2	0	1	0	5	0	1	0	4
3	0	1	0	3	0	1	0	4
4	0	1	0	0	0	1	0	4

P9=3 members

P	L	H	A1	QE
1	1	4	1	0
2	1	3	1	0
3	1	2	1	0

After input of data click Show two times and Calculate to get the print here below.

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	0	0	3	-20	0	0	0
2	0	1	0	5	0	1	0	4
3	0	1	0	3	0	1	0	4
4	0	1	0	0	0	1	0	4

P	L	H	A1	QE
1	1	4	1	0
2	1	3	1	0
3	1	2	1	0

UH1=	-2,02/EA		
UV1=	-35,94/EA		
UH2=	0,00/EA	RH2=	3,30 kN
UV2=	0,00/EA	RV2=	6,61 kN
UH3=	0,00/EA	RH3=	0,00 kN
UV3=	0,00/EA	RV3=	8,98 kN
UH4=	0,00/EA	RH4=	-3,30 kN
UV4=	0,00/EA	RV4=	4,41 kN

Click member end forces to see the results here below.

member 1			
D5(1,4)=	3,30 kN	D5(4,1)=	-3,30 kN
D6(1,4)=	-4,41 kN	D6(4,1)=	4,41 kN
NAA(1,4)=	-5,51 kN	NAA(4,1)=	5,51 kN
DAA(1,4)=	0,00 kN	DAA(4,1)=	0,00 kN
member 2			
D5(1,3)=	0,00 kN	D5(3,1)=	0,00 kN
D6(1,3)=	-8,98 kN	D6(3,1)=	8,98 kN
NAA(1,3)=	-8,98 kN	NAA(3,1)=	8,98 kN
DAA(1,3)=	0,00 kN	DAA(3,1)=	0,00 kN
member 3			
D5(1,2)=	-3,30 kN	D5(2,1)=	3,30 kN
D6(1,2)=	-6,61 kN	D6(2,1)=	6,61 kN
NAA(1,2)=	-7,39 kN	NAA(2,1)=	7,39 kN
DAA(1,2)=	0,00 kN	DAA(2,1)=	0,00 kN

Fig.3.

The three members with the member end forces drawn with their real directions.
member 1.
D5(1,4)= 3,3 kN at member end 1, positive value so directed as assumed to the right.
D5(4,1)=-3,30 kN at member end 4, negative value so not directed as assumed but opposite directed.
Positive value?, then directed as assumed, negative value?, then not directed as asuumed but opposite directed.

P	L	H	A1	QE
1	1	2	1	0
2	1	3	1	0
3	2	3	1	0

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	0	0	0	0	1	0	0
2	0	1	0	2	0	1	0	0
3	-15	0	0	0	0	0	0	4

```
UH1=   0,00/EA
UV1=   0,00/EA      RV1=  30,00 kN
UH2=   0,00/EA      RH2=  15,00 kN
UV2=   0,00/EA      RV2= -30,00 kN
UH3= -575,41/EA
UV3= -120,00/EA
```

```
NAA( 1,1)=   0,00   NAA( 1,2)=   0,00  kN
DAA( 1,1)=   0,00   DAA( 1,2)=   0,00  kN

NAA( 2,1)=  30,00   NAA( 2,2)= -30,00  kN
DAA( 2,1)=   0,00   DAA( 2,2)=   0,00  kN

NAA( 3,1)= -33,54   NAA( 3,2)=  33,54  kN
DAA( 3,1)=   0,00   DAA( 3,2)=   0,00  kN
```

Here NAA(P,1) and NAA(P,2), DAA(P,1) and DAA(P,2),
(P,1) is (L,H) and (P,2) is (H,L).

Changing some data, see fig.4.
Click CSE=0 to CSE=1 red, in TSTRING, PH(1)=1 Enter, PH(2)=1 Enter and FX(3)=15 Enter.

Click Again, Show 2 times, Calculate and NAADAA. See the results here below.

Member 1 is not anymore a zero member!
UH(3)=(575,41+30,00)/EA=605,41/EA

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	0
2	0	0	0	2	0	1	0	0
3	15	0	0	0	0	0	0	4

P	L	H	A1	QE
1	1	2	1	0
2	1	3	1	0
3	2	3	1	0

```
UH1=   0,00/EA      RH1= -15,00 kN
UV1=   0,00/EA      RV1= -30,00 kN
UH2=  30,00/EA
UV2=   0,00/EA      RV2=  30,00 kN
UH3= 605,41/EA
UV3= 120,00/EA
```

```
NAA( 1,1)= -15,00   NAA( 1,2)=  15,00  kN
DAA( 1,1)=   0,00   DAA( 1,2)=   0,00  kN

NAA( 2,1)= -30,00   NAA( 2,2)=  30,00  kN
DAA( 2,1)=   0,00   DAA( 2,2)=   0,00  kN

NAA( 3,1)=  33,54   NAA( 3,2)= -33,54  kN
DAA( 3,1)=   0,00   DAA( 3,2)=   0,00  kN
```

Example.   Click EX1b to EX1c.

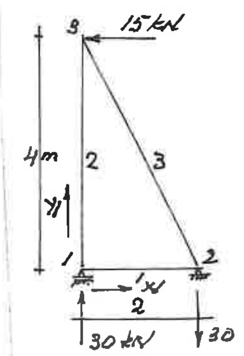


Fig.4.

N9=3 joints

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	0	0	0	0	1	0	0
2	0	1	0	2	0	1	0	0
3	-15	0	0	0	0	1	0	4

P9=7 members

P	L	H	A1	QE
1	1	2	1	0
2	1	3	1	0
3	2	3	1	0

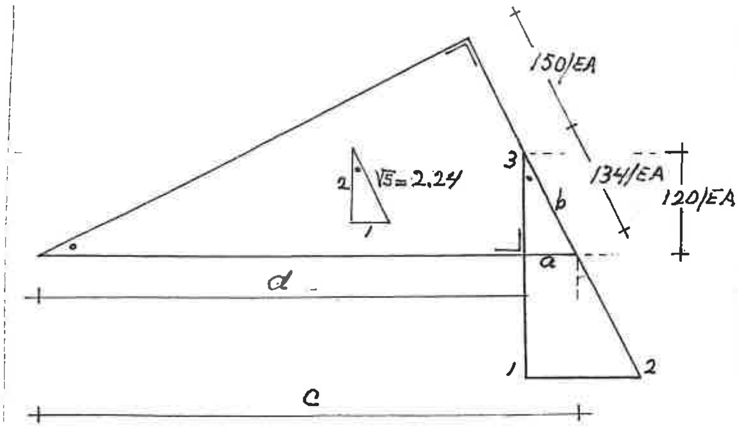


Fig.5.

Displacements of joint 3.
ΔL= FL/EA Hooke's law.

Member 3.
Length of member 3 is SQR(2^2+5^2)= 4,47 m.
NAA(3,2)= 33,54 kN, draws at end 3 of member 3, member 3 is a tension member, becomes longer, (33,54*4,47)/EA= 150/EA.

Member 2
Length of member 2 is 4 m.
NAA(3,1)= -30,00 kN, pushes at end 3, so member 2 is a compression member, becomes shorter, (30,00*4)/EA= 120/EA.

b/(120/EA)= 2,24/2   b=(2,24/2)*120/EA= 134/EA

a/1= b/2,24   a=1*(134/EA)/2,24= 60/EA

c/(150/EA+134/EA)= 2,24/1
c=(284/EA)*2,24/1= 636/EA

d= c-a= 636/EA-60/EA= 576/EA   is   UH(3)=-575/EA

P	L	H	A1	QE
1	1	2	1	0,2
2	2	3	1	0,2
3	4	5	1	0,2
4	1	4	1	0,2
5	2	4	1	0,2
6	2	5	1	0,2
7	3	5	1	0,2

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	0
2	0	0	0	6	-7	0	0	0
3	0	0	0	12	0	1	0	0
4	0	0	0	3	-5	0	0	4
5	0	0	0	9	0	0	0	4

UH1= 0,00/EA RH1= 0,00 kN
UV1= 0,00/EA RV1= 11,05 kN
UH2= 44,78/EA
UV2= -178,16/EA
UH3= 78,30/EA
UV3= 0,00/EA RV3= 8,55 kN
UH4= 71,89/EA
UV4= -131,65/EA
UH5= 12,04/EA
UV5= -107,90/EA

member 1

D5(1,2)=	-7,46 kN	D5(2,1)=	7,46 kN
D6(1,2)=	0,60 kN	D6(2,1)=	0,60 kN
NAA(1,2)=	-7,46 kN	NAA(2,1)=	7,46 kN
DAA(1,2)=	0,60 kN	DAA(2,1)=	0,60 kN

member 2

D5(2,3)=	-5,59 kN	D5(3,2)=	5,59 kN
D6(2,3)=	0,60 kN	D6(3,2)=	0,60 kN
NAA(2,3)=	-5,59 kN	NAA(3,2)=	5,59 kN
DAA(2,3)=	0,60 kN	DAA(3,2)=	0,60 kN

member 3

D5(4,5)=	9,98 kN	D5(5,4)=	-9,98 kN
D6(4,5)=	0,60 kN	D6(5,4)=	0,60 kN
NAA(4,5)=	9,98 kN	NAA(5,4)=	-9,98 kN
DAA(4,5)=	0,60 kN	DAA(5,4)=	0,60 kN

member 4

D5(1,4)=	7,46 kN	D5(4,1)=	-7,46 kN
D6(1,4)=	10,45 kN	D6(4,1)=	-9,45 kN
NAA(1,4)=	12,84 kN	NAA(4,1)=	-12,04 kN
DAA(1,4)=	0,30 kN	DAA(4,1)=	0,30 kN

member 5

D5(2,4)=	2,51 kN	D5(4,2)=	-2,51 kN
D6(2,4)=	-2,85 kN	D6(4,2)=	3,85 kN
NAA(2,4)=	-3,79 kN	NAA(4,2)=	4,59 kN
DAA(2,4)=	-0,30 kN	DAA(4,2)=	-0,30 kN

member 6

D5(2,5)=	-4,39 kN	D5(5,2)=	4,39 kN
D6(2,5)=	-5,35 kN	D6(5,2)=	6,35 kN
NAA(2,5)=	-6,91 kN	NAA(5,2)=	7,71 kN
DAA(2,5)=	0,30 kN	DAA(5,2)=	0,30 kN

member 7

D5(3,5)=	-5,59 kN	D5(5,3)=	5,59 kN
D6(3,5)=	7,95 kN	D6(5,3)=	-6,95 kN
NAA(3,5)=	9,71 kN	NAA(5,3)=	-8,91 kN
DAA(3,5)=	-0,30 kN	DAA(5,3)=	-0,30 kN

Example. Click EX2 to EX2a.

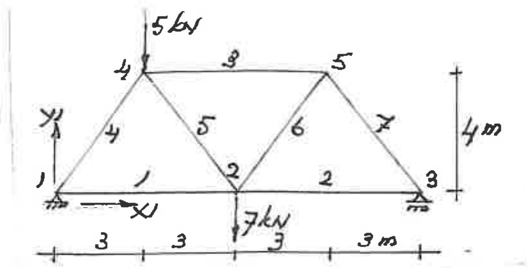


Fig.6.

N9=5 joints

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	0
2	0	0	0	6	-7	0	0	0
3	0	0	0	12	0	1	0	0
4	0	0	0	3	-5	0	0	4
5	0	0	0	9	0	0	0	4

P9=7 members

P	L	H	A1	QE	P	L	H	A1	QE
1	1	2	1	.2	5	2	4	1	.2
2	2	3	1	.2	6	2	5	1	.2
3	4	5	1	.2	7	3	5	1	.2
4	1	4	1	.2					

After input of joint and member data click Show 2 times and Calculate.
Next click member end forces,
first click shows the member end forces for the members 1 to 6, second click of forces of member 7.

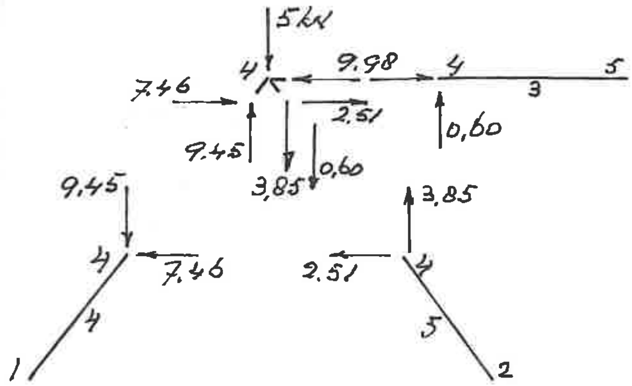


Fig.7.

Joint 4 is separated from the members. The member end forces D5(,) and D6(,) of the member ends 4 of the members 3, 4 and 5 are drawn with their real directions, values are added.

At joint 4 act forces as large as but opposite directed, values are added.

∑ hor.= 0 ? 7,46+2,51-9,89= -0,01 ok
or 9,86-7,46-2,51= 0,01 ok

∑ vert. =0 ? 5,00+3,85+0,60-9,45= 0,00 ok
or 9,45-5,00-3,85-0,60= 0,00 ok

(All done without sign conventions.)

UH1=	0,00/EA	RH1=	0,00 kN
UV1=	0,00/EA	RV1=	6,00 kN
UH2=	0,00/EA		
UV2=	-193,58/EA		
UH3=	0,00/EA		
UV3=	0,00/EA	RV3=	6,00 kN
UH4=	42,86/EA		
UV4=	-21,00/EA		
UH5=	0,00/EA		
UV5=	-193,58/EA		
UH6=	-42,86/EA		
UV6=	-21,00/EA		

member 1			
DS(1,2)=	0,00 kN	DS(2,1)=	0,00 kN
D6(1,2)=	0,00 kN	D6(2,1)=	0,00 kN
NAA(1,2)=	0,00 kN	NAA(2,1)=	0,00 kN
DAA(1,2)=	0,00 kN	DAA(2,1)=	0,00 kN
member 2			
DS(2,3)=	0,00 kN	DS(3,2)=	0,00 kN
D6(2,3)=	0,00 kN	D6(3,2)=	0,00 kN
NAA(2,3)=	0,00 kN	NAA(3,2)=	0,00 kN
DAA(2,3)=	0,00 kN	DAA(3,2)=	0,00 kN
member 3			
DS(1,4)=	0,00 kN	DS(4,1)=	0,00 kN
D6(1,4)=	6,00 kN	D6(4,1)=	-6,00 kN
NAA(1,4)=	6,00 kN	NAA(4,1)=	-6,00 kN
DAA(1,4)=	0,00 kN	DAA(4,1)=	0,00 kN
member 4			
DS(2,4)=	8,57 kN	DS(4,2)=	-8,57 kN
D6(2,4)=	-6,00 kN	D6(4,2)=	6,00 kN
NAA(2,4)=	-10,46 kN	NAA(4,2)=	10,46 kN
DAA(2,4)=	0,00 kN	DAA(4,2)=	0,00 kN
member 5			
DS(2,5)=	0,00 kN	DS(5,2)=	0,00 kN
D6(2,5)=	0,00 kN	D6(5,2)=	0,00 kN
NAA(2,5)=	0,00 kN	NAA(5,2)=	0,00 kN
DAA(2,5)=	0,00 kN	DAA(5,2)=	0,00 kN
member 6			
DS(2,6)=	-8,57 kN	DS(6,2)=	8,57 kN
D6(2,6)=	-6,00 kN	D6(6,2)=	6,00 kN
NAA(2,6)=	-10,46 kN	NAA(6,2)=	10,46 kN
DAA(2,6)=	0,00 kN	DAA(6,2)=	0,00 kN

member 7			
DS(3,6)=	0,00 kN	DS(6,3)=	0,00 kN
D6(3,6)=	6,00 kN	D6(6,3)=	-6,00 kN
NAA(3,6)=	6,00 kN	NAA(6,3)=	-6,00 kN
DAA(3,6)=	0,00 kN	DAA(6,3)=	0,00 kN
member 8			
DS(4,5)=	8,57 kN	DS(5,4)=	-8,57 kN
D6(4,5)=	0,00 kN	D6(5,4)=	0,00 kN
NAA(4,5)=	8,57 kN	NAA(5,4)=	-8,57 kN
DAA(4,5)=	0,00 kN	DAA(5,4)=	0,00 kN
member 9			
DS(5,6)=	8,57 kN	DS(6,5)=	-8,57 kN
D6(5,6)=	0,00 kN	D6(6,5)=	0,00 kN
NAA(5,6)=	8,57 kN	NAA(6,5)=	-8,57 kN
DAA(5,6)=	0,00 kN	DAA(6,5)=	0,00 kN

Example. Click EX2a to EX2b.

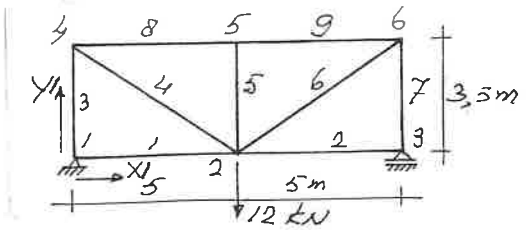


Fig.8.

N9=6 joints

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	0
2	0	0	0	5	**-12**	0	0	0
3	0	0	0	10	0	1	0	0
4	0	0	0	0	0	0	0	3.5
5	0	0	0	5	0	0	0	3.5
6	0	0	0	10	0	0	0	3.5

P9=9 members

P	L	H	A1	QE	P	L	H	A1	QE
1	1	2	1	0	6	2	6	1	0
2	2	3	1	0	7	3	6	1	0
3	1	4	1	0	8	4	5	1	0
4	2	4	1	0	9	5	6	1	0
5	2	5	1	0					

After input of joint and member data click Show 2 times and Calculate. Next click member end forces, first click shows the member end forces for the members 1 to 6, second click forces of member 7 to 9.

Construction is symmetric, load forces and equal strain stiffness of allmembers. Then e.g.

NAA(4,2)= 10,46 kN and NAA(6,2)= 10,46 kN.

UV(4)= -21,00/EA and UV(6)= -20,00/EA. Etc.

UV(I) is assumed upward, so a negative value means not as assumed but opposite direction, thus downward.

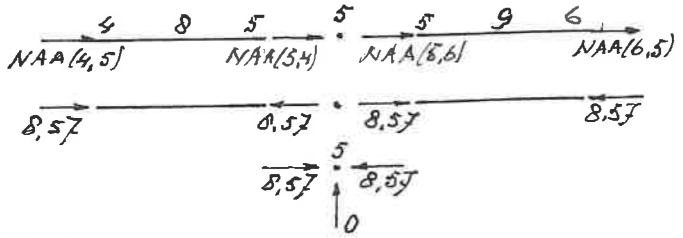


Fig.9.

Suppose no joint load force, FY(2)=0, UV(2) is prescribed PV(2)=1 and UV(2)=-193,58/EA. After the calculation here above click Again to make all displacements UH(I) and UV(I) zero.

Then click CSE=0 to CSE=1, getting red, and type in TSTRIBG, set the cursor there, and type FY(2)=0 Enter, PV(2)=1 Enter and UV(2)=-193.58 Enter (Show with -193,58 over-lapping with value of Y1, or type just -193). Click Show 2 times to check the data put in, click Calculate, the displacements are the same and click member end forces and see that the member end forces are the same as well.

UH1= 0,00/EA	RH1= 0,00 kN
UV1= 0,00/EA	RV1= 7,07 kN
UH2= 26,52/EA	
UV2=-152,51/EA	
UH3= 56,52/EA	
UV3=-168,96/EA	
UH4= 86,25/EA	
UV4= 0,00/EA	RV4= 7,93 kN
UH5= 65,58/EA	
UV5=-104,43/EA	
UH6= 43,48/EA	
UV6=-153,72/EA	
UH7= 14,35/EA	
UV7=-115,86/EA	

member 1	
DS(1,2)= -5,30 kN	DS(2,1)= 5,30 kN
D6(1,2)= 0,00 kN	D6(2,1)= 0,00 kN
NAA(1,2)= -5,30 kN	NAA(2,1)= 5,30 kN
DAA(1,2)= 0,00 kN	DAA(2,1)= 0,00 kN
member 2	
DS(2,3)= -7,50 kN	DS(3,2)= 7,50 kN
D6(2,3)= 0,00 kN	D6(3,2)= 0,00 kN
NAA(2,3)= -7,50 kN	NAA(3,2)= 7,50 kN
DAA(2,3)= 0,00 kN	DAA(3,2)= 0,00 kN
member 3	
DS(3,4)= -5,95 kN	DS(4,3)= 5,95 kN
D6(3,4)= 0,00 kN	D6(4,3)= 0,00 kN
NAA(3,4)= -5,95 kN	NAA(4,3)= 5,95 kN
DAA(3,4)= 0,00 kN	DAA(4,3)= 0,00 kN

UH1= -86,25/EA	
UV1= 0,00/EA	RV1= 7,93 kN
UH2= -56,52/EA	
UV2=-168,96/EA	
UH3= -26,52/EA	
UV3=-152,51/EA	
UH4= 0,00/EA	RH4= 0,00 kN
UV4= 0,00/EA	RV4= 7,07 kN
UH5= -14,35/EA	
UV5=-115,86/EA	
UH6= -43,48/EA	
UV6=-153,72/EA	
UH7= -65,58/EA	
UV7=-104,43/EA	

member 1	
DS(1,2)= -5,95 kN	DS(2,1)= 5,95 kN
D6(1,2)= 0,00 kN	D6(2,1)= 0,00 kN
NAA(1,2)= -5,95 kN	NAA(2,1)= 5,95 kN
DAA(1,2)= 0,00 kN	DAA(2,1)= 0,00 kN
member 2	
DS(2,3)= -7,50 kN	DS(3,2)= 7,50 kN
D6(2,3)= 0,00 kN	D6(3,2)= 0,00 kN
NAA(2,3)= -7,50 kN	NAA(3,2)= 7,50 kN
DAA(2,3)= 0,00 kN	DAA(3,2)= 0,00 kN
member 3	
DS(3,4)= -5,30 kN	DS(4,3)= 5,30 kN
D6(3,4)= 0,00 kN	D6(4,3)= 0,00 kN
NAA(3,4)= -5,30 kN	NAA(4,3)= 5,30 kN
DAA(3,4)= 0,00 kN	DAA(4,3)= 0,00 kN

Example. Click EX2b to EX2c.

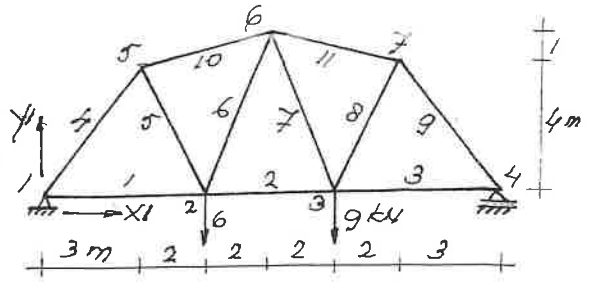


Fig.8.

N9=7 joints

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	0
2	0	0	0	5	-6	0	0	0
3	0	0	0	9	-9	0	0	0
4	0	0	0	14	0	1	0	0
5	0	0	0	3	0	0	0	4
6	0	0	0	7	0	0	0	5
7	0	0	0	11	0	0	0	4

P9=11 members

P	L	H	A1	QE	P	L	H	A1	QE
1	1	2	1	0	7	3	6	1	0
2	2	3	1	0	8	3	7	1	0
3	3	4	1	0	9	4	7	1	0
4	1	5	1	0	10	5	6	1	0
5	2	5	1	0	11	6	7	1	0
6	2	6	1	0					

Click Show 2 times, click Cls and Calculate to see the reactions and displacements printed, when N9>6 then it looks different than before. Further click member end forces, some results shown on the left.

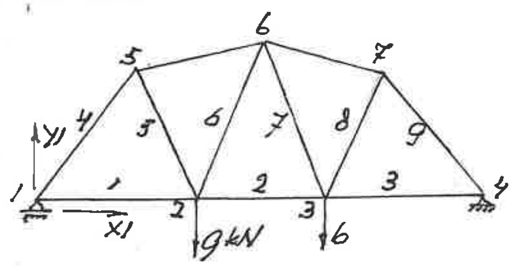


Fig.9.

Now FY(2)=9, FY(3)=6 i.s.o. FY(2)=6, FY(2)=9. When going on with the last results, click CLS=0 to CLS=1 red, and type in TSTRING

FY(2)=9 Enter, FY(3)=6 Enter.

The roller hinge now at joint 4, so PH(1)=0 Enter and at joint 4 PH(4)=1 Enter.

Click Again to make calculated displacements zero, next Show 2 times to check the data put in and Calculate, see the results on the left, to compare.

1st case	2nd case
UV(2)= -152,51/EA	UV(3)= -152,51
UH(5)= 65,58/EI	UH(7)= -65,58
NAA(1,2)= -5,30 kN	NAA(4,3)= 5,30 kN
NAA(4,3)= 5,95 kN	NAA(1,2)= -5,95 kN

UH1=	0,00/EA	RH1=	12,00 kN
UV1=	0,00/EA	RV1=	15,07 kN
UH2=	-12,36/EA		
UV2=	0,00/EA	RV2=	-8,93 kN
UH3=	-67,96/EA		
UV3=	-17,93/EA		
UH4=	-82,23/EA		
UV4=	18,07/EA		
UH5=	-31,64/EA		
UV5=	-1,79/EA		

NAA(1,1)=	4,12	NAA(1,2)=	-4,12	kN
DAA(1,1)=	0,45	DAA(1,2)=	0,45	kN
NAA(2,1)=	4,76	NAA(2,2)=	-4,76	kN
DAA(2,1)=	0,45	DAA(2,2)=	0,45	kN
NAA(3,1)=	6,43	NAA(3,2)=	-5,53	kN
DAA(3,1)=	0,00	DAA(3,2)=	0,00	kN
NAA(4,1)=	-5,57	NAA(4,2)=	6,47	kN
DAA(4,1)=	0,00	DAA(4,2)=	0,00	kN
NAA(5,1)=	11,37	NAA(5,2)=	-10,92	kN
DAA(5,1)=	0,23	DAA(5,2)=	0,23	kN
NAA(6,1)=	-5,60	NAA(6,2)=	6,05	kN
DAA(6,1)=	-0,23	DAA(6,2)=	-0,23	kN
NAA(7,1)=	-6,95	NAA(7,2)=	6,50	kN
DAA(7,1)=	0,22	DAA(7,2)=	0,23	kN
NAA(8,1)=	10,02	NAA(8,2)=	-10,47	kN
DAA(8,1)=	-0,23	DAA(8,2)=	-0,23	kN

member 1			
D5(1,2)=	4,12 kN	D5(2,1)=	-4,12 kN
D6(1,2)=	0,45 kN	D6(2,1)=	0,45 kN
NAA(1,2)=	4,12 kN	NAA(2,1)=	-4,12 kN
DAA(1,2)=	0,45 kN	DAA(2,1)=	0,45 kN
member 2			
D5(3,4)=	4,76 kN	D5(4,3)=	-4,76 kN
D6(3,4)=	0,45 kN	D6(4,3)=	0,45 kN
NAA(3,4)=	4,76 kN	NAA(4,3)=	-4,76 kN
DAA(3,4)=	0,45 kN	DAA(4,3)=	0,45 kN
member 3			
D5(1,3)=	0,00 kN	D5(3,1)=	0,00 kN
D6(1,3)=	6,43 kN	D6(3,1)=	-5,53 kN
NAA(1,3)=	6,43 kN	NAA(3,1)=	-5,53 kN
DAA(1,3)=	0,00 kN	DAA(3,1)=	0,00 kN

member 7			
D5(3,5)=	-4,76 kN	D5(5,3)=	4,76 kN
D6(3,5)=	5,08 kN	D6(5,3)=	-4,44 kN
NAA(3,5)=	-6,95 kN	NAA(5,3)=	6,50 kN
DAA(3,5)=	0,22 kN	DAA(5,3)=	0,23 kN
member 8			
D5(4,5)=	-7,24 kN	D5(5,4)=	7,24 kN
D6(4,5)=	-6,92 kN	D6(5,4)=	7,56 kN
NAA(4,5)=	10,02 kN	NAA(5,4)=	-10,47 kN
DAA(4,5)=	-0,23 kN	DAA(5,4)=	-0,23 kN

Example. Click EX3 to EX3a.

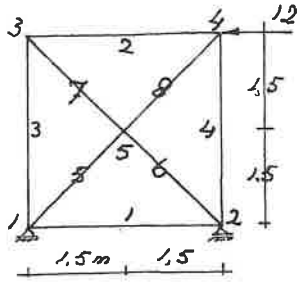


Fig.10.

N9=5 joints

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	0
2	0	0	0	3	0	1	0	0
3	0	0	0	0	0	0	0	3
4	-12	0	0	3	0	0	0	3
5	0	0	0	1.5	0	0	0	1.5

P9=8 members

P	L	H	A1	QE	P	L	H	A1	QE
1	1	2	1	.3	5	1	5	1	.3
2	3	4	1	.3	6	2	5	1	.3
3	1	3	1	.3	7	3	5	1	.3
4	2	4	1	.3	8	4	5	1	.3

After input click Show 2 times, Calculate and displacements and member end forces. Click NAADAA to get the member end forces on the left. and member end forces like shown until now. Some member end forces as example.

	NAA(P,1)		NAA(L,H)	
P=1	NAA(1,1)=	4,12 kN	NAA(1,2)=	4,12 kN
P=3	NAA(3,1)=	6,43 kN	NAA(1,3)=	6,43 kN
	NAA(P,2)		NAA(H,L)	
P=7	NAA(7,2)=	6,50 kN	NAA(5,3)=	6,50 kN
P=8	NAA(8,2)=	-10,47 kN	NAA(5,4)=	-10,47 kN

Same way with DAA(,). The calculations are carried out NAA(P,1) and NAA(P,2) page 25, for output into NAA(L,H) and NAA(H,L).

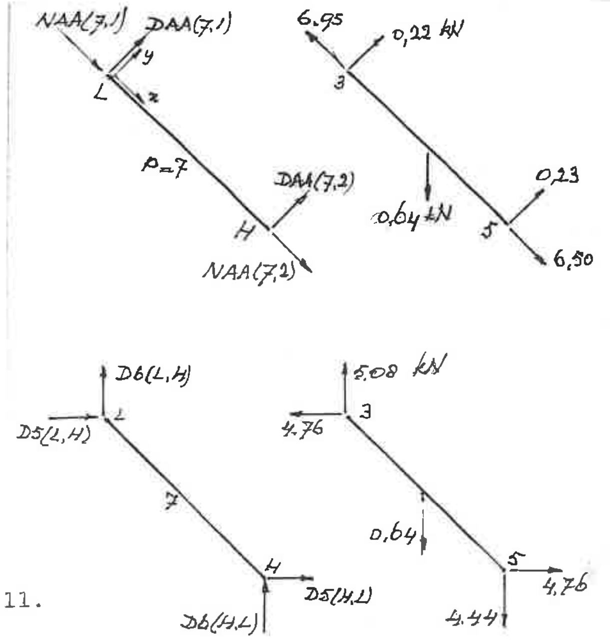


Fig.11.

Member 7. Member end forces drawn with their sssumed directions, and member end forces drawn with their real directions, values added.

```
UH1=     0,00/EA      RH1=    0,00  kN
UV1=     0,00/EA      RV1=    5,28  kN
UH2=    16,90/EA
UV2=   -63,72/EA
UH3=    33,79/EA
UV3=     0,00/EA      RV3=    5,28  kN
UH4=    27,72/EA
UV4=   -53,88/EA
UH5=     6,08/EA
UV5=   -53,88/EA
UH6=    16,90/EA
UV6=   -52,85/EA
```

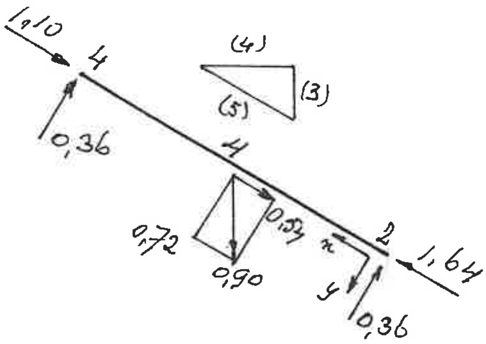


Fig.13.

NAA(1,1)=	-5,28	NAA(1,2)=	5,28	kN
DAA(1,1)=	0,72	DAA(1,2)=	0,72	kN
NAA(2,1)=	-5,28	NAA(2,2)=	5,28	kN
DAA(2,1)=	0,72	DAA(2,2)=	0,72	kN
NAA(3,1)=	6,96	NAA(3,2)=	-6,24	kN
DAA(3,1)=	0,48	DAA(3,2)=	0,48	kN
NAA(4,1)=	1,65	NAA(4,2)=	-1,11	kN
DAA(4,1)=	-0,36	DAA(4,2)=	-0,36	kN
NAA(5,1)=	-3,99	NAA(5,2)=	5,07	kN
DAA(5,1)=	0,00	DAA(5,2)=	0,00	kN
NAA(6,1)=	1,65	NAA(6,2)=	-1,11	kN
DAA(6,1)=	0,36	DAA(6,2)=	0,36	kN
NAA(7,1)=	6,96	NAA(7,2)=	-6,24	kN
DAA(7,1)=	-0,48	DAA(7,2)=	-0,48	kN
NAA(8,1)=	5,59	NAA(8,2)=	-4,87	kN
DAA(8,1)=	0,48	DAA(8,2)=	0,48	kN
NAA(9,1)=	5,59	NAA(9,2)=	-4,87	kN
DAA(9,1)=	-0,48	DAA(9,2)=	-0,48	kN

```
member 4
 D5(2,4)=   -1,10 kN    D5(4,2)=   1,10 kN
 D6(2,4)=    1,28 kN    D6(4,2)=  -0,38 kN
NAA(2,4)=    1,65 kN   NAA(4,2)=  -1,11 kN
DAA(2,4)=   -0,36 kN   DAA(4,2)=  -0,36 kN
```

Example.   Click EX3a to EX3b.

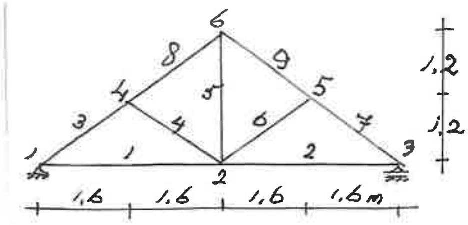


Fig.12.

N9=6 joints

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	0
2	0	0	0	3.2	0	0	0	0
3	0	0	0	6.4	0	1	0	0
4	0	0	0	1.6	0	0	0	1.2
5	0	0	0	4.8	0	0	0	1.2
6	0	0	0	3.2	0	0	0	2.4

P9=9 members   Only own weight.

P	L	H	A1	QE	P	L	H	A1	QE
1	1	2	1	.45	6	2	5	1	.45
2	2	3	1	.45	7	3	5	1.3	.6
3	1	4	1.3	.6	8	4	6	1.3	.6
4	2	4	1	.45	9	5	6	1.3	.6
5	2	6	1	.45					

After input click Show 2 times, Calculate and displacements and member end forces.

Calculate, the displacements and reactions are printed.
Click NAADAA to get the member end forces, NAA(P,1) and NAA(P,2).

Fig.13.

Member 4

See the preceding page for the assumed directions of the member end forces. Calculated value positive then directed as assumed, if negative then opposite directed. Drawn with their real directions, values (not + or -!), 'real' values added.

```
NAA(P,1)                 NAA(L,H)=  1,65 kN
NAA(4,1)=  1,65 kN       NAA(2,4)=  1,65 kN

NAA(P,2)                 NAA(H,L)
Naa(4,2)= -1,11 kN       NAA(4,2)= -1,11 kN
```

Length of member 4.
Sqr(1.6^2+1.2^2)= Sqr(2.56+1.44)= Sqr(4)= 2,0 m

Components of own weight of 0,45 kN/m.
(0,45/5)*3= 0,27 kN/m   (0,45/5)*4= 0,36 kN/m
Components of weight over 2 m,
0,27*2= 0,54 kN and 0,36*2= 0,72 kN.

Holding member end 4, as if a clamp.
The member lengthens by 0,27 kN/m along the member, formula Q*L^2/2EA,
(0,27*2^2)/2EA= 0,54/EA,
shortens by 1,64 kN, F*L/EA,
(1,64*2)/EA= 3,28/EA
Becoming (3,28-0,54)/EA= 2,74/EA shorter.

member 1			
DS(1,2)=	-7,30 kN	DS(2,1)=	7,30 kN
D6(1,2)=	0,00 kN	D6(2,1)=	0,00 kN
NAA(1,2)=	-7,30 kN	NAA(2,1)=	7,30 kN
DAA(1,2)=	0,00 kN	DAA(2,1)=	0,00 kN
member 2			
DS(1,3)=	7,30 kN	DS(3,1)=	-7,30 kN
D6(1,3)=	9,73 kN	D6(3,1)=	-9,73 kN
NAA(1,3)=	12,17 kN	NAA(3,1)=	-12,17 kN
DAA(1,3)=	0,00 kN	DAA(3,1)=	0,00 kN
member 3			
DS(2,3)=	0,00 kN	DS(3,2)=	0,00 kN
D6(2,3)=	9,73 kN	D6(3,2)=	-9,73 kN
NAA(2,3)=	9,73 kN	NAA(3,2)=	-9,73 kN
DAA(2,3)=	0,00 kN	DAA(3,2)=	0,00 kN
member 4			
DS(2,4)=	-7,30 kN	DS(4,2)=	7,30 kN
D6(2,4)=	-9,73 kN	D6(4,2)=	9,73 kN
NAA(2,4)=	-12,17 kN	NAA(4,2)=	12,17 kN
DAA(2,4)=	0,00 kN	DAA(4,2)=	0,00 kN
member 5			
DS(3,4)=	3,65 kN	DS(4,3)=	-3,65 kN
D6(3,4)=	0,00 kN	D6(4,3)=	0,00 kN
NAA(3,4)=	3,65 kN	NAA(4,3)=	-3,65 kN
DAA(3,4)=	0,00 kN	DAA(4,3)=	0,00 kN
member 6			
DS(4,5)=	-3,65 kN	DS(5,4)=	3,65 kN
D6(4,5)=	0,00 kN	D6(5,4)=	0,00 kN
NAA(4,5)=	-3,65 kN	NAA(5,4)=	3,65 kN
DAA(4,5)=	0,00 kN	DAA(5,4)=	0,00 kN

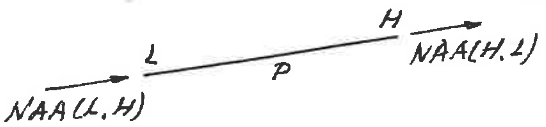


Fig.15

Drawn assumptions:
NAA(L,H) pushes at member end L, thus member P is a compression member,
NAA(H,L) pulls at member end H, thus member P is a tension member.

member 7			
DS(3,6)=	3,65 kN	DS(6,3)=	-3,65 kN
D6(3,6)=	4,87 kN	D6(6,3)=	-4,87 kN
NAA(3,6)=	6,08 kN	NAA(6,3)=	-6,08 kN
DAA(3,6)=	0,00 kN	DAA(6,3)=	0,00 kN
member 8			
DS(4,6)=	0,00 kN	DS(6,4)=	0,00 kN
D6(4,6)=	-9,73 kN	D6(6,4)=	9,73 kN
NAA(4,6)=	-9,73 kN	NAA(6,4)=	9,73 kN
DAA(4,6)=	0,00 kN	DAA(6,4)=	0,00 kN
member 9			
DS(5,6)=	-3,65 kN	DS(6,5)=	3,65 kN
D6(5,6)=	4,87 kN	D6(6,5)=	-4,87 kN
NAA(5,6)=	6,08 kN	NAA(6,5)=	-6,08 kN
DAA(5,6)=	0,00 kN	DAA(6,5)=	0,00 kN

Example. Click EX3b to EX3c.

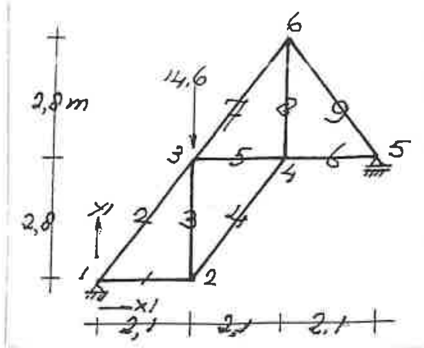


Fig.14.

N9=6 joints

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	0
2	0	0	0	2.1	0	0	0	0
3	0	0	0	2.1	-14.6	0	0	2.8
4	0	0	0	4.2	0	0	0	2.8
5	0	0	0	6.3	0	1	0	2.8
6	0	0	0	4.2	0	0	0	5.6

P9=9 members

P	L	H	A1	QE	P	L	H	A1	QE
1	1	2	1	0	6	4	5	1	0
2	1	3	1	0	7	3	6	1	0
3	2	3	1	0	8	4	6	1	0
4	2	4	1	0	9	5	6	1	0
5	3	4	1	0					

After input click Show 2 times to check the input of data, Calculate and click member end forces.

Fig.15.

The assumed direction of the member end forces NAA(L,H) and NAA(H,L).
When looking at member end L with NAA(L,H) one can determine if the member is a compression member or a tension member.

1 NAA(1,2)= -7,30 kN, not a compression mem- as assumed but a tension member.
2 NAA(1,3)= 12,17 kN, a compression member as assumed.

3 NAA(2,3)= 5,73 kN, a compression member.
4 NAA(2,4)= -12,17 kN, a tension member.
5 NAA(3,4)= 3,65 kN, a compression member.

6 NAA(4,5)= -3,65 kN, a tension member.
7 NAA(3,6)= 6,08 kN, a compression member.
8 NAA(4,6)= -9,73 kB, a tension member.
9 NAA(5,6)= 6,08 kN, a compression member.

And looking at member end H, same way.
1 NAA(2,1)= 7,30 kN, a tension member as assumed.
2 NAA(3,1)= -12,17 kN, not a tension member as assumed but a compression member.
3 NAA(3,2)= -9,73 kN, a compression member.
4 NAA(4,2)= 12,17 kN, a tension member.

These way to decide compression or tension can and is programmed as well.

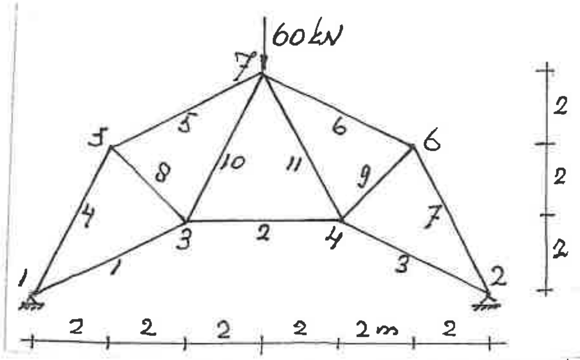


Fig.16.

```
UH1=     0,00/EA        RH1=  29,11 kN
UV1=     0,00/EA        RV1=  30,00 kN
UH2=     0,00/EA        RH2= -29,11 kN
UV2=     0,00/EA        RV2=  30,00 kN
UH3=    -2,67/EA
UV3= -204,97/EA
UH4=     2,67/EA
UV4= -204,97/EA
UH5=    19,28/EA
UV5= -124,77/EA
UH6=   -19,28/EA
UV6= -124,77/EA
UH7=     0,00/EA
UV7= -316,45/EA

member 1
 D5(1,3)=  18,81 kN    D5(3,1)=-18,81 kN
 D6(1,3)=   9,41 kN    D6(3,1)= -9,41 kN
NAA(1,3)=  21,03 kN   NAA(3,1)=-21,03 kN
DAA(1,3)=   0,00 kN   DAA(3,1)=  0,00 kN
```

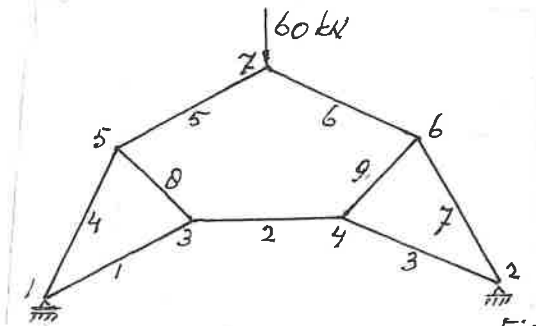


Fig.17.

```
3UH1=80000,00/EA
 UV1=    0,00/EA        RV1= -16,00 kN
 UH2=    0,00/EA        RH2=-160,00 kN
 UV2=    0,00/EA        RV2= 160,00 kN
8UH3=70000,00/EA
9UV3=25000,00/EA
8UH4=70000,00/EA
7UV4=40000,00/EA

member 1
 D5(1,3)=   0,00 kN    D5(3,1)=   0,00 kN
 D6(1,3)=   0,00 kN    D6(3,1)=   0,00 kN
NAA(1,3)=   0,00 kN   NAA(3,1)=   0,00 kN
DAA(1,3)=   0,00 kN   DAA(3,1)=   0,00 kN
```

Example.   Click EX3c to EX3d.

Fig.16.

N9=7 joints

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	0
2	0	1	0	12	0	1	0	0
3	0	0	0	4	0	0	0	2
4	0	0	0	8	0	0	0	2
5	0	0	0	2	0	0	0	4
6	0	0	0	10	0	0	0	4
7	0	0	0	6	-60	0	0	6

(input 60 kN, Show shows -60 kN)

P9=11 members

P	L	H	A1	QE	P	L	H	A1	QE
1	1	3	1	0	7	2	6	1	0
2	3	4	1	0	8	3	5	1	0
3	2	4	1	0	9	4	6	1	0
4	1	5	1	0	10	3	7	1	0
5	5	7	1	0	11	4	7	1	0
6	6	7	1	0					

After input of data click Show 2 times, Calculate and 'displacements...' 2 times to get the form print shown on the left. Symmetry, UH3= -2,67/EA, that's to the left and UH4= 2,67/EA, that's to the right. Etc.

Going on with this example.
Click Again to make all UH(I)=0 and UV(I)=0, click Calculate etc.

Now support 1 becomes a roller support, then click CSE=0 to CSE=1 red, and type in TSTRING PH(1)=0 Enter, click Again, next Show 2 times to check the change. Click Cls, Calculate etc., see some results here below.
UH(1)=-1202,58/EA, joint 1 moves to the left.

```
UH1=-1202,58/EA        RV1=  30,00 kN
UV1=     0,00/EA        RH2=   0,00 kN
UH2=     0,00/EA        RV2=  30,00 kN
UV2=     0,00/EA
UH3=  -691,29/EA
UV3=  -798,98/EA
UH4=  -511,29/EA
UV4=  -798,98/EA
UH5=  -553,57/EA
UV5=  -548,12/EA
UH6=  -649,02/EA
UV6=  -548,12/EA
UH7=  -601,29/EA
UV7=  -899,88/EA

member 1
 D5(1,3)= -20,00 kN    D5(3,1)= 20,00 kN
 D6(1,3)= -10,00 kN    D6(3,1)= 10,00 kN
NAA(1,3)= -22,36 kN   NAA(3,1)= 22,36 kN
DAA(1,3)=   0,00 kN   DAA(3,1)=  0,00 kN
```

Fig 17.
Suppose member 10 and 11 are removed, the number of members becomes P9=9.
Click CSE=0 to CSE=1 red and type in TSTRING P9=9 Enter.
Click Again and Show 2 times to check the change, click Cls.
Click Calculate etc. See some results on the left, chaos, the truss is not stable.

```
Private Sub TRUSSMAINCALC()    page 21
'1. Composition of construction ma-
'trix CC with member matrices S5.
CONSTRMATCCTRUSS

'2.Elements of force vector FF.
'2a. Joint load forces FX(I)
'and FY(I).
N=2*N9
For I=1 To N9
A=2*I-1 : B=2*I
FF(A)=FX(I)  : FF(B)=FY(I)
PP(A)=PH(I)  : PP(B)=PV(I)
UU(A)=UH(I)  : UU(B)=UV(I)
SS(A)=SH(I)  : SS(B)=SV(I)
Next I

'2b. Primary forces due to own
'weight.
For P=1 To P9 : L=LL(P)  : H=HH(P)
QE=QEE(P)
D1=X1(H)-X1(L)  : D2=Y1(H)-Y1(L)
L1=Sqr(D1^2+D2^2)  : L11(P)=L1
RT=QE*L1/2
D4(P,1)=-RT  : D4(P,2)=-RT
Next P

'2c. Alteration of force vector FF.
For I=1 To N9
B=2*I
For P=1 TO P9  : L=LL(P)  : H=HH(P)
If I=L Then
FF(B)=FF(B)+D4(P,1)
Else If I=H Then
FF(B)=FF(B)+D4(P,2)
End If
Next P
Next I

'3. Alteration of force vector FF and
'construction matrix CC.
'3a. Of FF in case of prescribed
'displacements <>0.
For I=1 To N
If UU(I)<>0 Then
For K=1 To N
FF(K)=FF(K)-CC(K,I)*UU(I)
Next K
End If
Next I
'3b. Of FF and CC in case of pres-
'cribed displacements.
For I=1 To N
If PP(I)=1 Then
For J=1 To N
CC(I,J)=0  : CC(J,I)=0
Next J
CC(I,I)=1  : FF(I)=UU(I)
End If
Next I
'3c. Of CC in case of springy
'supports.
For I=1 To N
If SS(I)>0 Then CC(I,I)=CC(I,I)+SS(I)
Next I

'4. Calculation of the unknown
'displacements UH(I) and UV(I).
For I=1 To N : BB(I)=FF(I)
For J=1 To N
AA(I,J)=CC(I,J)
Next J
Next I
'The solution of N=2*N9 equations.
GAUSS
For I=1 To N9
A=2*I-1 : B=2*I
UH(I)=XX(A)  : UV(I)=XX(B)
UU(A)=XX(A)  : UU(B)=XX(B)
Next I
'5. Calculation of the member end
'forces w.r.t. the construction
'axes system X-Y.
'5a. Due to the displacements
'alone.
For P=1 To P9 : L=LL(P)  : H=HH(P)
EA=EAA(P)
MEMBERMATS5TRUSS
TT(1)=2*L-1  : TT(2)=2*L
TT(3)=2*H-1  : TT(4)=2*H
For I=1 To 4  : FK(P,I)=0
For J=1 To 4  : A=TT(J)
FK(P,I)=FK(P,I)+S5(I,J)*UU(A)
Next J
Next I

'5b. Due to displacements and
'own weight.
D5(P,1)=FK(P,1)
D6(P,1)=FK(P,2)-D4(P,1)
D5(P,2)=FK(P,3)
D6(P,2)=FK(P,4)-D4(P,2)

'5c.Computation of the member end
'forces w.r.t. the member axes
'system x-y.
D1=X1(H)-X1(L)  : D2=Y1(H)-Y1(L)
L1=Sqr(D1^2+D2^2)
S=D2/L1  : C=D1/L1
NAA(P,1)=D5(P,1)*C+D6(P,1)*S
NAA(P,2)=D5(P,2)*C+D6(P,2)*S
DAA(P,1)=D6(P,1)*C-D5(P,1)*S
DAA(P,2)=D6(P,2)*C-D5(P,2)*S
Next P
'6. Computation of the joint for-
'ces KH(I) and KV(I).
'6a. Due to the displacements
'alone.
CONSTRMATCCTRUSS
For I=1 To N9
A=2*I-1  : B=2*I
KH(I)=0  : KV(I)=0
For J=1 To N
KH(I)=KH(I)+CC(A,J)*UU(J)
KV(I)=KV(I)+CC(B,J)*UU(J)
Next J
'6b. Due to the displacements
'and own weight of the members.
For P=1 To P9  : L=LL(P)  : H=HH(P)
If I=L Then
KV(I)=KV(I)-D4(P,1)
ElseIf I=H Then
KV(I)=KV(I)-D4(P,2)
End If
Next P
Next I
```

```
'7. Calculation of the reactions.
For I=1 To N9
If SH(I)>0 Then
RH(I)=-SH(I)*UH(I)
Else
RH(I)=KH(I)-FX(I)
End If
If SV(I)>0 Then
RV(I)=-SV(I)*UV(I)
Else
RV(I)=KV(I)-FY(I)
End If
Next I

End Sub
```

```
Private Sub CONSTRMATCCTRUSS()
N=2*N9
For I=1 To N : For J=1 To N
CC(I,J)=0 : Next J : Next I
For P=1 To P9 : L=LL(P) : H=HH(P)
EA=EAA(P)
MEMBERMATS5TRUSS
TT(1)=2*L-1 : TT(2)=2*L
TT(3)=2*H-1 : TT(4)=2*H
For I=1 To 4 : I1=TT(I)
For J=1 To 4 : J1=TT(J)
CC(I1,J1)=CC(I1,J1)+S5(I,J)
Next J
Next I
Next P
End Sub
```

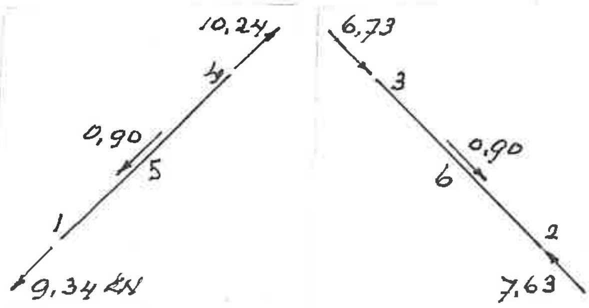
page 17

```
Private Sub MEMBERMATS5TRUSS()
D1=X1(H)-X1(L) : D2=Y1(H)-Y1(L)
L1=Sqr(D1^2+D2^2)
S=D2/L1 : C=D1/L1 : R=EA/L1
A1=R*C^2 : A2=R*S*C : A3=R*S^2

S5(1,1)=A1 : S5(1,2)=A2
S5(1,3)=-A1 : S5(1,4)=-A2
S5(2,2)=A3 : S5(2,3)=-A2
S5(2,4)=-A3
S5(3,3)=A1 : S5(3,4)=A2
S5(4,4)=A3
A=2 : For I=1 To 3
For J=A To 4
S5(J,I)=S5(I,J)
Next J
Next I

End Sub
```

page 20



And how will it looks like when support 1 is a roller support instead of support 2?

Example. See page 34.

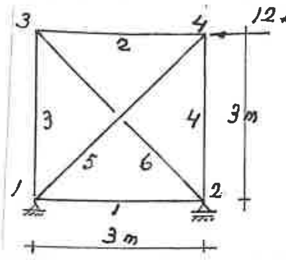


Fig.18a.

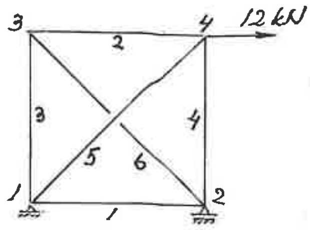


Fig.18b.

N9=**4** joints

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	0
2	0	0	0	3	0	1	0	0
3	0	0	0	0	0	0	0	3
4	**-12**	0	0	3	0	0	0	3

P9=6 members

P	L	H	A1	QE	P	L	H	A1	QE
1	1	2	1	.3	5	1	4	1	.3
2	3	4	1	.3	6	2	3	1	.3
3	1	3	1	.3					
4	2	4	1	.3					

After input clic Show 2 times, Calculatr, Cls, NAADAA, see the results here below.

	NAA(P,1)	NAA(L,H)
P=1	NAA(1,1)= 4,44 kN	NAA(1,2)= 4,44 kN
P=4	NAA(4,1)= -5.57 kN	NAA(2,4)= -5,57 kN
	NAA(P,2)	NAA(H,L)
P=5	NAA(5,2)=-10,24 kN	NAA(4,1)=-10,24 kN
P=6	NAA(6,2)= 6,73 kN	NAA(5,4)= 6,73 kN

Length of member 5 and 6 3*Sqr(2)= 4,24 m
Own weight vertical 0,3 kN/m
Along the member 4,24*(0,3/Sqr(2))= 0,90 kN

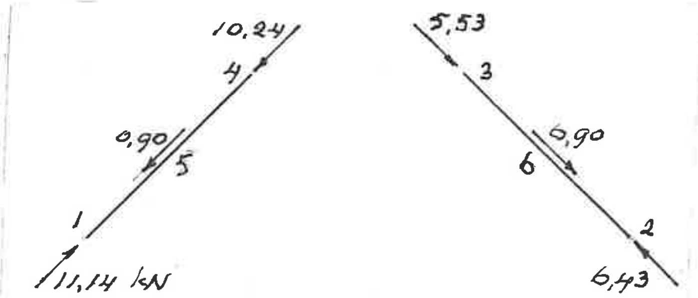


Fig.18b. 12 kN in opposite direction.

N9=5 joints

I	FX	PH	UH	X1	FY	PV	UV	Y1
1	0	1	0	0	0	1	0	0
2	0	0	0	3	0	1	0	0
3	0	0	0	0	0	0	0	3
4	**12**	0	0	3	0	0	0	3

	NAA(P,1)	NAA(L,H)
P=1	NAA(1,1)= -5,08 kN	NAA(1,2)= -5,08 kN
P=4	NAA(4,1)= 8,91 kN	NAA(2,4)= 8,91 kN
	NAA(P,2)	NAA(H,L)
P=5	NAA(5,2)= 10,24 kN	NAA(4,1)= 10,24 kN
P=6	NAA(6,2)= -6,73 kN	NAA(5,4)= -6,73 kN

the members 5 and 6 on the left.