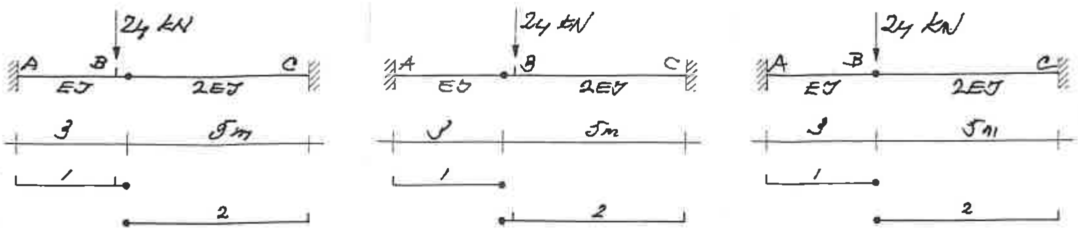
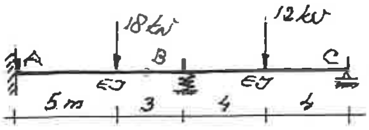
Part 6

The three following cases with determination of the three different member stiffness matrices S5 for a single beam.
Determination of the elements of the three different 4 x 4 stiffness matrices.





Composing construction matrix CC with member matrices S5.
Deterrmination of the S5's, matrix CC after composition, and CC before a main calculation of the unknows is to be carried out.

3.1. Beams over more than two supports, **without** vertical joint displacements and **without** internal hinges.

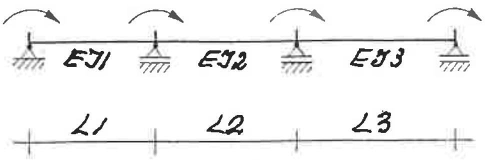


Fig.1.

Fig.1.
A continuous beam on four supports. Above the supports, which do not displace vertiacally, are joints assumed at which the beam ends are rigidly connected.
These joints can be indicated with short vertical little stripes, here thus above the supports. (Not yet joints between the beam ends, which can displace vertically.)
The joint can rotate, undergo a rotation.

The relation between member end moments and joint rotations of one single simple beam.

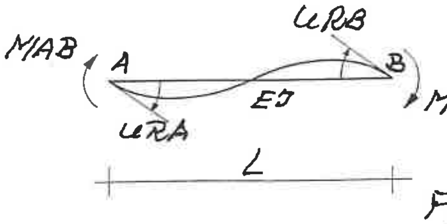


Fig.2a.

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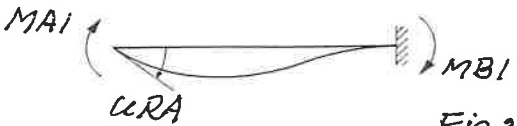

Fig.2b.

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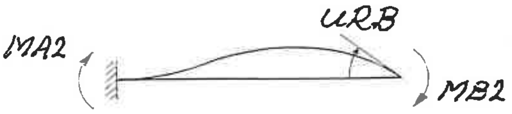


Fig.2c.

Fig.2a.
At the member ends of the from the joints seperated beam slope deflections, or rotations, can arise.
The direction of these rotations URA and URB is assumed to the right, as is drawn. (The joint rotations as well are assumed to the right.)
By deformation, bending, of the beam arise member end moments. These moments MAB and MBA are assumed to be directed to the right.

The beam of fig.2a can be seen as the sum of the figures 2b and 2c.

Fig.2b.
The beam is clamped on the right and hinged supported on the left.
Slope deflection URA at member end A is assumed to the right, from which follows that moment MA1 at A must be directed to the right.
By deformation of the beam arises at member end B a moment MB1 to the right. According to the page with formula, page 30, the slope deflection at A due to moment MA1,
URA=MA1*L/(4*EI). Then moment MA1 is

MA1=(4*EI/L)*URA. Further is MB1=MA1/2 so that

MB1=(2*EI/L)*URA.

Fig.2c.
In similar way one finds for this beam due to the slope deflection to the right URB the belonging moments MA2 and MB2.

MA2=(2*EI/L)*URB

MB2=(4*EI/L)*URB

Now fig.2a is the sum of fig.2b and 2c so that

MAB=MA1+MA2 or MAB=(4*EI/L)*URA+(2*EI/L)*URB

and

MBA=MB1+MB2 or MBA=(2*EI/L)*URA+(4*EI/L)*URB.

The relation between member end moments and joint rotations found this way is shown on the left in matrix form.

URA=MA1*L/(4*EI) MA1=(4*EI/L)*URA

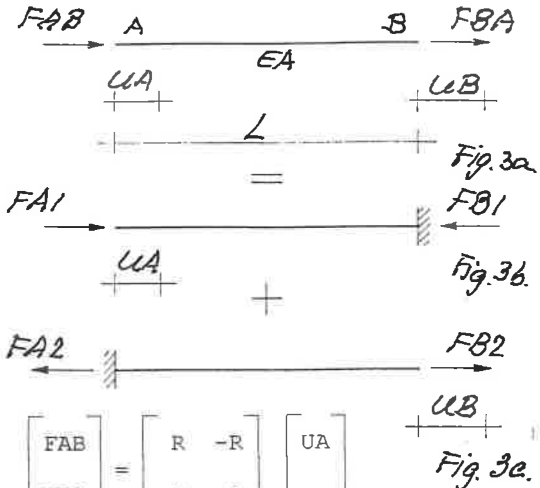
MB1=(2*EI/L)*URA

URB=MA2*L/(4*EI) MA2=(2*EI/L)*URB

MB2=(4*EI/L)*URB

$$\begin{bmatrix} MAB \\ MBA \end{bmatrix} = \begin{bmatrix} 4EI/L & 2EI/L \\ 2EI/L & 4EI/L \end{bmatrix} \cdot \begin{bmatrix} URA \\ URB \end{bmatrix}$$

$$\underset{\underline{f}}{\begin{bmatrix} MAB \\ MBA \end{bmatrix}} = \underset{S5}{\begin{bmatrix} D & E \\ E & D \end{bmatrix}} \cdot \underset{\underline{u}}{\begin{bmatrix} URA \\ URB \end{bmatrix}}$$



$$\begin{bmatrix} FAB \\ FBA \end{bmatrix} = \begin{bmatrix} R & -R \\ -R & R \end{bmatrix} \begin{bmatrix} UA \\ UB \end{bmatrix}$$

$\underline{f}$ S5 $\underline{u}$

Fig.3a, 3b en 3c.
For a moment back to part 4 page 1.
The same way as just before one can find the relation between member end forces and displacements for axially loaded members.
Fig. 3a is de som van fig.3b en 3c.

Fig. 3b.
The right member end is holded. To the assumed displacement UA to the right belongs a force FA1 to the right.

With Hooke ΔL=FL/EA follows UA=FA1*L/EA from which FA1=(EA/L)*UA,
and with R=EA/L becomes FA1=R*UA.

At B arises a to the left directed reaction force FB1, as large as FA1. FB1=FA1=R*UA.

Fig.3c.
With same reasoning follow
FB2=R*UB and FA2=FB2=R*UB.

Adding both cases gives

FAB= FA1-FA2 or <u>FAB= R*UA-R*UB</u>, and

FBA= FB2-FB1 or FBA=-FB1+FB2 so that

<u>FBA=-R*UA+R*UB</u>.

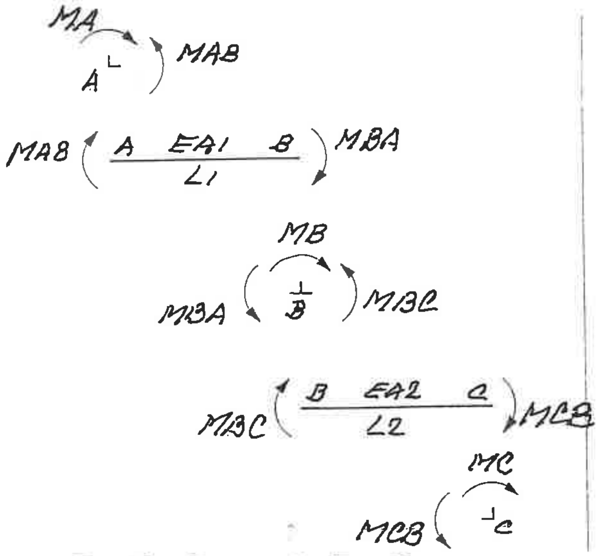


Fig.4.

$$\begin{bmatrix} MAB \\ MBA \end{bmatrix} = \begin{bmatrix} D1 & E1 \\ E1 & \underline{D1} \end{bmatrix} \cdot \begin{bmatrix} URA \\ URB \end{bmatrix}$$

$$\begin{bmatrix} MBC \\ MCB \end{bmatrix} = \begin{bmatrix} \underline{D2} & E2 \\ E2 & D2 \end{bmatrix} \cdot \begin{bmatrix} URB \\ URC \end{bmatrix}$$

$\underline{f}$ S5 $\underline{u}$

$$\begin{bmatrix} MAB \\ MBA+MBC \\ MCB \end{bmatrix} = \begin{bmatrix} D1 & E1 & 0 \\ E1 & \underline{D1+D2} & E2 \\ 0 & E2 & D2 \end{bmatrix} \cdot \begin{bmatrix} URA \\ URB \\ URC \end{bmatrix}$$

$\underline{f}$ C $\underline{u}$

$$\begin{bmatrix} D1 & E1 & 0 \\ E1 & \underline{D1+D2} & E2 \\ 0 & E2 & D2 \end{bmatrix} \cdot \begin{bmatrix} URA \\ URB \\ URC \end{bmatrix} = \begin{bmatrix} MA \\ MB \\ MC \end{bmatrix}$$

C $\underline{u}$ $\underline{f}$

Fig.4.
If the construction consists of two beams then there are two sets of equations f = S5 u.

MAB= D1*URA + E1*URA	D1=4*EI1/L1
MBA= E1*URA + D1*URB	E1=2*EI1/L1
MBC= D2*URB + E2*URC	D2=4*EI2/L2
MCB= E2*URB + D2*URC	E2=2*EI2/L2

Joined they deliver $\underline{f}$ = C $\underline{u}$.
The underlined elements of both S5's coincides in construction matrix C.

Joints and beams are seperated from each other.
On the joints act the joint load moments MA, MB en MC, assumption to the right.
On the member ends act the member end moments MAB and MBA for beam 1, and
MBC and MCB for beam 2, assumption to the right.
On the loosened joints act momemts as large as but opposite directed, thus to the left.
The elements of force vector f are found using equilibium of joints.

Σ mom. joint A =0
MA-MAB=0 ⇒ MAB=MA
Σ mom. joint B =0
MB-MBA-MBC=0 ⇒ MBA+MBC=MB
Σ mom. joint C =0
MC-MCB=0 ⇒ MCB=MC

Is none of the rotations, or slope deflections, prescribed then the unknowns URA, URB and URC can be solved from the three equations C $\underline{u}$ = $\underline{f}$.
(Unless there are member loads then $\underline{f}$ will be altered first; to be dealt with later.)

3.2. Beams over more than two supports **with** vertical joint displacements and **without** internal hinges.

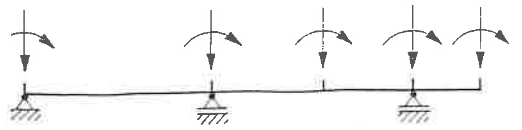


Fig.1.

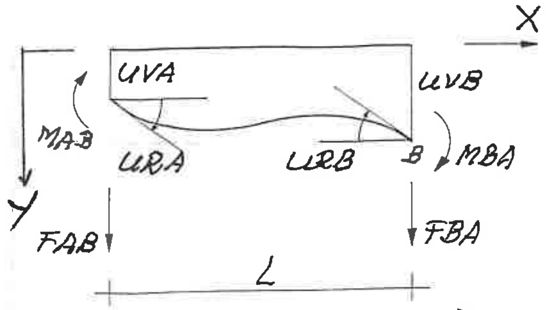


Fig.2a.

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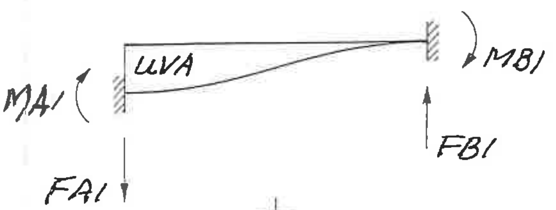

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Fig.2b.

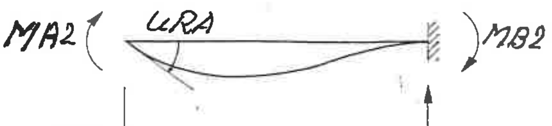

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Fig.2c.

Fig.1.
Each point of the beam can be considered as a joint. Always above the supports, and at places between the supports, and always at a member end (until now).
The joints are loaded by
joint load moments, assumption to the right, and
joint load forces, assumption downward.

Fig.2a.
Because joints between member ends and supports if they displace, mostly displace downward, the Y-axis of the construction axis system X-Y is assumed downward.

First the relation between member end forces and member end moments, and joint displacements and slope deflections (translations and rotations) of one single beam.

The rotations URA and URB of the joints, and thus of the member ends, are assumed to the right, then also the to them belonging member end momemts MAB and MBA.

The vertical displacements UVA and UVB are assumed downward like the Y-axis. So are the to them belonging vertical member end forces FAB and FBA.

The beam of fig.2a. can be seen as the sum of the figures 2b, 2c, 2d and 2e.

Fig.2b.
The member ends are clamped and holded after which a displacement UVA downward is applied. The beam deforms by it as drawn, through which the clamp moments MA1 and MB1 arise, to the right.
According to the formulas, page 30, follow

MA1=(6*EI/L^2)*UVA and MB1=(6*EI/L^2)*UVA.

Since both moments are directed to the right the beam can only be in equilibrium if the two member end forces FA1 and FB1 deliver a couple of forces to the left. Then FA1 is directed downwad and FB1 is directed upward. Their magnitudes are

FA1=(12*EI/L^3)*UVA and FB1=(12*EI/L^3)*UVA.

Fig.2c.
As on page 1 one finds the to the slope deflection URA belonging member end moments,

MA2=(4*EI/L)*URA and MB2=(2*EI/L)*URA.

The member end forces deliver again a couple of forces to the left from which their directions folloe. And with the formulas follow

FA2=(6*EI/L^2)URA and FB2=(6*EI/L^2)*URA.

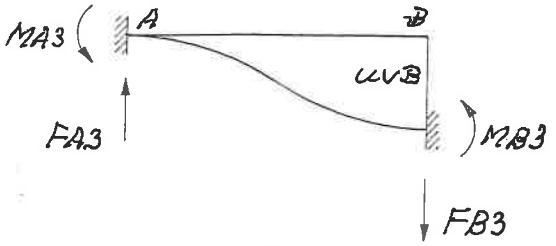

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Fig.2d.

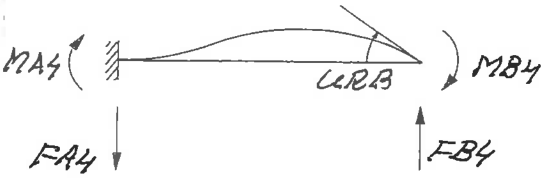

Fig.2e.

$$\begin{bmatrix} FAB \\ MAB \\ FBA \\ MBA \end{bmatrix} = S5 \begin{bmatrix} UVA \\ URA \\ UVB \\ URB \end{bmatrix}$$

$\underline{f}$ S5 $\underline{u}$

$$\begin{bmatrix} 12EI/L^3 & 6EI/L^2 & -12EI/L^3 & 6EI/L^2 \\ 6EI/L^2 & 4EI/L & -6EI/L^2 & 2EI/L \\ -12EI/L^3 & -6EI/L^2 & 12EI/L^3 & -6EI/L^2 \\ 6EI/L^2 & 2EI/L & -6EI/L^2 & 4EI/L \end{bmatrix}$$

S5

$$\begin{bmatrix} A & B & -A & B \\ B & D & -B & E \\ -A & -B & A & -B \\ B & E & -B & D \end{bmatrix}$$

$$A = \frac{(kN/m^2)(m^4)}{m^3} = kN/m$$

$$\begin{bmatrix} kN \\ kNm \\ kN \\ kNm \end{bmatrix} = \begin{bmatrix} kN/m & kN & kN/m & kN \\ kN & kNm & kN & kNm \\ kN/m & kN & kN/m & kN \\ kN & kNm & kN & kNm \end{bmatrix} \begin{bmatrix} m \\ 1 \\ m \\ 1 \end{bmatrix}$$

$\underline{f}$ S5 $\underline{u}$

Fig.2d.
Both member ends are clamped and holded when B is displaced over UVB downward. Due to the deformation/bending arise the moments MA3 and MB3 to the left. These are

MA3=(6*EI/L^2)*UVB and MB3=(6*EI/L^2)*UVB

Due to both moments to the left arise member end forces FA3 upward, and FB3 downward. With the formulas follow

FA3=(12*EI/L^3)*UVB and FB3=(12*EI/L^3)*UVB.

Fig.2e.
To the assumed slope deflection URB to the right belong the two moments MA4 and MB4 to the right. Their magnitudes are

MA4=(2*EI/L)*URB and MB4=(4*EI/L)*URB.

To make equilibrium with both moments to the right, member end force FA4 must be directed downward, and member end force FB4 directed upward, and equal in magnitude. Like found with fig.2c follow

FA4=(6*EI/L^2)*URB and FB4=(6*EI/L^2)*URB.

Member end forces and member end moments of fig.2a are the resultants of the figures 2b, 2c, 2d and 2e.
First member end A with FAB and MAB.

FAB= FA1+FA2+FA4-FA3 and in nice order,
FAB= FA1+FA2-FA3+FA4
= (12*EI/L^3)*UVA +(6*EI/L^2)*URA
-(12*EI/L^3)*UVB +(6*EI/L^2)*URB.

MAB= MA1+MA2+MA4-MA3 or
= MA1+MA2-MA3+MA4
= (6*EI/L^2)*UVA + (4*EI/L)*URA
-(6*EI/L^2)*UVB + (2*EI/L)*URB.

FBA= FB3-FB1-FB2-FB4 or
=-FB1-FB2+FB3-FB4
=-(12*EI/L^3)*UVA -(6*EI/L^2)*URA
(12*EI/L^3)*UVB -(6*EI/L^2)*URB.

MBA= MB1+MB2+MB4-MB3 or
= MB1+MB2-MB3+MB4
(6*EI/L^2)*UVA + (2*EI/L)*URA
-(6*EI/L^2)*UVB + (4*EI/L)*URB.

These equations are given on the left in matrix form, $\underline{f}$ = S5 $\underline{u}$.
The elements of stiffness maytrix S5 can be indicated with four letters, in conserning cases with a minus sign.

A=12*EI/L^3 B=6*EI/L^2 D=4*EI/L E=2*EI/L

The dimensions of the elements follow with modulus of elasticity in kN/m^2, moment of inertia I in m^4 and length L in m.

Each element of f is equal to a row of S5 times column u. Translations are found in m, rotations in (dimensionless)radians.

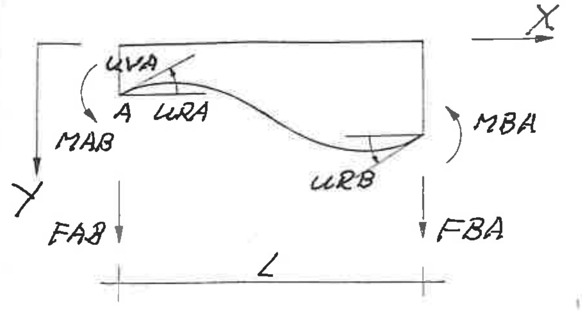

Fig. 3a

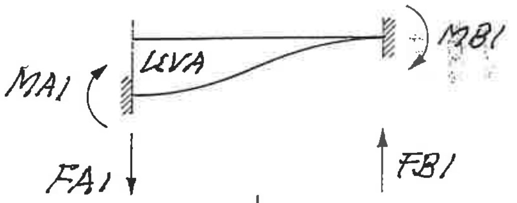

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Fig. 3b.

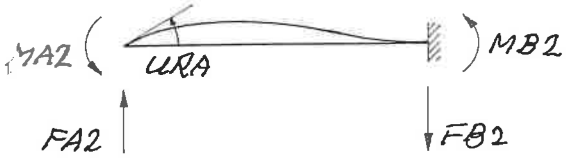

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Fig. 3c.

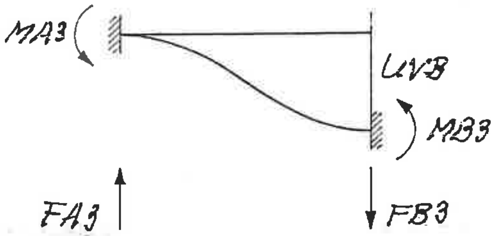

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Fig. 3d.

Fig. 3e

$$\begin{bmatrix} FAB \\ MAB \\ FBA \\ MBA \end{bmatrix} = \begin{bmatrix} A & -\underline{B} & -A & -\underline{B} \\ -\underline{B} & D & \underline{B} & E \\ -A & \underline{B} & A & \underline{B} \\ -\underline{B} & E & \underline{B} & D \end{bmatrix} \begin{bmatrix} UVA \\ URA \\ UVB \\ URB \end{bmatrix}$$

$\underline{f}$ S5 $\underline{u}$

Another stiffness matrix S5?

Fig.3a.
The slope deflections of the member ends which are rigidly connected with the joints. The joint rotations URA and URB now are assumed to the left. The to them belonging member end moments MAB and MBA are then also directed to the left.
Again four cases are considered of which fig.3a is the resultant.
The same magnitudes for forces and moments follow as on the preceding page, expressed in UVA, URA, UVB, and URB, now with the letters A, B, D and E. (C is used for Cos(inus).)

A=12*EI/L^3 B=6*EI/L^2 D=4*EI/L E=2*EI/L

Fig.3b as fig.2b.

FA1=A*UVA MA1=B*UVA FB1=A*UVA MB1=B*UVA

Fig.3c not like fig.2c.
Rotations, forces and moments have directions opposite to those of fig.2c.

FA2=B*URA MA2=D*URA FB2=B*URA MB2=E*URA

Fig.3d as fig.2d.

FA3=A*UVB MA3=B*UVB FB3=A*UVB MB3=B*UVB

Fig.3e not like fig.2e.
Rotations, forces and moments have directions opposite to those of fig.2e.

FA4=B*URB MA4=E*URB FB4=B*URB MB4=D*URB

Then the resultants can be determinated.

FAB= FA1 -FA2 -FA3 -FA4

= A*UVA -B*URA -A*UVB -B*URB

MAB= -MA1 +MA2 +MA3 +MA4

= -B*UVA +D*URA +B*UVB +D*URB

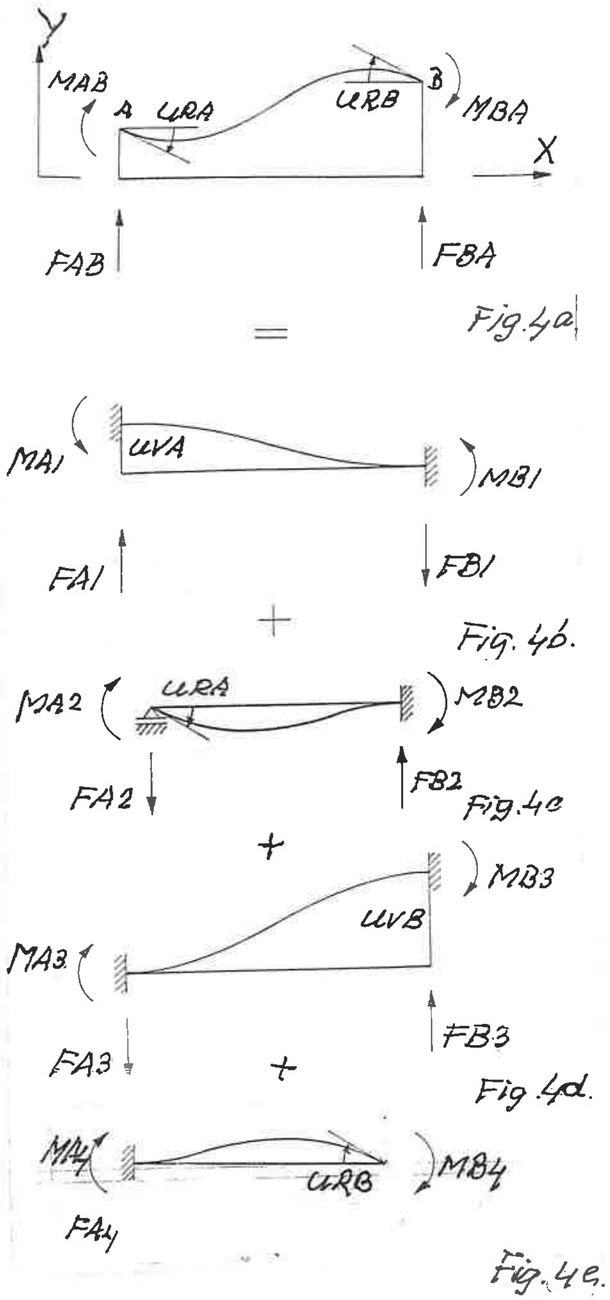
FBA= -FB1 +FB2 +FB3 +FB4

= -A*UVA +B*URA +A*UVB +B*URB

MBA= -MB1 +MB2 +MB3 +MB4

= -B*UVB +E*URA +B*UVB +D*URB

On the left the equations are given in matrix form. The magnitudes of the elements of stiffness matrix S5 are like those of the preceding page but the values have got an opposite sign because the directions of forces and moments of fig.3c are opposite to those of fig.2c, and of fig.3e opposite to those of fig.2e.
Now it is possible to go on consequently with these asumptions, ofcourse..., but it is not worked out further here.
(The joint rotations were assumed to the left, not necessarily 'from X to Y'..., that does not matter at all.)



A	B	-A	B
B	D	-B	E
-A	-B	A	-B
B	E	-B	D

fig.2a page.4.

A	-B	-A	-B
-B	D	B	E
-A	B	A	B
-B	E	B	D

fig.3a page.5

A	-B	-A	-B
-B	D	B	E
-A	B	A	B
-B	E	B	D

fig.4a

A	B	-A	B
B	D	-B	E
-A	-B	A	-B
B	E	-B	D

S5

Another possible stiffness matrix S5.

Fig.4a.
The Y-axis is assumed upward, so the displacements UVA and UVB are, and so are the to them belonging member end forces FAB and FBA.
The joint rotations are assumed to the right, so are the to them belonging member end moments MAB and MBA.

With these assumptions for the displacements UVA and UVB, and for the joint rotations URA and URB, follow the figures 4b, 4c, 4d and 4e. Displacement and joint rotation determine the deformation of the beam from which the directions of momemts and forces follow.

Fig. 4a is wqual to the sum of the figures 4b t/m 4e.

FAB= FA1 -FA2 -FA3 -FA4

= A*UVA -B*URA -A*UVB -B*URB

MAB= -MA1 +MA2 +MA3 +MA4

= -B*UVA +D*URA +B*UVB +E*URB

FBA= -FB1 +FB2 +FB3 +FB4

= -A*UVA +B*URA +A*UVB +B*URB

MBA= -MB1 +MB2 +MB3 +MB4

= -B*UVA +E*URA +B*UVB +D*URB

The same stiffness matrix S5 is found as on the preceding page. The equations are the same.

Once again fig.4a but now with joint rotations to the left, then also the to them belonging member end moments to the left.
The direction of forces and moments of fig.4c and 4e are then opposite directed. Then the equations look a bit different.

FAB= A*UVA +B*URA -A*UVB +B*URB

MAB= B*UVA +D*URA -B*UVB +E*URB

FBA= -A*UVA -B*URA +A*UVB -B*URB

MBA= B*UVA +E*URA -B*UVB +D*URB

Member matrix S5 of this case looks like that of the first matrix of page 4 .
The four matrices found are given on the left. The first and last one are equal, also the second and the third are equal.

Now the cases 2, 3 and 4 will be 'forgotten' and explanation goes further with the first relation found.

(By the way, there are more possibilities with different assumptions, but rather difficult to be maintained.)

$$\begin{bmatrix} FAB \\ MAB \\ FBA \\ MBA \end{bmatrix} = \begin{bmatrix} A1 & B1 & -A1 & B1 \\ B1 & D1 & -B1 & E1 \\ -A1 & -B1 & \underline{A1} & \underline{-B1} \\ B1 & E1 & \underline{-B1} & \underline{D1} \end{bmatrix} \cdot \begin{bmatrix} UVA \\ URA \\ UVB \\ URB \end{bmatrix}$$

$$\begin{bmatrix} FBC \\ MBC \\ FCB \\ MCB \end{bmatrix} = \begin{bmatrix} \underline{A2} & \underline{B2} & -A2 & B2 \\ \underline{B2} & \underline{D2} & -B2 & E2 \\ -A2 & -B2 & A2 & -B2 \\ B2 & E2 & -B2 & D2 \end{bmatrix} \cdot \begin{bmatrix} UVB \\ URB \\ UVC \\ URC \end{bmatrix}$$

$$\begin{bmatrix} FBA+FBC \\ MBA+MBC \\ FCB \\ MCB \\ FAB \\ MAB \end{bmatrix} \qquad \begin{bmatrix} UVB \\ URB \\ UVC \\ URC \\ UVA \\ URA \end{bmatrix} \qquad \begin{bmatrix} FYB \\ MZB \\ FYC \\ MZC \\ FYA \\ MZA \end{bmatrix}$$

$\underline{f}$ CC $\underline{u}$

$$\begin{bmatrix} \underline{A1+A2} & \underline{-B1+B2} & -A2 & B2 & -A1 & -B1 \\ \underline{-B1+B2} & \underline{D1+D2} & -B2 & E2 & B1 & E1 \\ -A2 & -B2 & A2 & -B2 & 0 & 0 \\ B2 & E2 & -B2 & D2 & 0 & 0 \\ -A1 & B1 & 0 & 0 & A1 & B1 \\ -B1 & E1 & 0 & 0 & B1 & D1 \end{bmatrix}$$

construction matrix CC

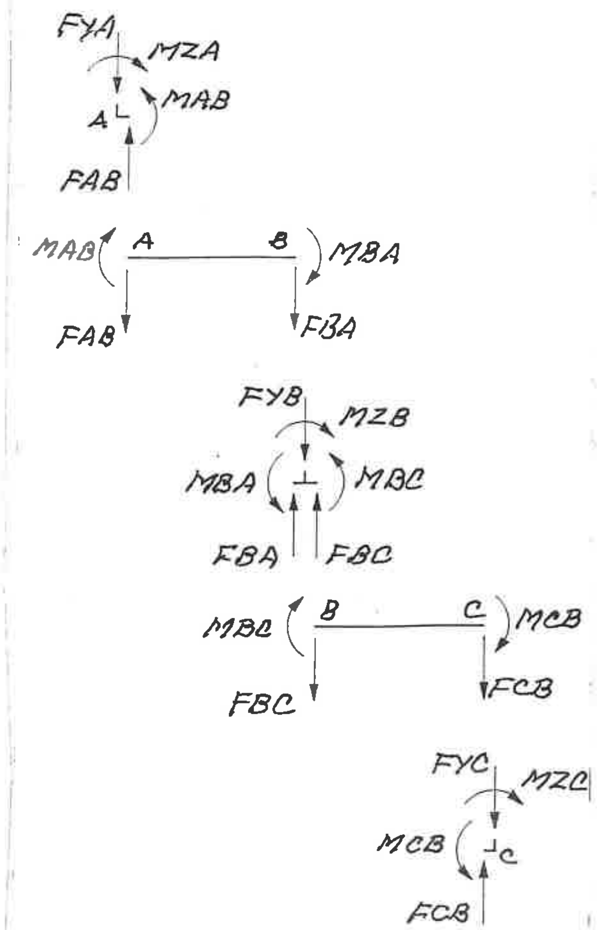


Fig. 3.

Fig.5.
If the construction consists of two beams, two members, then there are two sets of equations, $\underline{f}$ = S5 $\underline{u}$.

Member A-B with elements A1, B1, D1 and E1, and member B-C with A2, B2, D2 and E2.
They are placed in the siffness matrices S5 with a minus sign when needed.
Both sets of equations can be composed to a single set f = CC u.
Both S5's are put in CC, here with joint order B, C and A, a possibility, see next page. Written CC instead of C; CC is used in the code.
The underlined elements of the S5-s coincide in construction matrix CC.

FBA= -A1*UVA -B1*URA +A1*UVB -B1*URB

FBC= A2*UVB +B2*URB -A2*UVC +B2*URC

FBA+FBC= (A1+A2)*UVB +(-B1+B2)*URB

-A2*UVC +B2*URC

-A1*UVA -B1*URA and so on.

The member end forces FAB, FBA, FBC and FCB are assumed downward. On the seperated joints act forces as large as but bopposite directed, so directed upward.

The on the member end acting member end moments MAB, MBA, MBC and MCB are assumed to the right. On the seperated joints act moments as large as but opposite directed, so to the left.

On the joints act joint load forces FYA, FYB and FYC, assumed direction downward, and joint load moments MZA, MZB and MZC, assumed direction to the right.

To be able to solve the set of equations CC u = f the force vector is filled with joint load forces and joint load moments, which follow from equilibrium of the joints.

Σ vert. joint B =0 FYB-FBA-FBC=0	⇒	FBA+FBC =	FYB
Σ mom. joint B =0 MZB-MBA-MBC=0	⇒	MBA+MBC =	MZB
Σ vert. joint C =0 FYC-FCB=0	⇒	FCB =	FYC
Σ mom. joint C =0 MZC-MCB=0	⇒	MCB =	MZC
Σ vert. joint A =0 FYA-FAB=0	⇒	FAB =	FYA
Σ mom. joint A =0 MZA-MAB=0	⇒	MAB =	MZA
			f

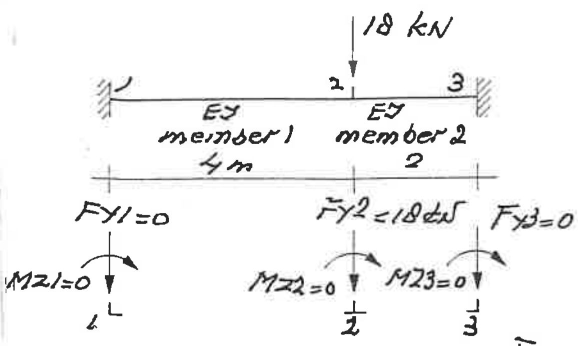


Fig. 1.

$$\begin{bmatrix} F12 \\ M12 \\ F21 \\ M21 \end{bmatrix} = \begin{bmatrix} 188 & 375 & -188 & 375 \\ 375 & 1000 & -375 & 500 \\ -188 & -375 & 188 & -375 \\ 375 & 500 & -375 & 1000 \end{bmatrix} \begin{bmatrix} UV1 \\ UR1 \\ UV2 \\ UR2 \end{bmatrix}$$

S5 member 1

$$\begin{bmatrix} F23 \\ M23 \\ F32 \\ M32 \end{bmatrix} = \begin{bmatrix} 1500 & 1500 & -1500 & 1500 \\ 1500 & 2000 & -1500 & 1000 \\ -1500 & -1500 & 1500 & -1500 \\ 1500 & 1000 & -1500 & 2000 \end{bmatrix} \begin{bmatrix} UV2 \\ UR2 \\ UV3 \\ UR3 \end{bmatrix}$$

$\underline{f}$ S5 member 2 $\underline{u}$

$$\begin{bmatrix} 188 & 375 & -188 & 375 & 0 & 0 \\ 375 & 1000 & -375 & 500 & 0 & 0 \\ -188 & -375 & \underline{1688} & \underline{1125} & -1500 & 1500 \\ 375 & 500 & \underline{1125} & \underline{3000} & -1500 & 1000 \\ 0 & 0 & -1500 & -1500 & 1500 & -1500 \\ 0 & 0 & 1500 & 1000 & -1500 & 2000 \end{bmatrix}$$

times EI/1000

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1688 & 1125 & 0 & 0 \\ 0 & 0 & 1125 & 3000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} UV1 \\ UR1 \\ UV2 \\ UR2 \\ UV3 \\ UR3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 18 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

CC $\underline{u}$ $\underline{f}$

Example.

Fig.1.
The on both ends clamped beam is loaded by a force of 18 kN. Member end 1 or member end 2 can displace horizontally.
The construction is divided into two beams/members and three joints.

$$\begin{bmatrix} A & B & -A & B \\ B & D & -B & E \\ -A & -B & A & -B \\ B & E & -B & D \end{bmatrix}$$

S5

A=12*EI/L^3
B= 6*EI/L^2
D= 4*EI/L
E= 2*EI/L

By using the formulas found earlier the elements of the member stiffness matrices are determinated.

Beam 1 with L=4 m and bending stiffness EI.
A=12*EI/4^3=0,188EI B=6*EI/4^2=0,375EI
D= 4*EI/4 =1,000EI E=2*EI/4 =0,500EI

Beam 2 with L=2 m and bending stiffness EI.
A=12*EI/2^3=1,500EI B=6*EI/2^2=1,500EI
D= 4*EI/2 =2,000EI E=2*EI/2 =1,000EI

In the S5-s these values are multiplied by 1000/EI so that S5 has to be multiplied by EI/1000.
Both sets of equations $\underline{f}$ = S5 $\underline{u}$ are composed to the set $\underline{f}$ = CC $\underline{u}$. The underlined elements of the S5-s coincide in construction matrix CC.
All joint load forces and joint load moments are zero except FY2=18 kN.
With vertical equilibrium of joint 2 follows FY2-F21-F23=0 so that F21+F23=FY2 is 18 kN, the third element of the force vector.
The vertical displacements UV1 and UV3, and the joint rotations UR1 and UR3 are prescribed and all equal zero. The concerning rows and columns of matrix CC are made zero, but the diagonal elements become 1.
Two equations remain,
(EI/1000)(1688UV2+1125UR2)=18 and
(EI/1000)(1125UV2+3000UR2)= 0 from which follow
UV2=14,2/EI and UR2=-5,33/EI.
The beam end forces and momemts of beam 1:
UV1=0 and UR1=0.

F12=(EI/1000)(-188(14,2/EI)+375(-5,33/EI))

=(EI/1000)(-2670/EI-1999/EI)=-4,67 kN

M12=(EI/1000)(-375(14,2/EI)+500(-5,33/EI))

=(EI/1000)(-5325/EI-2665/EI)=-7,99 kNm

F21=(EI/1000)(188(14,2/EI)-375(-5,33/EI))

= 4,67 kN

M21=(EI/1000)(-375(14,2/EI)+1000(-5,33/EI))

=-10,66 kNm

$$\begin{bmatrix} KV1 \\ KM1 \\ KV2 \\ KM2 \\ KV3 \\ KM3 \end{bmatrix} = \begin{bmatrix} F12 \\ M12 \\ F21+F23 \\ M21+M23 \\ F32 \\ M32 \end{bmatrix} = CC\ \underline{u}$$

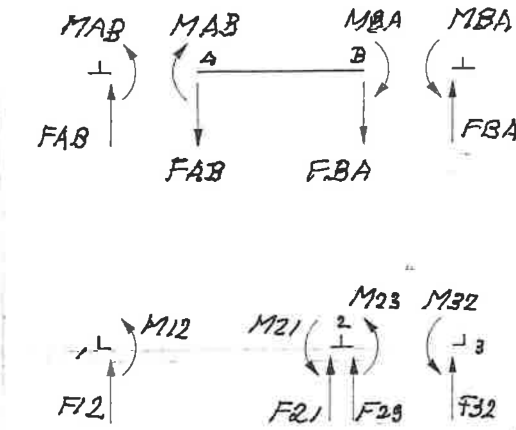


Fig.2a.

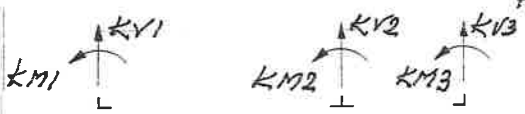


Fig.2b.

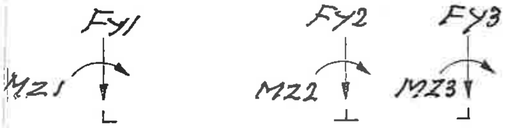


Fig.2c.

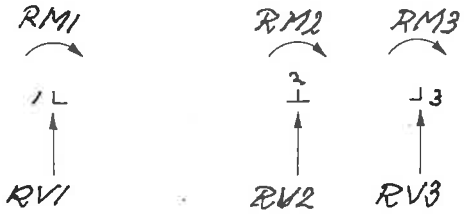


Fig.2d.

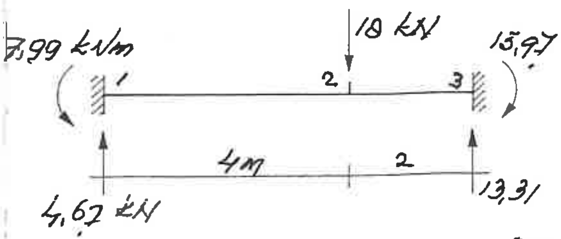


Fig.3.

The beam end forces and moments of beam 2.
UV3=0 and UR3=0.
F23=(EI/1000)(1000(14,2/EI)+1500(-5,33/EI)
= 13,31 kN
M23=(EI/1000)(1500(14,2/EI)+2000(-5,33/EI)
= 10,64 kNm
F32=(EI/1000)(-1500(14,2/EI)-1500(-5,33/EI)
=-13,31 kN
M32=(EI/1000)(1500(14,2/EI)+1000(-5,33/EI)
= 15,97 kNm

Fig.2a.
On the beam ends act according assumption downward durected forces FAB and FBA, and according assumption to the right directed beam end moments MAB and MBA.
On the seperated joints act forces and moments as large as but opposite directed, forces upward and moments to the left.
Fig.2b.
On the joints act vertical joint forces KV1, Kv2 and KV3, assumption upward, and the joint moments KM1, KM2 and KM3, assumption to the left.
These joint forces and joint moments form the elements of force vector f = CC u. Such an element is equal a row of the original construction matrix CC times column u.

KV1=F12 =- 4,67 kN
KM1=M12 =- 7,99 kNm
KV2=F21+F23= 4,67+13,31= 17,98 kN
KM2=M21+M23=-10,66+10,64≈ 0 kNm
KV3=F32 =-13,31 kN
KM3=M32 = 15,97 kNm

Fig.2c.
On the joints also act
joint load forces FY1, FY2 and FY3, according assumption downward, and the
joint load moments MZ1, MZ2 and MZ3, according assumption directed to the right.
FY1=0 FY2=18,00 kN FY3=0 MZ1=0 MZ2=0 MZ3=0
Fig.2d.
For each joint a vertical reaction is assumed, RV1, RV2 and RV3, assumption upward, and reaction moment, RM1, RM2 and RM3, assumption to the right.

Fig.2b, 2c and 2d.

Σ vert. joint 1 =0 RV1+KV1-FY1=0
RV1=-KV1+FY1= -(-4,67)+0= 4,67 kN
Σ mom. joint 1 =0 RM1-KM1+MZ1=0
RM1= KM1-MZ1= -7,99-0= -7,99 kNm

Σ vert. joint 2 =0 RV2+KV2-FY2=0
RV2=-KV2+FY2=-17,98+18,00≈ 0 kN
Σ mom. joint 2 =0 RM2-KM2+MZ2=0
RM2= KM2-MZ2= 0 - 0 = 0 kNm

Σ vert. joint 3 =0 RV3+KV3-FY3=0
RV3=-KV3+FY3=-(-13,31)+0 = 13,31 kN
Σ mom. joint 3 =0 RM3-KM3+MZ3=0
RM3= KM3-MZ3= 15,97 -0 = 15,97 kNm

Fig.3.
The reactions are drawn with their real directions. The whole is in equilibrium.

3.3. Primary forces and moments due to loads perpendicular to the beam axis.

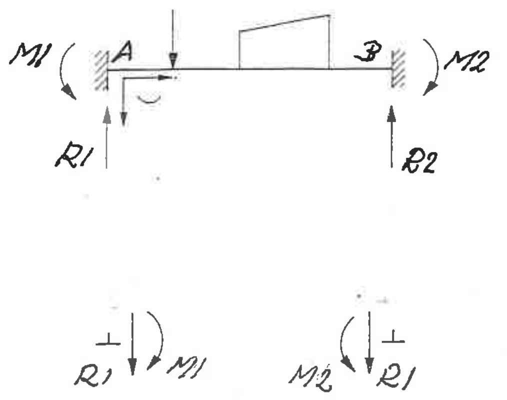


Fig. 1a.

Fig.1a.
Starting point is the beam clamped at both ends with the given assumed directions of the beam loads and rections, and the assumed place of the beam axis system at beam end A.
The reactions due to the loads are the forces R1 and R2 and the moments M1 and M2.
On the separated joints act forces and moments as large as but opposite directed.

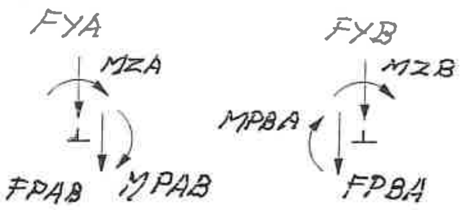


Fig. 1b.

Fig.1b.
The loads on the beam deliver on the joints acting primary forces FPAB and FPBA with an assumed direction like that of the joint load forces FYA and FYB, that is downward.

Also arise the on the joints acting primary moments MPAB and MPBA with an assumed direction like that of the joint load moments MZA and MZB, that is to the right.

Fig.1a and 1b.
At joint A.
FPAB= R1   FPAB and R1 have the same direction.
MPAB= M1   MPAB and M1 have the same direction.

At joint B.
FPBA= R2   FPBA and R2 have the same direction.
MPBA=-M2   MPBA and M2 are opposite directed, so the minus sign is needed to indicate that.
First force vector f is filled with joint load forces and joint load moments, after that the primary forces will be added.
For joint A with   FYA+FPAB=FYA+R1 and
MZA+MPAB=MZA+M1.
For joint B with   FYB+MPBA=FYB+R2 and
MZB+MPBA=MZB+(-M2).
(Plus eventual primary forces and moments left of A and/or right of B in case of a continous beam.)

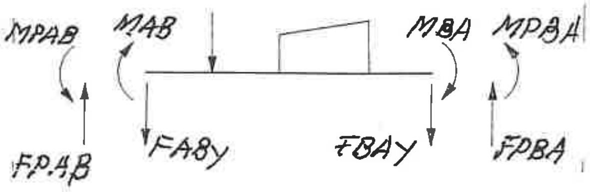


Fig. 1c.

Fig.1c.
The beam end forces FABY and FBAY are according assumption directed downward. (The letter Y indicates only! that they are vertical forces. One could have assumed them to be directed upward.)
The beam end moments MAB and MBA are assumed directed to the right.

These beam end forces and moments are first calculated due to the displacements alone, translations and rotations of the joints.

When there are beam loads then the primary forces and primary moments must be added to get the final beam end forces and beam end moments. See fig.1b for the assumed directions of FPAB, MPAB, FPBA and MPBA.

FABY	FYA+FPAB	FYA+R1
MAB	MZA+MPAB	MZA+M1
FBAY	FYB+FPBA	FYB+R2
MBA	MBA+MPBA	MZB-M2

FABY   becomes   FABY-FPAB=FABY-R1
MAB   becomes   MAB-MPAB= MAB-M1

FBAY   becomes   FBAY-FPBA=FBAY-R2
MBA   becomes   MBA-MPBA= MBA-(-M2)

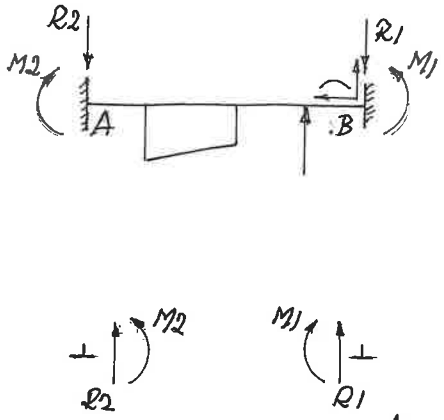


Fig.2a.

Fig.2a.
One can also place the beam axis system ⌐ at beam end B.
Then arise at B the reactions R1 and M1 and at A the reactions R2 and M2.
R1 and R2 are now directed downward.
Like fig.1a is M1 directed to the left, and M2 to the right.
On the joints act forces and moments as large as but opposite directed.

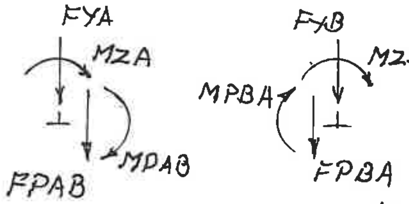


Fig.2b.

Fig.2b.
The assumption for the direction of the on the joints acting forces FPAB and FPBA is like fig.1b downward, and of the primary moments MPAB and MPBA also no on the joints to the right. Their magnitudes are the calculated reactions R2, R1, M2 and M1.
Fig.2a and 2b.
At joint A.
FPAB=-R2 FPAB and R2 are opposit directed.
MPAB=-M2 MPAB and M2 are opposit directed.
At joint B.
FPBA=-R1 FPBA and R1 are opposit directed.
MPBA= M1 MPBA and M1 have the same direction.
Force vector f is now filled with joint load forces and moments, and primary forces and moments.

For joint A with FYA+FPAB=FYA+(-R2) and
MZA+MPAB=MZA+(-M2).

For joint B with FYB+FPBA=FYB+(-R1) and
MZB+MPBA=MZB+M1.

FABY	FYA+FPAB	FYA-R2
MAB	MZA+MPAB	MZA-M2
FBAY	FYB+FPBA	FYB-R1
MBA	MZB+MPBA	MZB+M1

At last follow the final beam end forces and moments.

FABY becomes FABY-FPAB=FABY-(-R2)
MAB becomes MAB-MPAB= MAB-(-M2)

FBAY becomes FBAY-FPBA=FBAY-(-R1)
MBA becomes MBA-MPBA= MBA-M1

The beam end forces w.r.t. member axis y.

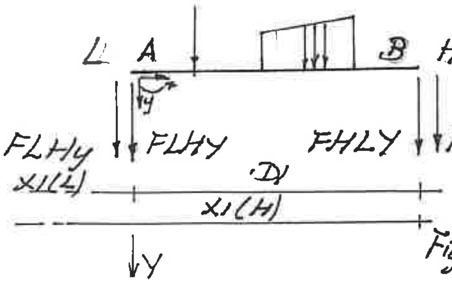


Fig.3a.

Fig.3a.
The lowest member end number L is assumed at A, the highest member end number H at B. The beam axis system ⌐ is placed at A (later it can be placed at B as well.)
(The member axis system ⌐ x y is always placed at the lowest member end number.)
The member end forces FABy and FBAy are assumed downward, like the y-axis.

D1=X1(H)-X1(L) L1=SQR(D1^2)=D1 C=D1/L1=+1

Now are FLHy= FLHY·C and FHLy=FHLY·C.
C is positief so that FLHy and FHLy are directed downward like FLHY and FHLY.

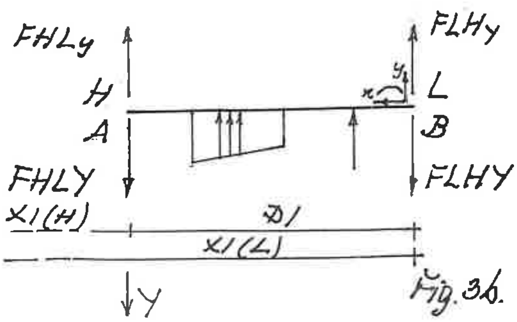


Fig.3b.

Fig.3b.
L and H are exchanged. (Now A is H and B is L.)
The member axis system is again placed at L.
FLHy and FHLy are now directed upward according to the member axis y.
D1=X1(H)-X1(L) L1=SQR(D1^2)=D1 C=D1/L1=-1

Now are FLHy= FLHY·C and FHLy=FHLY·C.
C is negative so that FLHy and FHLy are not directed upward but downward like FLHY and FHLY.

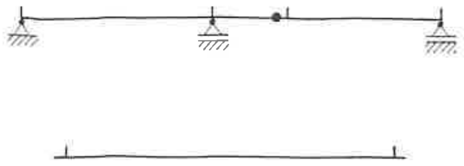


Fig.1a.

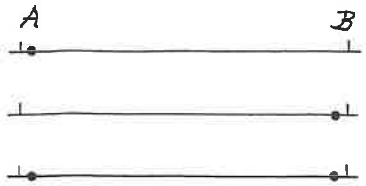


Fig.1b.

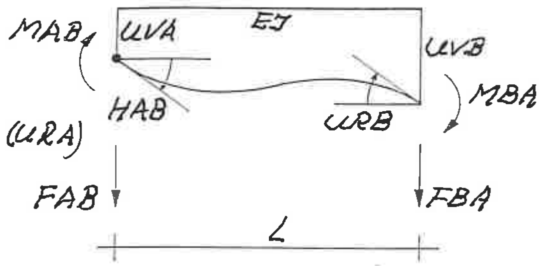


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Fig.2a.

Fig.2b.

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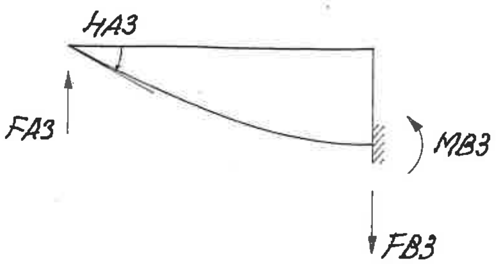

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Fig.2c.

3.4. Beams over more than two supports **with** vertical joint displacements and **with** internal hinges.

Fig.1a.
Until now it was assumed that the beam ends were rigidly connected with the joints.
The beam delivers resistance against the vertical joint displacements and joint rotations. The belonging beam stiffness matrix was found on page 4.

Fig.1b.
In case of an internal hinge the concerning beam end is hingy connected with the joint. Now there are three possibilities.

1) Beam end A is hinged connected with the adjoining joint.
2) Beam end B is hinged connected with the adjoining joint.
3) Beam ends A and B are hinged connected with the adjoining joints.

The resistance of these beams against joint displacements and joint rotations is smaller than that of a beam of which both beam ends are rigidly connected with the adjoining joints.
Each ofv these three member has an own beam stiffness matrix S5.

1) Beam end A is a hinge.

Fig.2a.
The assumptions are like those of fig.2a of page 3 . There's no joint rotation URA because of the hinge.
Slope deflection HAB, assumption to the right, shall be calculated seperatedly after calculation of UVA, UVB and URB.
Fig.2a can be seen as the sum of the figures 2b, 2c and 2d.

Fig.2b.
If the beam en on the right is holded and A is displaced over UVA downward, then arises at B a clamp moment MB1 to the right.
The formulas on page 30 give

MB1=(3*EI/L^2)*UVA

From equilibrium follows a reaction FA1 at A downward, and a reaction FB1 at B upward.

FA1=(3*EI/L^3)*UVA   en   FB1=(3*EI/L^3)*UVA.

At beam end A arises a slope of deflection

HA1 to the left,   HA1=(3/(2*L))*UVA.

Fig.2c.
If one holds beam end A at its place and is the clamped beam end B is displaced over UVB downward then arises momemt MB3 to the left.

MB3=(3*EI/L^2)*UVB   MA3=0   Further arise

FA3=(3*EI/L^3)*UVB   and   FB3=(3*EI/L^3)*UVB.

At beam end A arises a slope of deflection HA2 to the right, HA3=(3/(2*L))*UVB.

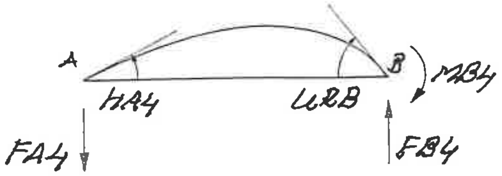

Fig.2d.

$$\begin{bmatrix} FAB \\ MAB \\ FBA \\ MBA \end{bmatrix} = S5 \begin{bmatrix} UVA \\ URA \\ UVB \\ URB \end{bmatrix}$$

$\underline{f}$ S5 $\underline{u}$

$$\begin{bmatrix} 3EI/L^3 & 0 & -3EI/L^3 & 3EI/L^2 \\ 0 & 0 & 0 & 0 \\ -3EI/L^3 & 0 & 3EI/L^3 & -3EI/L^2 \\ 3EI/L^2 & 0 & -3EI/L^2 & 3EI/L \end{bmatrix}$$

S5

$$\begin{bmatrix} A & 0 & -A & B \\ 0 & 0 & 0 & 0 \\ -A & 0 & A & -B \\ B & 0 & -B & D \end{bmatrix}$$

Fig.2d.
This beam is statically determinate supported.
Is a slope deflection URB to the right applied at B then belongs to it a moment MB4 to the right. The formula, page 30 , gives

URB=(MB4*L)/(3*EI) so that <u>MB4=(3*EI/L)*URB</u>.

At A arises reaction FA4 downward, and at B arises reaction FB4 upward so that they together make equilibrium with MB4.

<u>FA4=(3*EI/L^2)*URB</u> and <u>FB4=(3*EI/L^2)*URB</u>.

At beam en A arises a slope HA4 to the left which is half as large as URB,

Beam end forces and beam end moments of fig.2a are the resultants of the figures 2b, 2c and 2d. One! of the figures is not drawn, the beam like fig.2d but without slope deflection URA so that FA2=0, MA2=0, FB2=0 en MB2=0.

First beam end A with FAB and MAB.

FAB= FA1+FA2-FA3+FA4

= (3*EI/L^3)*UVA + 0 *URA
-(3*EI/L^3)*UVB + (3*EI/L^2)*URB

The left beam end is a hinge, MAB=0.
MAB= 0*UVA + 0*URA + 0*UVB + 0*URB

Then beam end B with FBA and MBA.
FBA=-FB1+FB2+FB3-FB4

=-(3*EI/L^3)*UVA + 0 *URA
+(3*EI/L^3)*UVB - (3*EI/L^2)*URB

The right beam end is **no** hinge!

MBA= MB1+MB2-MB3+MB4

= (3*EI/L^2)*UVA + 0 *URA
-(3*EI/L^2)*UVB + (3*EI/L)*URB

The equations are given on the left in matrix form, $\underline{f}$ = S5 $\underline{u}$.
The elements of S5 can be indicated with three letters.

A=3*EI/L^3 B=3*EI/L^2 D=3*EI/L

Joint rotation URA on the left of the hinge does exist but delivers nothing for the elements of force vector $\underline{f}$. The second column of S5 gives then four times 0*URA.

The slope deflection at the hinged beam end A is HAB, assumption to the right, see fig.2a.

HAB=-H1+HA2+HA3-HA4

=-1,5*UVA/L + 0*URA
+1,5*UVB/L - URB/2 or

<u>HAB= 1,5*(UVB-UVA)/L-URB/2</u>.

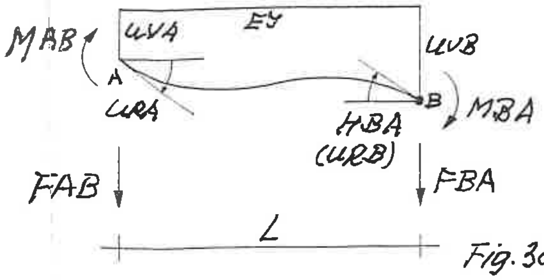

Fig.3a.

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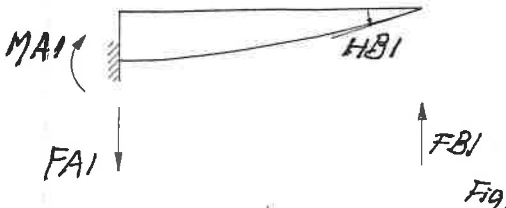


Fig.3b.

+

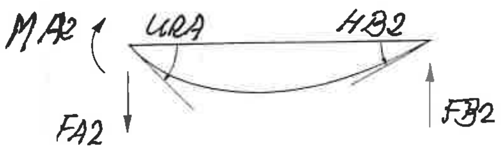


Fig.3c.

+

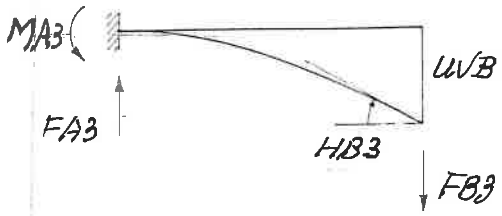


Fig.3d.

$$\begin{bmatrix} 3EI/L^3 & 3EI/L^2 & -3EI/L^3 & 0 \\ 3EI/L^2 & 3EI/L & -3EI/L^2 & 0 \\ -3EI/L^3 & -3EI/L^2 & 3EI/L^3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underset{\underline{f}}{\begin{bmatrix} FAB \\ MAB \\ FBA \\ MBA \end{bmatrix}} = \underset{S5}{\begin{bmatrix} A & B & -A & 0 \\ B & D & -B & 0 \\ -A & -B & A & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} \cdot \underset{\underline{u}}{\begin{bmatrix} UVA \\ URA \\ UVB \\ URB \end{bmatrix}}$$

2) Beam end B is a hinge.

Fig.3a.
Assumptions again like those of fig.2a, page .
In this case slope deflection HBA, assumption to the right, does not equal the joint rotation URB of joint B to which the hinge is connected.
Slope HBA is seperatedly calculated.
Fig.3b.
Due to displacement UVA arises at A reaction MA1 to the right.
MA1=(3*EI/L^2)*UVA

The reactions FA1 and FB1 become
FA1=(3*EI/L^3)*UVA and FB1=(3*EI/L^3)*UVA.

And slope HB1 is HB1=1,5*UVA/L.

Fig.3c.
To roatation URA belongs MA2 to the right.
URA=(MA2*L)/(3*EI) so that MA2=(3*EI/L)*URA.
And the reactions FA2 and FB2 become
FA2=(3*EI/L^2)*URA and FB2=(3*EI/L^2)*URA.

And slope HB2 is HB2=URA/2.

Fig.3d.
Due to UVB arises at A moment MA3 to the left,
MA3=(3*EI/L^2)*UVB and FA3 at A and FB3 at B.

FA3=(3*EI/L^3)*UVB and FB3=(3*EI/L^3)*UVB.

Further is slope HB3=1,5*UVB/L.

The fourth case, like fig.3c but now with URB=0, gives FA4=0, MA4=0 and MB4=0, and HB4=0.
Fig.3a is equal the sum of the figures 3b, 3c and 3d (and 3e not drawn).

```
FAB= FA1+FA2-FA3+FA4
   = (3*EI/L^3)*UVA + (3*EI/L^2)*URA
    -(3*EI/L^3)*UVB +       0    *URB

MAB= MA1+MA2-MA3+MA4
   = (3*EI/L^2)*UVA +   (3*EI/L)*URA
    -(3*EI/L^2)*UVB +       0    *URB

FBA=-FB1-FB2+FB3+FB4
   =-(3*EI/L^3)*UVA - (3*EI/L^2)*URA
    +(3*EI/L^3)*UVB +       0    *URB

MBA= 0*UVA + 0*URA + 0*UVB + 0*URB
```

See on the left how beam matrix S5 looks like.
Again the elements can be indicated by letters, now A,B and D.

A=3*EI/L^3   B=3*EI/L^2   D=3*EI/L

The slope at beam end B is HBA, assumption to the right.
HBA=-HB1-HB2+HB3+HB4

```
   =-1,5*UVA - URA/2
    +1,5*UVB + 0*URB     or
```

HBA= 1,5*(UVB-UVA)-URA/2.

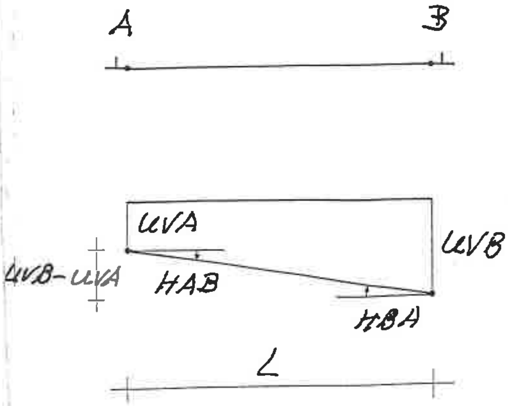


Fig. 4.

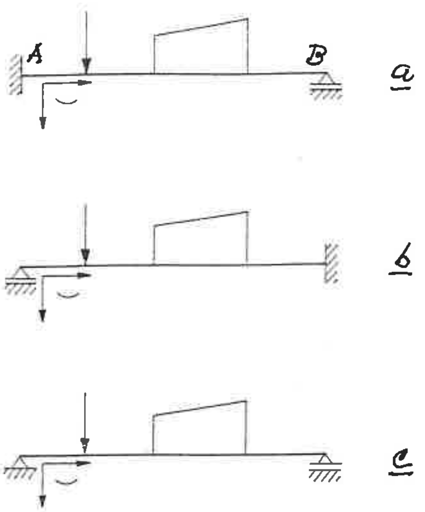
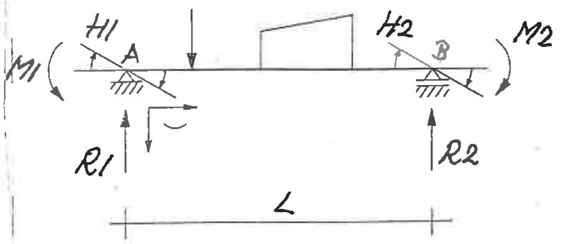


Fig. 5.



Beam ends A and B are hinges.
Fig.4.
Slope deflection HAB at beam end A and slope deflection HBA at beam end B are seperatedly calculated.
It is assumed that UVB is larger than UVA.
Length L is large compairing with UVB-UVA, then the tangent of angle HAB is (UVB-UVA)/L equal to angle HAB.
HAB=(UVB-UVA)/L and HBA=(UVB-UVA)/L.
The elements of beam stiffness matrix S5 are all zero.
Due to the displacements UVA and UVB alone! do not arise beam end forces and beam end moments.

Primary forces and primary moments due to beam loads perpendicular to the beam axis.
Fig.5.
With the assumed beam axis sytem at A the reactions R1,R2, M1 and M2 are calculated with the subroutines, see part **8**,

BEAM1() page 1, or
BEAM2() page 4, or
BEAM3() page 7.

These reactions deliver their contribution to force vector f, and to the final beam end forces and beam end moments as shown on page 8 to 11.

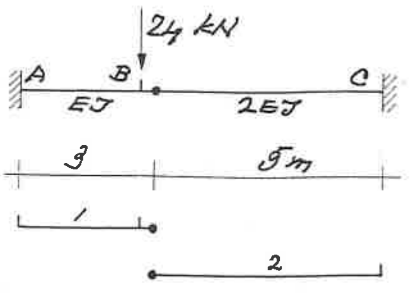
The final slope deflections at the hinged beam ends.
Fig.6a and figures 5.
The beam axis system ⌐ is placed at A.
With the mentioned subroutines are calculated slope deflection H1 at A, and
slope deflection H2 at B.
To these are added the slope deflections due to only the displacements, displacements alone.

a) Beam end B is a hinge, preceding page.
   HBA becomes HBA=H2+1,5*(UVB-UVA)/L-URA/2.
b) Beam end A is a hinge, see page .
   HAB becomes HAB=H1+1,5*(UVB-UVA)/L-URB/2.
c) The beam ends A en B are hinges.
   HAB becomes HAB=H1+(UVB-UVA)/L and
   HBA becomes HBA=H2+(UVB-UVA)/L.

Fig.6b and figures 5.
The beam axis system ⌐ is placed at B.

a) Beam end B is a hinge.
   HBA becomes HBA=H1+1,5*(UVB-UVA)/L-URA/2.
b) Beam end A is a hinge.
   HAB becomes HAB=H2+1,5*(UVB-UVA)/L-URB/2.
c) Beam ends A en B are hinges.
   HAB becomes HAB=H2+(UVB-UVA)/L and
   HBA becomes HBA=H1+(UVB-UVA)/L.

When considering horizontal beams one shall usually place the beam axis system at the beam end on the left. But what is 'left' when considering e.g. frames? Then the place of the beam axis system is placed at one of the beam ends which is indicated by a beam array variable for the program. Chosen the place, one knows how the beam data have to be put in.



EX2 CBEAMMAT page 26 .

Fig.1.

A	B	-A	B
B	D	-B	E
-A	-B	A	-B
B	E	-B	D

S51 page 4

A	0	-A	B
0	0	0	0
-A	0	A	-B
B	0	-B	D

S52 page 13

FAB	=	444	667	-444	667	UVA
MAB		667	1333	-667	667	URA
FBA		-444	-667	444	-667	UVB
MBA		667	667	-667	1333	URB

S51
times EI/1000

FBC	=	48	0	-48	240	UVB
MBC		0	0	0	0	URB
FCB		-48	0	48	-240	UVC
MCB		240	0	-240	1200	URC

S52

444	667	-444	667	0	0	UVA
667	1333	-667	667	0	0	URA
-444	-667	492	-667	-48	240	UVB
667	667	-667	1333	0	0	URB
0	0	-48	0	48	-240	UVC
0	0	240	0	-240	1200	URC

CC

1	0	0	0	0	0	UVA	0
0	1	0	0	0	0	URA	0
0	0	492	-677	0	0	UVB	24
0	0	-667	1333	0	0	URB	0
0	0	0	0	1	0	UVC	0
0	0	0	0	0	1	URC	0

CC u f

Example.

Fig.1.
At B the beam has as internal hinge. The construction is divided into two beams, or members, and three joints.
Just left of the hinge is assumed a joint, the vertical short little stripe. This is one possibility, after this example follow another two.

All joint load forces and joint load moments are zero except joint load force FYB= 24 kN.

Beam 1 with L=3 m and bending-stiffness EI. The beam ends are rigidly connected with the joints. Then applies stiffness matrix S51.

A=12*'EI'/L^3=12*EI/3^3=0,444EI
B= 6*'EI'/L^2= 6*EI/3^2=0,667EI

D= 4*'EI'/L=4*EI/3=1,333EI
E= 2*'EI'/L=2*EI/3=0,667EI

Beam 2 with L=5 m and bending-stiffness 2EI. Het linker staafeind is een scharnier, dan geldt matrix S52.

A= 3*'EI'/L^3=3*2EI/5^3=0,048EI
B= 3*'EI'/L^2=3*2EI/5^2=0,240EI
D= 3*'EI'/L =3*2EI/5 =1,200EI

Like on page 7/8 the elements of S51 and S52 are placed in construction matrix CC. The underlined elements coincide in CC, and are added.
The displacements UVA and UVC, and the rotations URA and URC are prescribed and equal zero. One finds CC u = f of which the third element is 24 kN is. There are two equations left.

(EI/1000)(492UVB- 667URB)=24
(EI/1000)(-667UVB+1333URB)= 0 from which

UVB=151,4/EI and URB=75,8/EI.
Now the beam end forces and beam end moments can be calculated.

FAB=(EI/1000)(-444(151,4/EI)+667(75,8/EI))
=(EI/1000)(-16663/EI)=-16,7 kN

MAB=(EI/1000)(-667(151,4/EI)+667(75,8/EI))
=(EI/1000)(-50425/EI)=-50,4 kNm

FBA=16,7 kN

MBA=(EI/1000)(-667(151,4/EI)+1333(75,8))
=(EI/1000)(-57/EI)=-0,057≈ 0 kNm

FBC=(EI/1000)(48(151,4/EI)+0(75,8/EI)= 7,3 kN
MBC= 0 kNm
FCB=-48*UVB + 0*URB=-7,3 kN
MCB=240*UVB + 0*URB=36,3 knM

Slope HBC at beam end B of member 2 is seperatedly calculated, see page 13 .

HBC=1,5(UVC-UVB)/L-URC/2
=1,5(0-151,4/EI)/5-0/2=-45,4/EI

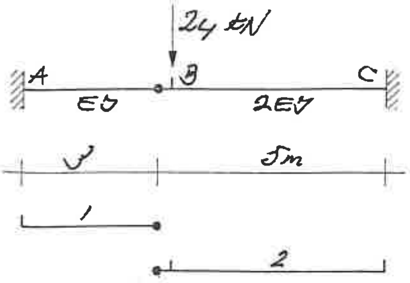


Fig.2.

$$\begin{bmatrix} A & B & -A & 0 \\ B & D & -B & 0 \\ -A & -B & A & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} A & B & -A & B \\ B & D & -B & E \\ -A & -B & A & -B \\ B & E & -B & D \end{bmatrix}$$

S51 page 14 S52 page

$$\begin{bmatrix} FAB \\ MAB \\ FBA \\ MBA \end{bmatrix} = \begin{bmatrix} 111 & 333 & -111 & 0 \\ 333 & 1000 & -333 & 0 \\ -111 & -333 & 111 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} UVA \\ URA \\ UVB \\ URB \end{bmatrix}$$

S51
times EI/1000

$$\begin{bmatrix} FBC \\ MBC \\ FCB \\ MCB \end{bmatrix} = \begin{bmatrix} 192 & 480 & -192 & 480 \\ 480 & 1600 & -480 & 800 \\ -192 & -480 & 192 & -480 \\ 480 & 800 & -480 & 1600 \end{bmatrix} \begin{bmatrix} UVB \\ URB \\ UVC \\ URC \end{bmatrix}$$

S52

$$\begin{bmatrix} 111 & 333 & -111 & 0 & 0 & 0 \\ 333 & 1000 & -333 & 0 & 0 & 0 \\ -111 & -333 & 303 & 480 & -192 & 480 \\ 0 & 0 & 480 & 1600 & -480 & 800 \\ 0 & 0 & -192 & -480 & 192 & -480 \\ 0 & 0 & 480 & 800 & -480 & 1600 \end{bmatrix} \begin{bmatrix} UVA \\ URA \\ UVB \\ URB \\ UVC \\ URC \end{bmatrix}$$

CC

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 303 & 480 & 0 & 0 \\ 0 & 0 & 480 & 1600 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} UVA \\ URA \\ UVB \\ URB \\ UVC \\ URC \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 24 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

CC $\underline{u}$ $\underline{f}$

Fig.2.
The same beam construction. Now not left of the hinge but right of the hinge joint B is assumed. The right beam end of beam 1 is a hinge, to this member belongs beam stiffness matrix S51 shown on the left.

Beam 1 with L=3 m and bending-stiffness EI.

A= 3*'EI'/L^3=3*EI/3^3=0,111EI
B= 3*'EI'/L^2=3*EI/3^2=0,333EI
D= 3*'EI'/L =3*EI/3 =1,000EI

Beam 2 with L=5 m and bending stiffness 2EI.
Now applies matrix S52.

A=12*'EI'/L^3=12*2EI/5^3=0,192EI
B= 6*'EI'/L^2= 6*2EI/5^2=0,480EI

D= 4*'EI'/L=4*2EI/5=1,600EI
E= 2*'EI'/L=2*2EI/5=0,800EI

Construction matrix CC is formed and $\underline{f}$ = CC $\underline{u}$ is transformed into CC $\underline{u}$ = $\underline{f}$.

Again two equations remain.

(EI/1000)(303UVB+ 480URB)=24
(EI/1000)(480UVB+1600URB)= 0 from which

UVB=150,8/EI en URB=-45,2/EI.
This UVB=150,8/EI is the same as UVB=151,4/EI of the preceding calculation, a little difference because of rounding.
Joint rotation URB=-45,2/EI is the on the preceding page separately calculated slope HBC=-45,4/EI. Next the beam end forces and beam end moments.
Zero multiplications are also now omitted. One finds values as on the preceding page.

FAB=(EI/1000)(-111(150,8/EI))=-16,7 kN

MAB=(EI/1000)(-333(150,8/EI))=-50,2 kNm

FBA=(EI/1000)(111(150,8/EI))= 16,7 kN

MBA=0 because of the hinge.

FBC=(EI/1000)(192(150,8/EI)+480(-45,2/EI))
=(EI/1000)(-7258/EI)=-7,3 kN

MBC=(EI/1000)(480(150,8/EI)+1600(-45,2/EI))
=(EI/1000)(-64/EI) ≈ 0 kNm

FCB=(EI/1000)(-192(150,8/EI)-480(-45,2/EI))
=(EI/1000)(7258/EI) = -7,3 kN

MCB=(EI/1000)(480(150,8/EI)+800(-45,2/EI))
=(EI/1000)(36224/EI)=36,2 kNm

Slope HBA at beam end B of beam 1 is seperatedly calculated.

HBA=1,5*(UVB-UVA)/L-URA/2
=1,5*(150,8/EI-0)/3-0/2=75,4/EI

This slope was before joint rotation URB=75,8/EI.

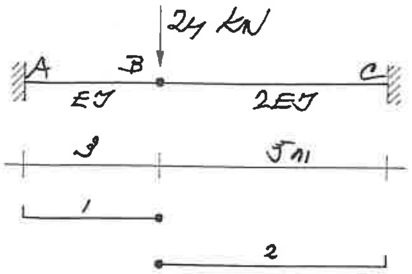


Fig.3.

$$\begin{bmatrix} A & B & -A & 0 \\ B & D & -B & 0 \\ -A & -B & A & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} A & 0 & -A & B \\ 0 & 0 & 0 & 0 \\ -A & 0 & A & -B \\ B & 0 & -B & D \end{bmatrix}$$

S51 page S52 page

$$\begin{bmatrix} FAB \\ MAB \\ FBA \\ MBA \end{bmatrix} = \begin{bmatrix} 111 & 333 & -111 & 0 \\ 333 & 1000 & -333 & 0 \\ -111 & -333 & \underline{111} & \underline{0} \\ 0 & 0 & \underline{0} & \underline{0} \end{bmatrix} \begin{bmatrix} UVA \\ URA \\ UVB \\ URB \end{bmatrix}$$

S51
times EI/1000

$$\begin{bmatrix} FBC \\ MBC \\ FCB \\ MCB \end{bmatrix} = \begin{bmatrix} \underline{48} & \underline{0} & -48 & 240 \\ \underline{0} & \underline{0} & 0 & 0 \\ -48 & 0 & 48 & -240 \\ 240 & 0 & -240 & 1200 \end{bmatrix} \begin{bmatrix} UVB \\ URB \\ UVC \\ URC \end{bmatrix}$$

S52

$$\begin{bmatrix} 111 & 333 & -111 & 0 & 0 & 0 \\ 333 & 1000 & -333 & 0 & 0 & 0 \\ -111 & -333 & \underline{159} & \underline{0} & -48 & 240 \\ 0 & 0 & \underline{0} & \underline{0} & 0 & 0 \\ 0 & 0 & -48 & 0 & 48 & -240 \\ 0 & 0 & 240 & 0 & -240 & 1200 \end{bmatrix} \begin{bmatrix} UVA \\ URA \\ UVB \\ URB \\ UVC \\ URC \end{bmatrix}$$

CC

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 159 & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} UVA \\ URA \\ UVB \\ URB \\ UVC \\ URC \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 24 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

CC u f

Fig.3.

Once again the same construction. Now the hinge itself is the 'joint', but it is not a real joint because no joint rotation URB is/can be calculated.
But the hinge can be loaded with a 'joint' load force FYB= 24 kN, but not with a 'joint' load moment MZB!
The beam matrices S51 and S52 were found before and copied and placed in construction matrix CC.
The four underlined elements of S51 and S52 are added and form the the four underlined elements of matrix CC.

Force vector f is filled with joint load forces and joint load moments, they are all zero except FYB= 24 kN which becomes the third element of force vector f.

From f = CC u to CC u = f on behalf of the solution of the unknowns with subroutine GAUSS , one more time as follows.

Is displacement UVA prescribed then the first row and first column of matrix CC are filled with zeros and is the element on the main diagonal made one with CC(1,1)=1, and gets the first element of f the value of displacement UVA. Is the prescribed displacement unequal zero then all elements of f will change,

And so in the same way for
URA, second row and second column,
UVC, fourth row and fifth column, and
URC, sixth row and sixth column.

By summing the underlined elements of S51 and S52 the diagonal element CC(4,4) has become zero. (With trusses there will never arise a zero on the main diagonal.
S51(3,3)+S52(1,1)=111+48=159=C(3,3)
S51(3,4)+S52(1,2)= 0 + 0= 0 =C(3,4)
S51(4,3)+S52(2,1)= 0 + 0= 0 =C(4,3)
S51(4,4)+S52(2,2)= 0 + 0= 0 =C(4,4)
To be able to solve the six equations with GAUSS this fourth element has to be made 1, C(4,4)=1. The fourth element of f is made zero so that 1*URB=0, but! URB does not exist!

Here below without GAUSS. There's only one equation left.
(EI/1000)(159UVB)=24 from which UVB=150,9/EI.

Slope HAB at beam end B of beam 1, and slope HBC at beam end B of beam 2 are separately calculated.

HBA=1,5*(UVB-UVA)/L-URA/2

=1,5*(150,9/EI-0)/3-0/2= 75,5/EI

HBC=1,5*(UVC-UVB)/L-URC/2

=1,5*(0-150,9/EI)/5-0/2=-45,3/EI

URB= 75,8/EI page and HBA= 75,4/EI page
URB=-45,2/EI page and HBC=-45,4/EI page

Example.

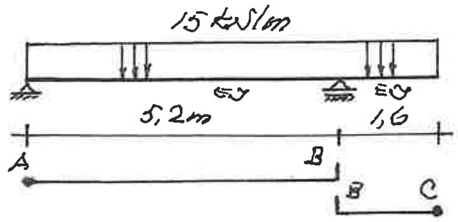


Fig.1.

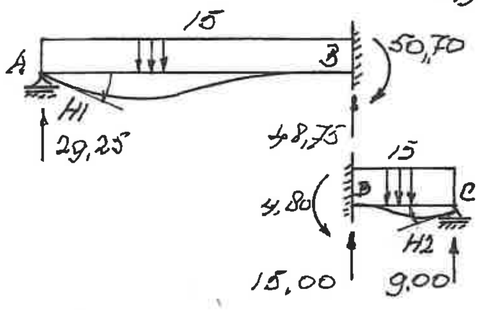


Fig.2.

Fig.1.
A statically determinate overhanging beam.
A and C are hinges and B a real joint.

A	0	-A	B
0	0	0	0
-A	0	A	-B
B	0	-B	D

S51 page 13

A	B	-A	0
B	D	-B	0
-A	-B	A	0
0	0	-0	0

S52 page 14

Beam 1 with L1=5,2 m and bending-stiffness EI.

A= 3*'EI'/L1^3=3*EI/(5,2^3)=0,021EI
B= 3*'EI'/L1^2=3*EI/(5,2^2)=0,111EI
D= 3*'EI'/L1   =3*EI/(5,2)   =0,577EI

Beam 2 with L1=1,6 m and bending stiffness EI.

A= 3*'EI'/L1^3=3*EI/(1,6^3)=0,732EI
B= 3*'EI'/L1^2=3*EI/(1,6^2)=1,172EI
D= 3*'EI'/L1   =3*EI/(1,6)   =0,577EI

Construction matrix CC is formed and $\underline{f}$ = CC $\underline{u}$ is transformed into CC $\underline{u}$ = $\underline{f}$.

FAB	=	21	0	-21	111	UVA
MAB		0	0	0	0	URA
FBA		-21	0	21	-111	UVB
MBA		111	0	-111	577	URB

S51
times EI/1000

FBC	=	732	1172	-732	0	UVB
MBC		1172	1875	-1172	0	URB
FCB		-732	-1172	732	0	UVC
MCB		0	0	0	0	URC

S52

21	0	-21	111	0	0	UVA
0	0	0	0	0	0	URA
-21	0	754	1061	-732	0	UVB
111	0	1061	2452	-1172	0	URB
0	0	-732	-1172	732	0	UVC
0	0	0	0	0	0	URC

CC

The elements of force vector $\underline{f}$.
Fig.2.
There are no joint load forces, but primary forces and moments due to the beam loads.
Beam 1. The left end is a hinge. The right end is a real joint which is fixed after which the distrubuted load of 15 kN is applied. The reactions can be found with BEAM2 of BEAMPROGRAM1. Or with the formulas of page 30.
R1=(3/8)*15*5,2= 29,25 kN
R2=(5/8)*15*5,2= 48,75 kN
M1=0 and M2=(1/8)*15*5,2^2= 50,70 kNm.
Beam 2. Now the left end is fixed and right end is a hinge.
R1=(5/8)*15*1,6= 15,00 kN
R2=(3/8)*15*1,6= 9,00 kN
M1=(1/8)*15*1,6^2= 4,80 kNm and M2=0.

The reactions of the two beams are drawn with their real directions, at the beam ends.
On the next page will be shown how the the force vector gets its values, already given below.

FAB	FYA	29,25
MAB	MZA	0
FBA+FBC	FYB	63,95
MBA+MBC	MZB	-45,90
FCB	FYC	9,00
MCB	MZC	0

1	0	0	0	0	0	UVA	0
0	0	.0	0	0	0	URA	0
0	0	1	0	0	0	UVB	0
0	0	0	2452	-1172	0	URB	-45,90
0	0	0	-1172	732	0	UVC	9,00
0	0	0	0	0	0	URC	0

CC $\underline{u}$ $\underline{f}$

Only displacement/rotation URB of joint B and the vertical displacement UVC of joint C are the unknowns to be calculated. The two equations to be solved then become

(EI/1000)(2452URB-1172UVC=-45,90 and
(EI/1000)(-1172URB+ 732UVC= 9,00.

Solution with program GAUSS

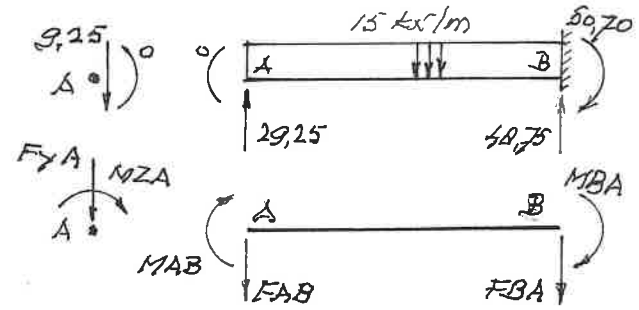
URB= -54,7/EI and UVC=-75,3/EI.

Determination of the elements of the force vector f.

The primary forces and moments acting on the beam ends are drawn with their real directions. On the joints act forces and moments equal in magnitude but ooposit directed.

The joint load forces are assumed to act downward, the joint load moments to the right. See page .

The beam end forces and moments to be calculated are drawn with their assumed directions.

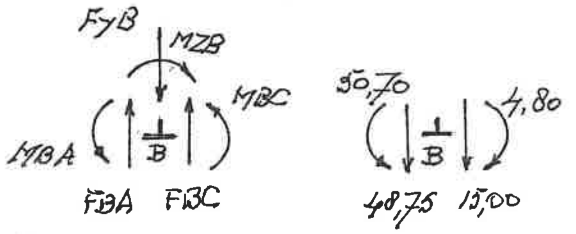


Joint A. FYA= 29,25 kN

Σ vert.=0 FYA-FAB=0 FAB=FYA

MZA=0

Σ mom.=0 MZA-MAB=0 MAB=MZA

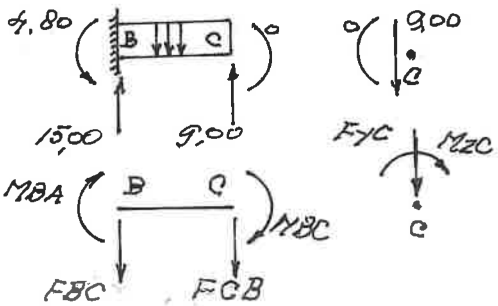


Joint B. FYB= 48,75+15,00= 63,75 kNm

Σ vert.=0 FYB-FBA-FBC=0 FBA+FBC=FYB

MZB=-50,70+4,80=-45,90 kNm

Σ mom.=0 MZB-MBA-MBC=0 MBA+MBC=MZB



Joint C. FYC= 9,00 kN

Σ vert.=0 FYC-FCB=0 FCB=FYC

MZC=0

Σ mom.=0 MZC-MCB=0 MCB=MZC

Now the beam end forces and beam end moments due to displacements alone can be calculated. That was done in the preding examples without beam loads.

Beam 1, UVA=0, URA=0, UVB=0 and URB=-54,7/EI. Zero multiplications are omitted.

FAB=(EI/1000)(111URB)
 =(EI/1000)(111(-54,7/EI))=-6,07 kN
MAB=0
FBA=(EI/1000)(-111URB)
 =(EI/1000)(-111(-54,7/EI))= 6,07 kN
FBA=(EI/1000)(577URB)
 =(EI/1000)(577(-54,7/EI))=-31,56 kNm

The final member end forces and member end moments are found by adding the primary forces and moments. Take into account the assumed directions, see fig.3.,

FAB becomes FAB-29,25=-6,07-29,25=-35,32 kN,
MAB becomes MAB+0= 0 kNm,
FBA becomes FBA-48,75= 6,07-48,75=-42,68 kN,
MBA becomes MBA+50,70=-31,56+50,70= 19,14 kNm.

Beam 2, UVB=0, URB=-54,7/EI, UVC=-75,3/EI and URC=0. Now using matrix S52.

FBC=(EI/1000)(1172URB-732UVC)
 =(EI/1000)(1172(-54,7/EI)-732(-75,3/EI))
 =(EI/1000)(-8988/EI)=-9,00 kN
MBC=(EI/1000)(1875URB-1172UVC)
 =(EI/1000)(1875(-54,7/EI)-1172(-75,3/EI))
 =(EI/1000)(-14311/EI)=-14,31 kNm
FCB=(EI/1000)(-1172URB+732UVC)
 =(EI/1000)(-1172(-54,7/EI)=&32(-75,3/EI))
 =(EI/1000)(8988/EI)= 9,00 kN
MCB=0

FBC becomes FBC-15,00=-9,00-15,00=-24,00 kN,
MBC becomes MBC-4,80=-14,31-4,80=-19,11 kNm,
FCB becomes FCB-9,00=9,00-9,00= 0 kN,
MBC becomes MBC+0= 0 kNm.

Fig.2.
Slope HAB is seperately calculated because joint a is a hinge, see page 13

HAB=1,5(UVB-UVA)/L1-URB/2
 =1,5(0/EI-0/EI)/1,6-(-54,7/EI)/2= 27,4/EI

Slope H1 can be calculated using the formula of page 30, or BEAMPROGRAM111 part 7.

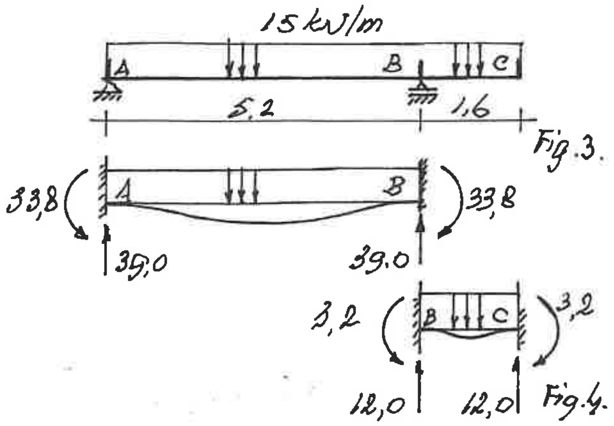
H1=(15*5,2^3)/(48EI)= 43,9/EI, and finally

HAB becomes 43,9/EI+27,4/EI= 71,3/EI.

Next slope HCB at member end C of beam 2, page fig.5a.
HCB=1,5(UVC-UVB)/L1-URB/2
 =1,5(-75,3/EI-0)/1,6-(-54,7/EI)/2
 =-70,38/EI+27,30/EI=-43,08/EI

H2=-(15*1,6^3)/48EI=-1,28/EI

HCB becomes -1,28/EI-43,08/EI=-44,4/EI



A	B	-A	B
B	D	-B	E
-A	-B	A	-B
B	E	-B	D

A=12*EI/L1^3
B= 6*EI/L1^2
D= 4*EI/L1
E= 2*EI/L1

S51 and S52 page

Beam 1. A=12*EI/(5,2^3)=0,085EI
B= 6*EI/(5,2^2)=0,222EI
D= 4*EI/5,2 =0,769EI
E= 2*EI/5,2 =0,385EI

Beam 2. A=12*EI/(1,6^3)=2,930EI
B= 6*EI/(1,6^2)=2,344EI
D= 4*EI/1,6 =2,500EI
E= 2*EI/1,6 =1,250EI

FAB	=	85	222	-85	222		UVA
MAB		222	769	-222	385		URA
FBA		-85	-222	85	-222		UVB
MBA		222	385	-222	769		URB

S51
times EI/1000

FBC	=	2930	2344	-2930	2344		UVB
MBC		2344	2500	-2344	1250		URB
FCB		-2930	-2344	2930	-2344		UVC
MCB		2344	1250	-2344	2500		URC

S52

85	222	-85	222	0	0
222	769	-222	385	0	0
-85	-222	3015	2122	-2930	2344
222	385	2122	3269	-2344	1250
0	0	-2930	-2344	2930	-2344
0	0	2344	1250	-2344	2500

CC

Fig.3.
The same structure but now all joints are real joints.

							u	f
1	0	0	0	0	0		UVA	0
0	769	0	385	0	0		URA	33,8
0	0	1	1	0	0		UVB	0
0	385	0	3269	-2344	1250		URB	-30,6
0	0	0	-2344	2930	-2344		UVC	12,0
0	0	0	1250	-2344	2500		URC	-3,2

CC u f

The elements of the force vector are found like before, after calculating the primary forces and moments. Since UVA and UVB are prescribed, four equations remain to solve the unknowns URA, URB, UVC and URC.

0,769*URA+0,385*URB +0*UVC +0*URC= 33,8
0,385*URA+3,269*URB-2,344*UVC+1,250*URC=-30,6
0*URA-2,344*URB+2,930*UVC-2,344*URC= 12,0
0*URA=1,250*URB-2,344*UVC+2,500*URC= -3,2

Solution with GAUSS. delivers
URA=71,4/EI, URB=-54,8/EI, UVC=-75,4/EI and URC=-44,6/EI (Little differences comparing with the values found before.)
The beam(member) end forces and moments for beam 1 and 2 follow below.

FAB=(EI/1000)(222URA+222URB)
=(EI/1000)(222(71,4/EI)+222(-54,8/EI))
= 3,69 kN
MAB=(EI/1000)(769URA+385URB)
=(EI/1000)(769(71,4/EI)+385(-54,8/EI))
= 33,81 kNm
FBA=-3,69 kN (like FAB, + is - now)
MBA=(EI/1000)(385URA+769URB)
=(EI/1000)(385(71,4/EI)+769(-54,8/EI))=
=-14,65 kNm

FAB becomes FAB-39,00= 3,69-39,00=-35,31 kN
MAB becomes MAB-33,80= 33,81-33,80= 0,01 is 0.
FBA becomes FBA-39,00=-3,69-39,00=-42,69 kN
MBA becomes MBA+33,80=-14,65+33,80= 19,15 kNm

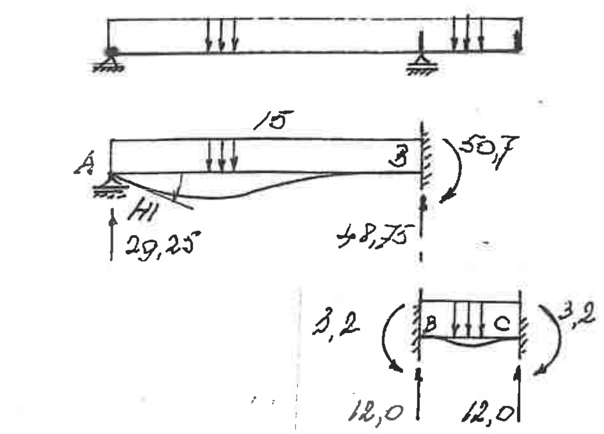
Found before, -35,32 kN, 0 kNm, -42,68 kN and 19,14 kNm.

FBC=(EI/1000)(2344URB-2930UVC+2344URC)
=(EI/1000)(2344(-54,8/EI)-2930(-75,4/EI)
+2344(-44,6/EI))=-12,07 kN
MBC=(EI/1000)(2500URB-2344UVC+1250UVR)
=(EI/1000)(2500(-54,8/EI)-2344(-75,4/EI)
+1250(-44,6/EI))=-16,01 kNm
FCB= 12,07 kN (like FCB, + is - and - is +)
MCB=(EI/1000)(1250URB-2344UVC+2500URC)
=(EI/1000)(1250(-54,8/EI)-2344(-75,4/EI)
+2500(-44,6/EI)=-3,26 kNm

FBC becomes FBC-12,00=-12,07-12,00=-24,07 kN
MBC becomes MBC-3,20=-16,01-3,20=-19,21 kNm
FCB becomes FCB-12,00= 12,07-12,00=0,07 is 0 kN.
MCB becomesMCB+3,20=-3,26+3,20=-0,06 is 0.

Found before -24,00 kN, -19,11 kNm, 0 kN and 0 kNm.

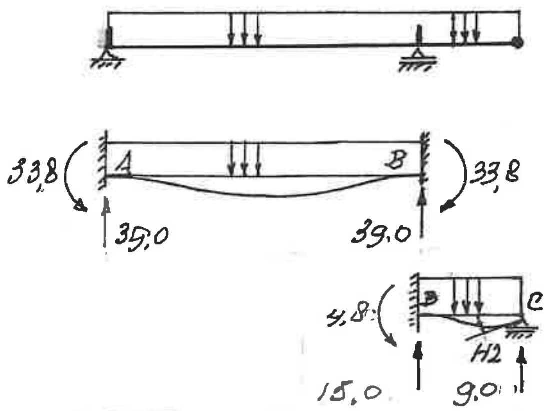
There are yet two other possibilities.



MZB=-50,7+3,2=-47,kNm
FYC= 12,0 kN
MZC=-3,2 kNm

21	0	-21	111	0	0
0	0	0	0	0	0
-21	0	2951	2233	-2930	2344
111	0	2233	3077	-2344	1250
0	0	-2930	-2344	2930	-2344
0	0	2344	1250	-2344	2500

CC



MZA= 33,8 kNm
MZB=-33,8+4,8=-30,6 kNm
FYC= 12,0 kN

85	222	-85	222	0	0
222	769	-222	385	0	0
-85	-222	807	950	0	0
222	385	950	2644	-1172	0
0	0	-732	-1172	732	0
0	0	0	0	0	0

CC

Fig.5.
For this case stiffness matrices S51 of fig.1 and S52 of fig.3 are combined. The unknowns to ber solved are now URB, UVC and URC. URA can be seperately calculated like done before.

S51:

21	0	-21	111
0	0	0	0
-21	0	21	-111
111	0	-111	577

S52:

2930	2344	-2930	2344
2344	2500	-2344	1250
-2930	-2344	2930	-2344
2344	1250	-2344	2500

S51 times EI/1000 S52

CC						u	f
1	0	0	0	0	0	UVA	0
0	0	0	0	0	0	URA	0
0	0	1	0	0	0	UVB	0
0	0	0	3077	-2344	1250	URB	-47,5
0	0	0	-2344	2930	-2344	UVC	12,0
0	0	0	1250	-2344	2500	URC	-3,2

CC u f

Then the equations are the following.

3,077*URB -2,344*UVC +1,250*URC= -47,5
-2,344*URB +2,930*UVC -2,344*URC= 12,0
1,250*URB -2,344*UVC +2,500*URC= -3,2

Solution with GAUSS1 page gives
URB=-54,7/EI, UVC=-75,4/EI and URC=-44,5/EI.

Fig.6.
Now joint A and B are real joints and joint C is a hinge. Matrices S51 of fig.3 and S52 of fig.1 are combined. The unknows to be solved are now URA, URB, and UVC are calculated. Slope dflection URC can be seperately calculated.

S51:

85	222	-85	222
222	769	-222	385
-85	-222	85	-222
222	385	-222	769

S52:

732	1172	-732	0
1172	1875	-1172	0
-732	-1172	732	0
0	0	0	0

S51 times EI/1000 S52

CC						u	f
1	0	0	0	0	0	UVA	0
0	769	0	385	0	0	URA	33,8
0	0	1	0	0	0	UVB	0
0	385	0	2644	-1172	0	URB	-30,6
0	0	0	-2344	2930	0	UVC	12,0
0	0	0	0	0	0	URC	0

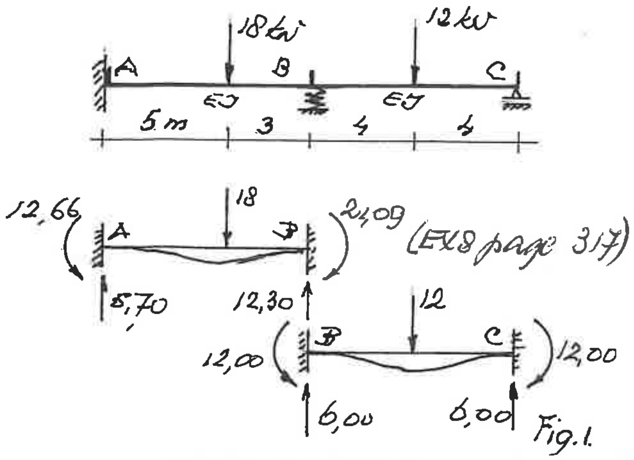
CC u f

Also this time three equations.

0,769*URA +0,385*URB +0*UVC= 33,8
0,385*URA +2,644*URB -1,172*UVC= -29,0
0*URA -1,172*URB +0,732*UVC= 9,0

And GAUSS1 delivers
URA=71,4/EI, URB=-54,8/EI and UVC=-75,5/EI.

And again the beam end forces and moments can be calculated using the concerning stiffness matrices S51 for beam 1 and S52 for beam 2.



Beam 1 and beam 2.

A=12*EI/(8^3)=0,023EI
B= 6*EI/(8^2)=0,094EI
D= 4*EI/8 =0,500EI
E= 2*EI/8 =0,250EI

FAB	=	23	94	-23	94		UVA
MAB		94	500	-94	250		URA
FBA		-23	-94	23	-94		UVB
MBA		94	250	-94	500		URB

S51
times EI/1000

FBC	=	23	94	-23	94		UVB
MBC		94	500	-94	250		URB
FCB		-23	-94	23	-94		UVC
MCB		94	250	-94	500		URC

S52

23	94	-23	94	0	0
94	500	-94	250	0	0
-23	-94	1146	0	-23	94
94	250	0	1000	-94	250
0	0	-23	-94	23	-94
0	0	94	250	-94	500

CC

1	0	0	0	0	0	UVA		0
0	1	0	0	0	0	URA		0
0	0	1146	0	0	94	UVB		18,30
0	0	0	1000	0	250	URB		-9,09
0	0	0	0	1	0	UVC		17,00
0	0	94	250	0	500	URC		-12,00

CC u f

Example.

Fig.1.
The structure has three real joints. Joint 2 is springy supported, spring constant 1,1EI, and therefore 1100 mus be added to CC(3,3), and becomes CC(3,3)=23+23+1100=1146.
Joint3 has a prescribed displacement UVC=17/EI.
The elements of force vector follow from the primary forces and moments like done before.

A	B	-A	B	A=12*EI/L1^3
B	D	-B	E	B= 6*EI/L1^2
-A	-B	A	-B	D= 4*EI/L1
B	E	-B	D	E= 2*EI/L1

S51 and S52 page 8
Because of the prescibed displacements UVA=0 and UVB=0, and the prescribed displacement UVC=17,00/EI, there are three equations left to solve. But first the elements of force vector f concerning these equations have to be changed because UVC is unequal zero.
These element have to be lessened by an element CC(I,5) times UVC, that is CC(I,5) of the orignal not altered construction matrix CC.

FYB= 18,30-CC(3,5)*UVC FF(3)
= 18,30-(EI/1000)(-23)(17,00/EI)
= 18,30+0,39= 18,69 kN

MZB=-9,09-CC(4,5)*UVC FF(4)
=-9,09-(EI/1000)(-94)(17,00/EI)
=-9,09+1,60=-7,49 kNm

MZC=-12,00-CC(6,5)*UVC FF(6)
=-12,00-(EI/1000)(-94(17,00/EI)
=-12,00+1,60=-10,40 kNm

Then the equations to be solved are

1,146*UVB + 0*URB + 0,094*URC= 18,69
0*UVB + 1,000*URB + 0,250*URC= -7,49
0,094*UVB + 0,250*URB + 0,500*URC=-10,40

Solution with GAUSS

UVB= 18,21/EI, URB=-1,64/EI, URC=-23,40/EI.
UVC is prescribed and is 17,00/EI.
Zero multiplications will be omitted.

FAB=(EI/1000)(-23UVB+94URB)
=(EI/1000)(-23(18,21/EI)+94(-1,64/EI))
=-0,57 kN

MAB=(EI/1000)(-94UVB+250URB)
=(EI/1000)(-94(18,21/EI)+250(-1,64/EI))
=-2,13 kNm

FBA= 0,57 kN (like FAB, - is + and + is -)

MBA=(EI/1000)(-94UVB+500URB)
=(EI/1000)(-94(18,21/EI)+500(-1,64/EI))
=-2,54 kNm

FAB becomes FAB-5,70 = -0,57-5,70= -6,27 kN
MAB becomes MAB+12,66=-2,13-12,66=-14,79 kNm
FBA becomes FBA-12,30= 0,57-12,30=-11,73 kN
MBA becomes MBA+21,09=-2,54+21,09= 16,55 kNm

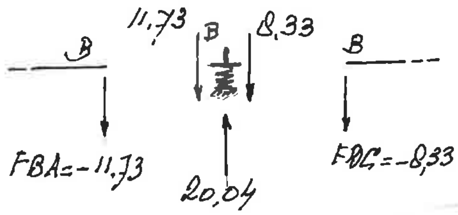


Fig.2.

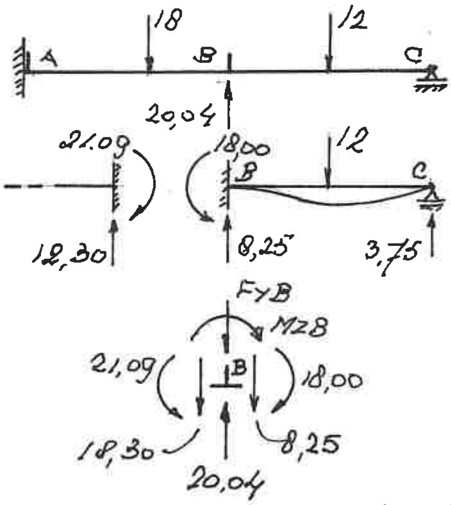


Fig.3.

FBC	=	6	47	-6	0		UVB
MBC		47	375	-47	0		URB
FCB		-6	-47	6	0		UVC
MCB		0	0	- 0	0		URC

S52
times EI/1000

23	94	-23	94	0	0
94	500	-94	250	0	0
-23	-94	29	-47	-6	0
94	250	-47	875	-47	0
0	0	-6	-47	6	0
0	0	0	0	0	0

CC

1	0	0	0	0	0	UVA	0
0	1	0	0	0	0	URA	0
0	0	1129	-47	0	0	UVB	0,61
0	0	-47	875	0	0	URB	-2,29
0	0	0	0	1	0	UVC	17,00
0	0	0	0	0	0	URC	0

CC u f

For beam 2 four displacements, UVB, URB, UVC and URC are used.
FBC=(EI/1000)(23UVB+94URB-23UVC=94URC)
=(EI/1000)(23(18,21/EI(+94(-1,64/EI)
-23(17,00/EI)+94(-23,40)/EI))=-2,33 kN

MBC=(EI/1000)(94UVB+500URB-94UVC+250URC)
(EI/1000)(94(18,21/EI)+500(-1,64/EI)
-94(17,00/EI)+250(-23,40/EI))
=-6,56 kNm

FCB= 2,33 kN (like FBC, + is - and - is +)

MCB=(EI/1000)(94UVB+250URB-94UVC+500URC)
=(EI/1000)(94(18,21/EI)+250(-1,64/EI)
-94(17,00/EI)+500(-23,40/EI)
=-12,00 kNm

FBC becomes FBC-6,00 = -2,33-6,00= -8,33 kN
MBC becomes MBC-12,00= -6,56-12,00=-18,56 kNm
FCB becomes FCB-6,00 = 2,33-6,00= -3,67 kN
MCB becomes MCB+12,00=-12,00+12,00= 0 kNm

Fig.2.
With UVB= 18,21/EI, downward, and spring constant of 1,1EI the spring force becomes 1,1EI*18,21/EI= 20,04 kN.
Now using CC with CC(3,3)= 1146, then is
FB1=FBA+FBC=(EI/1000)(1146UVB-23UVC+94URC)
= 18,61 kN
And with CC(3,3)= 46 follows
FB2=FBA+FBC=(EI/1000)(46UVB-23UVC+94URC)
=-1,75 kN
The difference is, ofcourse,
FB1-FB2= 18,61-(-1,75)= 20,03 kN

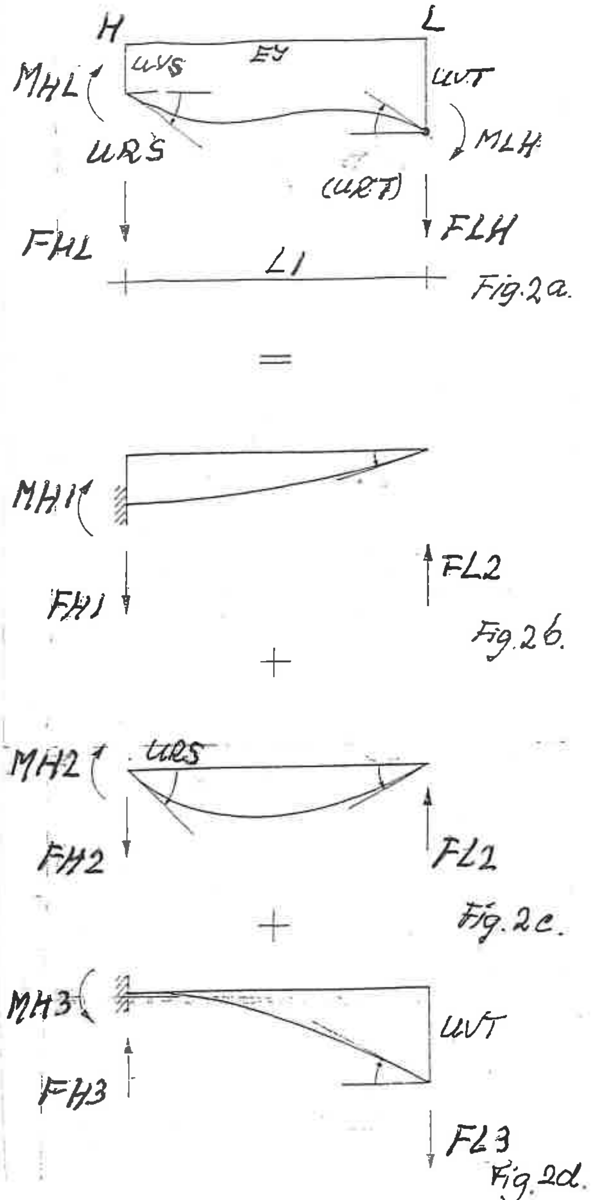
Fig.3.
The spring is omitted and the joint load force of joint B is -20,04 kN, that's upward, opposite to the assumed direction of FYB.
Joint C is supposed to be a hinge, so another stiffness matrix for beam 2, and, other primary forcws and moments.
FYB= 12,30+8,25-20,04= 0,51 kN
MZB=-21,09+18,00=-3,09 kNm

They have to be changed because of UVC=17,00/EI.
FYB= 0,51-CC(3,5)*UVC
= 0,51-(EI/1000)(-6)(17,00/EI)
= 0,51+0,10= 0,61 kN
MZB=-3,09-CC(4,5)*UVC
=-3,09-(EI/1000)(-47)(17,00/EI)
=-3,09+0,80=-2,29 kNm
Now two equations have to be solved.

0,029*UVB - 0,047*URB = 0,61
-0,047*UVB + 0,875*URB =-2,29

With GAUSS1 follow
UVB= 18,39/EI and URB=-1,63/EI.

The member end forces and moments of beam 1 are like calculated before. Of beam 2 will follow
FBC=-0,07 kN, MBC=-0,55kNm, FCB= 0,07 kN, MCB=0.
And finally, like found before,
FBC becomes FBC-8,25=-0,07-8,25=-8,32 kN
MBC becomes MBC-18,00=-0,55-18,00=-18,55 kNm
FCB becomes FCB-3,75=0,07-3,75=-3,68 kN
MCB becomes MCB+0=0+0=0 kNm



$$\begin{bmatrix} 3EI/L1^3 & 0 & -3EI/L1^3 & -3EI/L1^2 \\ 0 & 0 & 0 & 0 \\ 3EI/L1^3 & 0 & 3EI/L1^3 & 3EI/L1^3 \\ -3EI/L1^2 & 0 & 3EI/L1^3 & 3EI/L \end{bmatrix}$$

$$\underline{f} = \begin{bmatrix} FLH \\ MLH \\ FHL \\ MHL \end{bmatrix} \quad S5 = \begin{bmatrix} A & 0 & -A & -B \\ 0 & 0 & 0 & 0 \\ -A & 0 & A & B \\ -B & 0 & B & D \end{bmatrix} \quad \underline{u} = \begin{bmatrix} UVS \\ URS \\ UVT \\ URT \end{bmatrix}$$

X1(H)-X1(L)<0 If
member end numbers L and H are exchanged.

Fig.2a to 2d.
Considering the case like that on page 14 with a hinge at the right member end, then likewise the following equations can be written.

Fig.2b.
MH1=(3*EI/L1^2)*UVS and the reactions
FH1=(3*EI/L1^3)*UVS and FL1=(3*EI/L1^3)*UVS.
Fig.2c.
MH2=(3*EI/L1)*URS and the reactions
FH2=(3*EI/L1^2)*URS and FL2=(3*EI/L1^2)*URS.
Fig.2d.
MH3=(3*EI/L1^2)*UVT and the reactions
FH3=(3*EI/L1^3)*UVT and FL3=(3*EI/L1^3)*UVT.

And the fourth case, like fig.1c but now with URT=0, the beam is not bent, which gives FH4=0, MH4=0, FL4=0 and ML4=0.
The four matrix equations become, with follwing order, FLH, MLH, FHL and MHL.

```
FLH= -FL1-FL2+FL3+FL4
   = -(3*EI/L1^3)*UVS - (3*EI/L1^2)*URS
     +(3*EI/L1^3)*UVT +      0     *URT

MLH=  0*UVS + 0*URS + 0*UVT + 0*URT

FHL=  FH1+FH2-FH3+FH4
   =  (3*EI/L1^3)*UVS + (3*EI/L1^2)*URS
     -(3*EI/L1^3)*UVT +      0     *URT

MHL=  MH1+MH2-MH3+MH4
      (3*EI/L1^2)*UVS +   (3*EI/L1)*URS
     -(3*EI/L1^2)*UVT +      0     *URT
```

See the results on the left in matrix form and at the third matrix at the bottom.

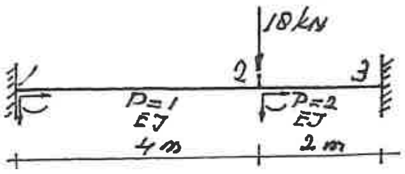
The second row of matrices here below arise if the sub matrices on the diagonals are exchanged.
In the first case B becomes -B.
Case a) becomes d) with B=-B, both NL(P)=1, and case b) becomes c) with B=-B, both NH(P)=1.

X1(H)-X1(L)>0

$$\left[\begin{array}{cc|cc} A & B & -A & B \\ B & D & -B & E \\ \hline A & -B & A & -B \\ B & E & -B & D \end{array}\right] \quad a)\ \left[\begin{array}{cc|cc} A & 0 & -A & B \\ 0 & 0 & 0 & 0 \\ \hline -A & 0 & A & -B \\ B & 0 & -B & D \end{array}\right] \quad b)\ \left[\begin{array}{cc|cc} A & B & -A & 0 \\ B & D & -B & 0 \\ \hline -A & -B & A & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

X1(H)-X1(L)<0

$$\left[\begin{array}{cc|cc} A & -B & -A & -B \\ -B & D & B & E \\ \hline -A & B & A & B \\ -B & E & B & D \end{array}\right] \quad c)\ \left[\begin{array}{cc|cc} A & -B & -A & 0 \\ -B & D & B & 0 \\ \hline -A & B & A & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad d)\ \left[\begin{array}{cc|cc} A & 0 & -A & -B \\ 0 & 0 & 0 & 0 \\ \hline -A & 0 & A & B \\ -B & 0 & B & D \end{array}\right]$$



N9=3	I	PV	PR	X1		EX1
	1	1	1	0		
	2	0	0	4		
	3	1	1	6		
P9=2	P	LL	HH	NL	NH	EI
	1	1	2	0	0	1
	2	2	3	0	0	1

S5-1	1	2	3	4
1	188	375	-188	375
2	375	1000	-375	500
3	-188	-375	188	-375
4	375	500	-375	1000

S5-2	1	2	3	4
1	1500	1500	-1500	1500
2	1500	2000	-1500	1000
3	-1500	-1500	1500	-1500
4	1500	1000	-1500	2000

	1	2	3	4	5	6
1	188	375	-188	375	0	0
2	375	1000	-375	500	0	0
3	-188	-375	1688	1125	-1500	1500
4	375	500	1125	3000	-1500	1000
5	0	0	-1500	-1500	1500	-1500
6	0	0	1500	1000	-1500	2000

	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	1688	1125	0	0
4	0	0	1125	3000	0	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1

N9= Calc.S5/CC Show S5 Show CC

P9= OK Again Cls

STORE NR ? GET Show End

Prf EX1 EX2 EX3 EX4 EX5 EX6 EX7

	1	2	3	4	5	6
1	1688	1125	0	0	0	0
2	1125	3000	0	0	0	0
3	0	0	1	0	0	0
4	0	0	0	1	0	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1

CBEAMMATRd for member stiffness matrices S5 and construction stiffness matrix CC.
Example of page 9.

Fig.1.

N9=3 supports.

I	PV	PR	X1
1	1	1	0
2	0	0	4
3	1	1	6

P9=2 members.

P	L	H	NL	NH	EI
1	1	2	0	0	1
2	2	3	0	0	1

Type 3 in TN9, Tab, cursor in TSTRING, type 1,1,1,0 Enter, 2,0,0,4 Enter, 3,1,1,6 Enter, cursor appears in TP(.
Type 2 in TP9, Tab, cursor in TSTRING, type 1,1,2,0,0,1 Enter and 2,2,3,0,0,1 Enter.

Click Show to see the data put in, next Calc.S5/CC to calculate the matrices S5 and CC. In first CC the two matrices S5 are combined. In second CC the modification of the first CC for calculation of the two equations with the unknown joint displacement UV2 and joint rotation UR2.

Here below the same construction with different joint and member numbering.

N9=3 supports.

I	PV	PR	X1
1	0	0	4
2	1	1	6
3	1	1	0

P9=2 members.

P	L	H	NL	NH	EI
1	1	2	0	0	1
2	1	3	0	0	1

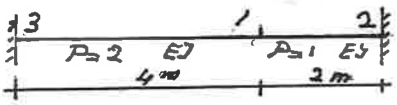


Fig.2.

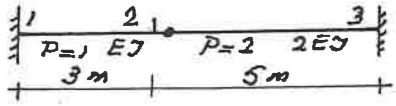
N9=3	I	PV	PR	X1		
	1	0	0	4		
	2	1	1	6		
	3	1	1	0		
P9=2	P	LL	HH	NL	NH	EI
	1	1	2	0	0	1
	2	1	3	0	0	1

S5-1	1	2	3	4
1	1500	1500	-1500	1500
2	1500	2000	-1500	1000
3	-1500	-1500	1500	-1500
4	1500	1000	-1500	2000

S5-2	1	2	3	4
1	188	-375	-188	-375
2	-375	1000	375	500
3	-188	375	188	375
4	-375	500	375	1000

	1	2	3	4	5	6
1	1688	1125	-1500	1500	-188	-375
2	1125	3000	-1500	1000	375	500
3	-1500	-1500	1500	-1500	0	0
4	1500	1000	-1500	2000	0	0
5	-188	375	0	0	188	375
6	-375	500	0	0	375	1000

EX2



S5-1	1	2	3	4
1	444	667	-444	667
2	667	1333	-667	667
3	-444	-667	444	-667
4	667	667	-667	1333

S5-2	1	2	3	4
1	48	0	-48	240
2	0	0	0	0
3	-48	0	48	-240
4	240	0	-240	1200

	1	2	3	4	5	6
1	444	667	-444	667	0	0
2	667	1333	-667	667	0	0
3	-444	-667	492	-667	-48	240
4	667	667	-667	1333	0	0
5	0	0	-48	0	48	-240
6	0	0	240	0	-240	1200

	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	492	-667	0	0
4	0	0	-667	1333	0	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1

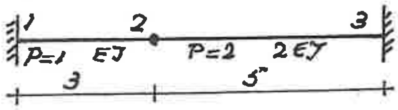
N9= Calc.S5/CC Show S5 Show CC

P9= OK Again Cls

STORE NR ? GET Show End

Prf EX1 EX2 EX3 EX4 EX5 EX6 EX7

EX4



	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	159	0	0	0
4	0	0	0	1	0	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1

Example EX2 page 16. A-B-C is 1-2-3.

Click EX2, Show, Calc.S5/CC, Show S5, Show CC to get the print shown on the left.
Calculated are joint displacement UV2 and joint rotation UR2.
Slope/angle H23 of beam end 2 of beam 1 is separately calculated.

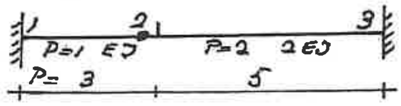
Example EX3 page 17.

Calculated are joint displacement UV2 and joint rotation UR2.
Slope H21 of beam 2 of beam 2 is separately calculated.

Example EX4 page 18.

Joint displacement UV2 is calculated and slope H21 of beam end 2 of beam 1 and slope H23 of beam end 2 of beam 2 are separately calculated.

EX3



EX3

N9=3	I	PV	PR	X1
	1	1	1	0
	2	0	0	3
	3	1	1	8

P9=2	P	LL	HH	NL	NH	EI
	1	1	2	0	1	1
	2	2	3	0	0	2

S5-1	1	2	3	4
1	111	333	-111	0
2	333	1000	-333	0
3	-111	-333	111	0
4	0	0	0	0

S5-2	1	2	3	4
1	192	480	-192	480
2	480	1600	-480	800
3	-192	-480	192	-480
4	480	800	-480	1600

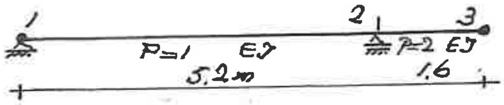
	1	2	3	4	5	6
1	111	333	-111	0	0	0
2	333	1000	-333	0	0	0
3	-111	-333	303	480	-192	480
4	0	0	480	1600	-480	800
5	0	0	-192	-480	192	-480
6	0	0	480	800	-480	1600

	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	303	480	0	0
4	0	0	480	1600	0	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1

Example EX5   page 19.

Joint 1 is a hinge, joint 3 is a hinge and joint 2 is a 'real' joint.

Click EX5, Show, Calc.S5/CC, Show S5 and Show CC.



N9=3	I	PV	PR	XI	EX5
	1	1	0	0	
	2	1	0	5,2	
	3	0	0	6,8	

P9=2	P	LL	HH	NL	NH	EI
	1	1	2	1	0	1
	2	2	3	0	1	1

S5-1	1	2	3	4
1	21	0	-21	111
2	0	0	0	0
3	-21	0	21	-111
4	111	0	-111	577

S5-2	1	2	3	4
1	732	1172	-732	0
2	1172	1875	-1172	0
3	-732	-1172	732	0
4	0	0	0	0

	1	2	3	4	5	6
1	21	0	-21	111	0	0
2	0	0	0	0	0	0
3	-21	0	754	1061	-732	0
4	111	0	1061	2452	-1172	0
5	0	0	-732	-1172	732	0
6	0	0	0	0	0	0

	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	1	0	0	0
4	0	0	0	2452	-1172	0
5	0	0	0	-1172	732	0
6	0	0	0	0	0	1

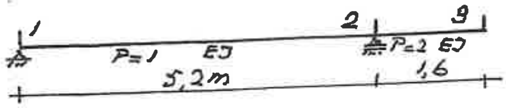
3	-21	0	2951	2233	-2930	2344
4	111	0	2233	3077	-2344	1250
5	0	0	-2930	-2344	2930	-2344
6	0	0	2344	1250	-2344	2500

	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	1	0	0	0
4	0	0	0	3077	-2344	1250
5	0	0	0	-2344	2930	-2344
6	0	0	0	1250	-2344	2500

Example EX6   page 21.

Joint 1, 2 and 3 are 'real' joints.
In both S5's no rows and columns with zeros.

Click EX5, Show, Calc.S5/CC, Show S5 and Show CC.



S5-1	1	2	3	4
1	85	222	-85	222
2	222	769	-222	385
3	-85	-222	85	-222
4	222	385	-222	769

S5-2	1	2	3	4
1	2930	2344	-2930	2344
2	2344	2500	-2344	1250
3	-2930	-2344	2930	-2344
4	2344	1250	-2344	2500

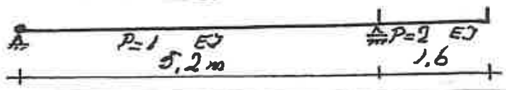
	1	2	3	4	5	6
1	85	222	-85	222	0	0
2	222	769	-222	385	0	0
3	-85	-222	3015	2122	-2930	2344
4	222	385	2122	3269	-2344	1250
5	0	0	-2930	-2344	2930	-2344
6	0	0	2344	1250	-2344	2500

	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	769	0	385	0	0
3	0	0	1	0	0	0
4	0	385	0	3269	-2344	1250
5	0	0	0	-2344	2930	-2344
6	0	0	0	1250	-2344	2500

Example EX7   page 22.

Joint 1 is a hinge, joint 3 is a hinge and joint 2 is a 'real' joint.

Click EX5, Show, Calc.S5/CC, Show S5 and Show CC.



N9=3	I	PV	PR	XI	EX7
	1	1	0	0	
	2	1	0	5,2	
	3	0	0	6,8	

P9=2	P	LL	HH	NL	NH	EI
	1	1	2	1	0	1
	2	2	3	0	0	1

S5-1	1	2	3	4
1	21	0	-21	111
2	0	0	0	0
3	-21	0	21	-111
4	111	0	-111	577

X1(H)-X1(L)>0

```
N9=2   I  PV  PR  X1
       1  1   0   0
       2  1   0   6
P9=1   P  LL  HH  NL  NH  EI
       1  1   2   0   0   1

     S5-1    1     2     3     4
        1   56   167   -56   167
        2  167   667  -167   333
        3  -56  -167    56  -167
        4  167   333  -167   667

        1     2     3     4
1      56   167   -56   167
2     167   667  -167   333
3     -56  -167    56  -167
4     167   333  -167   667

        1     2     3     4
1       1     0     0     0
2       0   667     0   333
3       0     0     1     0
4       0   333     0   667
```

$$\left[\begin{array}{cc|cc} A & B & -A & B \\ B & D & -B & E \\ \hline -A & -B & A & -B \\ B & E & -B & D \end{array}\right]$$

A= 12EI/L^3   B= 6EI/L^2

D= 4EI/L   E= 2EI/L

```
N9=2   I  PV  PR  X1
       1  1   0   0
       2  1   0   6
P9=1   P  LL  HH  NL  NH  EI
       1  1   2   1   0

     S5-1    1     2     3     4
        1   83     0   -83   500
        2    0     0     0     0
        3  -83     0    83  -500
        4  500     0  -500  3000

        1     2     3     4
1      83     0   -83   500
2       0     0     0     0
3     -83     0    83  -500
4     500     0  -500  3000

        1     2     3     4
1       1     0     0     0
2       0     1     0     0
3       0     0     1     0
4       0     0     0  3000
```

$$\left[\begin{array}{cc|cc} A & 0 & -A & B \\ 0 & 0 & 0 & 0 \\ \hline -A & 0 & A & -B \\ B & 0 & -B & D \end{array}\right]$$

A= 3EI/L3   B= 3EI/L^2

D= 3EI/L

```
N9=2   I  PV  PR  X1
       1  1   0   0
       2  1   0   6
P9=1   P  LL  HH  NL  NH  EI
       1  1   2   0   1

     S5-1    1     2     3     4
        1   83   500   -83     0
        2  500  3000  -500     0
        3  -83  -500    83     0
        4    0     0     0     0

        1     2     3     4
1      83   500   -83     0
2     500  3000  -500     0
3     -83  -500    83     0
4       0     0     0     0

        1     2     3     4
1       1     0     0     0
2       0  3000     0     0
3       0     0     1     0
4       0     0     0     1
```

$$\left[\begin{array}{cc|cc} A & B & -A & 0 \\ B & D & -B & 0 \\ \hline -A & -B & A & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

A= 3EI/L3   B= 3EI/L^2

D= 3EI/L

X1(H)-X1(L)<0

```
N9=2   I  PV  PR  X1
       1  1   0   6
       2  1   0   0
P9=1   P  LL  HH  NL  NH  EI
       1  1   2   0   0   1

     S5-1    1     2     3     4
        1   56  -167   -56  -167
        2 -167   667   167   333
        3  -56   167    56   167
        4 -167   333   167   667

        1     2     3     4
1      56  -167   -56  -167
2    -167   667   167   333
3     -56   167    56   167
4    -167   333   167   667

        1     2     3     4
1       1     0     0     0
2       0   667     0   333
3       0     0     1     0
4       0   333     0   667
```

$$\left[\begin{array}{cc|cc} A & -B & -A & -B \\ -B & D & B & E \\ \hline -A & B & A & B \\ -B & E & B & D \end{array}\right]$$

A= 12EI/L^3   B= 6EI/L^2

D= 4EI/L   E= 2EI/L

```
N9=2   I  PV  PR  X1
       1  1   0   6
       2  1   0   0
P9=1   P  LL  HH  NL  NH  EI
       1  1   2   0   1   1

     S5-1    1     2     3     4
        1   14   -83   -14     0
        2  -83   500    83     0
        3  -14    83    14     0
        4    0     0     0     0

        1     2     3     4
1      14   -83   -14     0
2     -83   500    83     0
3     -14    83    14     0
4       0     0     0     0

        1     2     3     4
1       1     0     0     0
2       0   500     0     0
3       0     0     1     0
4       0     0     0     1
```

$$\left[\begin{array}{cc|cc} A & -B & -A & 0 \\ -B & D & B & 0 \\ \hline -A & B & A & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

A= 3EI/L3   B= 3EI/L^2

D= 3EI/L

```
N9=2   I  PV  PR  X1
       1  1   0   6
       2  1   0   0
P9=1   P  LL  HH  NL  NH  EI
       1  1   2   1   0   1

     S5-1    1     2     3     4
        1   14     0   -14   -83
        2    0     0     0     0
        3  -14     0    14    83
        4  -83     0    83   500

        1     2     3     4
1      14     0   -14   -83
2       0     0     0     0
3     -14     0    14    83
4     -83     0    83   500

        1     2     3     4
1       1     0     0     0
2       0     1     0     0
3       0     0     1     0
4       0     0     0   500
```

$$\left[\begin{array}{cc|cc} A & 0 & -A & -B \\ 0 & 0 & 0 & 0 \\ \hline -A & 0 & A & B \\ -B & 0 & B & D \end{array}\right]$$

A= 3EI/L3   B= 3EI/L^2

D= 3EI/L

Standard formulas for simple beams.

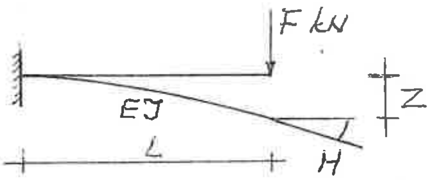
E is modulus of elasticity in kN/m^2
EI is bending stiffness, EI is E*I with
I is moment of inertia in m^4

EI is (kN/m^2)*m^4 is kNm^2

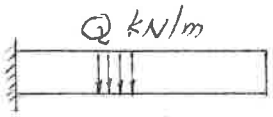
EA is strain stiffness, EA is E*A with
A is cross sectial area in m^2

EA is (kN/m^2)*m^2 is kN

Displcement Z in m, angle H in radians

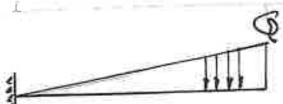


H= F*L^2/(2*EI)   Z= F*L^3/(3*EI)



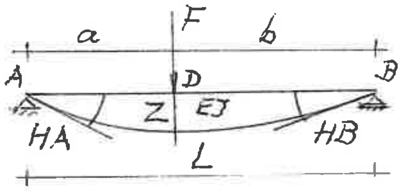
H= Q*L^3/(6*EI)   Z= Q*L^4/(8*EI)

H= Q*L^3/(24*EI)   Z= Q*L^4/(30*EI)



H= Q*L^3/(8EI)   Z= 11Q*L^3/(120EI)

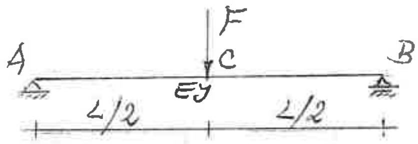
H= M*L/EI   Z= M*L^2/(2*EI)



HA= F*a*b*(L+b)/(6*L*EI)

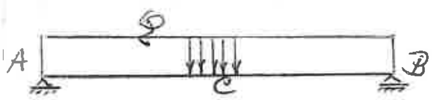
HB= F*a*b*(L+a)/(6*L*EI)

ZD= F*a^2*b^2/(3*L*EI)



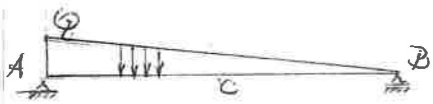
HA= HB= F*L^2/(16*EI)

ZC= F*L^3/(48*EI)



HA= HB= Q*L^3/(24*EI)

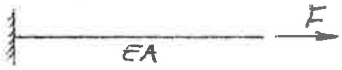
ZC= 5*Q*L^4/(384*EI)



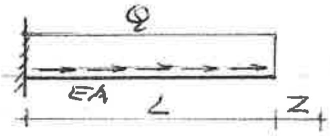
HA= Q*L^3/(45*EI)

HB= 7*Q*L^3/(360*EI)

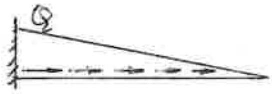
ZC= (5*Q*L^4/(384*EI))/2



Z=F*L/EA

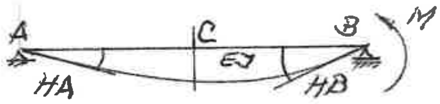


Z=Q*L^2/(2*EA)



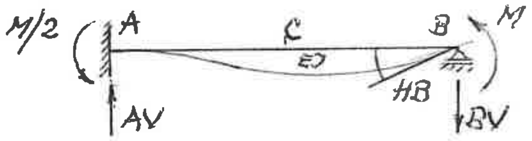
Z=Q*L^2/(6*EA)

Z=Q*L^2/(3EI)



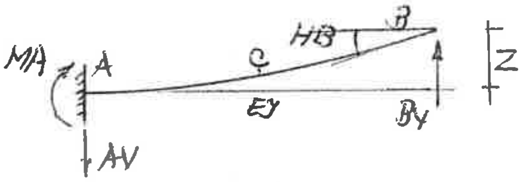
HA= M*L/(6*EI)   HB= M*L/(3*EI)

ZC= M*L^2/(16*EI)



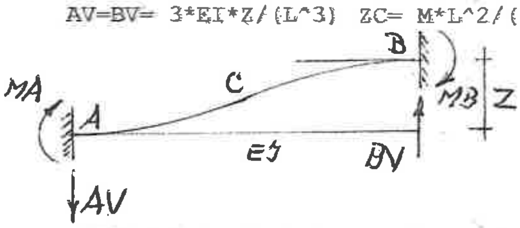
HB= M*L/(4*EI)   ZC= M*L^2/(32*EI)

AV=BV= 3*M/(2*EI)



MA=3*EI*Z/(L^2)   HB= 3*Z/(2*L)

AV=BV= 3*EI*Z/(L^3) ZC= M*L^2/(32*EI)



MA=MB= 6*EI*Z/(L^2)   ZC= Z/2

AV=BV= 12*EI*Z/(L^3)